5- Optimal Group Incentives with Social Preferences and Self-Selection*

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Abstract

This paper describes optimal group incentives when a certain fraction of agents are inequity-averse, as defined by Fehr and Schmidt (1999). We show that highly-competitive compensation schemes provide strong incentives only to agents who do not care about their relative payoff. In contrast, inequity-averse agents exert greater effort under more egalitarian payment schemes. If the market is perfectly flexible, there exists a unique separating equilibrium in agents' inequity aversion. Two contracts are offered by *ex ante* identical firms in equilibrium, each of which is optimal for one agent type. Each agent will work for the firm that offers his optimal contract, whether information about agent types is complete or incomplete. When an agent's type is private information, self-selection leads to homogeneous-group formation, and allows the *ex post* identification of personality types. Efficiency gains result from agents' self-selection of their payment scheme.

JEL Classification: D63, D82, J31, J33, M52.

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1 Introduction

A variety of different performance-based payment schemes are commonly used in companies, as underlined by O'Dell and McAdams (1987), Prendergast (1999) and Pfeffer (2007). O'Dell and McAdams (1987) surveyed reward schemes in 1,598 U.S. organizations and found that around 75% of companies used variable pay in order to motivate their employees. Overall, 59% of firms apply group incentives such as profit-sharing, gain-sharing and small-group incentives, 30% used lump-sum bonuses, and 28% individual incentives.[1] One explanation for this diversity of reward schemes is heterogeneity between companies with respect to, for example, the control costs of performance, the presence of multi-tasking, or complementarity between employees' effort levels. A second argument appeals to worker heterogeneity. Recent empirical work on incentives has underlined the importance of agents' self-selection between different payment schemes (see Chiappori and Salanié, 2003, for a survey of empirical tests of contract theory). This self-selection is shown to depend partly on agents' ability levels.[2] However, agent heterogeneity not only concerns ability levels but also individual preferences, such as social preferences (Bandiera, Barankay and Rasul, 2005, Carpenter and Seki, forthcoming, and Falk and Fehr, 2005).

These social preferences will affect agents' firm choice. For example, doctors, nurses, lawyers, solicitors, real-estate agents and accountants often decide to start their own business or to work in teams. Salesmen and, more generally, all employees whose salary includes a variable component decide to work for companies with individual or collective performance-based payment schemes. This paper will provide a theoretical rationale of this self-selection of agents into different group incentive payment schemes according to their social preferences. We will define optimal incentive contracts for agents in groups depending on their degree of inequity aversion, and consider whether a separating equilibrium exists when companies are *ex ante* identical. We then ask whether such agent segregation yields greater efficiency in organizations and on the market.

Recent work has suggested that people do indeed have social preferences, and that

[1] The sum of these percentages is greater than 100%, as firms may use several different reward systems.

[2] For example, Lazear (2000) has shown that the switch from a fixed wage to piece-rate pay in a large American auto glass company increased average output per worker by 44 percent. This rise is due to both incentive and sorting effects in terms of ability, with higher-ability workers attracted by the piece-rate pay and working at a higher effort levels, while low-ability workers prefer the fixed wage. Both effects are of the same magnitude. Hamilton, Nickerson and Owan (2003) show that heterogeneous ability can explain why a shift from individual to collective piece rates might improve worker productivity, provided that agents can self-select.

these differ across individuals. In standard theory, individuals are assumed to be selfish in the sense that they pursue their own material payoff. However, according to Fehr and Schmidt (1999), "*this may be true for some (maybe many) people, but it is certainly not true for everybody*" (p. 817).[3] They construct a utility function for the inequity-averse that is decreasing in the gap between agents' payoffs. This suggests that different individuals will have different preferences for different organizations. There could therefore be Pareto gains from offering agents a choice of compensation schemes or organizations.

Lazear (1989) suggests that "*it may be important to sort workers in different groups depending on their personality*" (p. 562). However, we may not need to actually observe personality types in order to allocate agents efficiently. A flexible market that allows people to select their payment scheme may yield self-selection, and hence the *ex post* identification of personality types. Our model shows how such self-selection of agents occurs when they are freely allowed to choose their firm according to the payment scheme it proposes.

In our model, we suppose *ex ante* identical firms that compete via the payment scheme they offer agents. We assume that people are either selfish or inequity-averse. Agents are matched together in firms in groups of two. To make the model more general, we assume an individual moral hazard problem: the joint production of each group is supposed to be observable by the firm, but individual outputs are only imperfectly measurable.[4] Each payment scheme corresponds to a specific distribution of the output between the agents in a group. The variable payment attributed to agents is a positive share of group joint production. The allocation between agents within groups depends on their relative performance. When this distribution is unequal, the payment scheme is competitive. A special case is the provision of equal prizes to the winner and the loser. This outcome, which we call revenue-sharing, has some of the same properties as a public-goods game.

Our first finding is that selfish people are motivated by highly-competitive environments, such as the winner-take-all scheme. In this environment, however, inequity-averse agents suffer substantial disutility due to the considerable *ex post* inequality between the final net payoffs; these agents prefer to work under a contest structure with a more equal

[3]Recent experimental work has demonstrated the existence of agent types with respect to social preferences (Fehr and Fischbacher, 2002, Burlando and Guala, 2005, Gächter and Thöni, 2005, and Fischbacher and Gächter, 2010) and has underlined the necessity of taking this heterogeneity into account to infer the appropriate incentives.

[4]For a theoretical analysis of team incentives with moral hazard, see Holmström (1982). Nalbantian and Schotter (1997) experimentally investigate the effect of various group incentive payment schemes on worker performance.

distribution of group production. For sufficiently high degrees of inequity aversion, the optimal contract of an inequity-averse agent is revenue-sharing, and inequity-averse agents may, in this case, exert greater effort than at the free-riding equilibrium. Under complete information about agent types, two kinds of firms co-exist on the labor market, each of which offers the optimal contract of one type of agent. There exists a unique and separating equilibrium in which all agents choose to work under their optimal contract.

When the agent type is private information, the two equilibrium contest structures continue to correspond to the optimal contract of each agent type. As different agents have different motivations, incentives in organizations are not efficient when the self-selection of agents does not pertain. Via a forward induction process, we show that inequity-averse agents working under revenue-sharing may behave co-operatively at the equilibrium. The separating equilibrium holds if the resulting level of co-operation is not too high, so that selfish agents are not attracted to the revenue-sharing scheme. Compared to the situation in which agents cannot self-select, the average outcome and global efficiency of the market rise with the formation of homogeneous groups. The cost of the asymmetry of information regarding types is borne by the inequity-averse agents, who have to reduce their co-operative behavior for the self-selection condition to hold. Mono-organizational outcomes, in which all organizations use the same payment scheme, are unstable. With incomplete information, our model then produces a unique and separating equilibrium. The results emphasize that the payment schemes offered by *ex ante* identical firms will differ in equilibrium: some firms pay competitively, while others favor co-operation.

The remainder of the paper is organized as follows. Section 2 presents the related literature. Section 3 describes the model: the timing of the game, agents' expected utility, and the different agent types. The subgame perfect equilibrium and the optimal contest structure for each agent type under complete information are characterized in Section 4. Section 5 presents the results when the agent type is private information, and underlines the differences from the complete-information solution. Finally, Section 6 concludes.

2 Related Literature

Our paper adds to the existing literature on both the incentive and sorting effects of payment schemes when agents have social preferences. The first contribution of our model is that the disutility from inequality differs across agents. The second is the endogeneity of group formation, via the self-selection of agents into different payment schemes.

Recent theoretical work has assumed that agents are inequity-averse, but homogeneous in their degree of inequity aversion. This work has shown that the degree of inequity aversion influences agents' effort decisions. Considering a tournament structure of compensation with exogenous prizes, Grund and Sliwka (2005) and Demougin and Fluet (2003) show that an agent with positive disadvantageous inequity aversion, as in the case of envy, exerts greater effort than a selfish agent. The predictions of our model are different. In the symmetric equilibrium, inequity-averse agents exert less effort than selfish agents in a competitive environment.[5] This result reflects that the prizes awarded to workers are endogenous to effort in our model. By decreasing their effort, they also reduce the gap between the prizes, whereas prizes are fixed in the other two papers cited above. For empirical evidence supporting our findings, Bandiera, Barankay and Rasul (2005) emphasize that "*workers internalize the negative externality their effort imposes on others under relative incentives*" (page 920). They show that the productivity of the employees of a large fruit farm in Great Britain is lower when their salary depends on relative incentives characterized by endogenous rewards compared to a piece-rate payment scheme. They then suggest that average performance is lower under a competitive scheme because some employees feel concerned about others' payoffs. The endogeneity of prizes in our model corresponds to the payment structure in this fruit farm, which is certainly also used in other firms.[6]

The model developed by Rey Biel (2008) determines the optimal reward structure that a principal should choose when agents are sensitive to inequity in a framework where individual effort is perfectly observable and verifiable. He suggests that this optimal contract may be a team-work contract implementing substantial inequality in payoffs between

[5] If we consider asymmetric equilibria, it is possible that inequity-averse agents exert greater effort than selfish agents, but at the same time the other agent in the group exerts less effort.

[6] Contest structures with endogenous prizes produce incentives that coincide with those in contests with fixed prizes in the sense that an increase in effort leads to a higher probability of winning the contest and thus to a greater reward along with a higher effort cost. However, although inequity aversion does not have the same impact on agents' effort in these two set-ups, the separation of agent types in equilibrium occurs in both cases. When prizes are fixed exogenously, greater disadvantageous inequity-aversion leads to more effort, as this raises the probability of winning the contest. Inequity aversion reduces effort when the size of the prizes depends on agents' effort, since increasing effort also increases the absolute inequality between the prizes. Nevertheless, the optimal contract of each agent type is the same under the two frameworks. Thus, selfish agents prefer contests providing the highest expected payoff, whereas inequity-averse agents prefer contests with less inequality. The separation of agents into different firms is thus not particular to our choice of endogenous prizes.

agents when they deviate from the co-operative situation. Under this contract, inequity-averse agents may work more than selfish agents. Although both his model and ours relate optimal contracts to inequity aversion, it is difficult to compare the ensuing results. The fundamental difficultly concerns strategic interactions. In Rey Biel's model, it is assumed that monetary payoffs are individual and do not depend on other group members' behavior. In our model, payment schemes are determined by strategic interactions, as they are based on the distribution of payoffs between agents in the group: the effect of one agent's behavior on the other's monetary payoff matters, while this is not the case in Rey Biel (2008).

Finally, although previous work has improved our understanding of the effect of inequity aversion on incentive efficiency, one restriction in these models is their use of homogeneous preferences across agents and exogenously-imposed payment schemes. Here we have two different types of agents according to their social preferences, and payment schemes that are endogenous to agents' choices. The endogeneity of group formation in our model follows the original work of Wooders (1978) and Bennett and Wooders (1979) applied to local public goods or clubs. In a general setting, these two articles show that equilibrium jurisdiction memberships are homogeneous for small group sizes, i.e. agents in each jurisdiction are identical in terms of their preferences. We apply this idea to a specific public good of small size that is teamwork with agents who choose to work for a firm who is characterized by a specific payment scheme.

Closely related to our work, two existing theoretical models have analyzed agent self-selection between firms when the former have social preferences. Cabrales and Calvó-Armengol (2008) consider that agents are heterogeneous in ability, and show that even relatively weak social preferences produce the segregation of workers into different firms according to ability. Agents wish to avoid payment inequality in this model. However, this work does not address agents' self-selection based on social preferences. Second, Kosfeld and von Siemens (2007) model the self-selection of agents who differ in their conditional reciprocity. Their topic is similar to ours, although the two papers were written independently at the same time. Kosfeld and von Siemens suggest that firms may differ in their intensity of teamwork and cooperation due to the self-selection of agents, who are either selfish or conditionally reciprocal, into firms according to their incentives. Each firm decides whether to base employees' payoffs only on their individual production, or also on their joint production. Individual effort levels are perfectly observable.

Although the main result of their model is of the same nature as ours, emphasizing the market coexistence of different payment schemes even when firms are *ex ante* identical, the two models are not constructed in the same way. The first difference concerns the type of social preferences considered. We consider the influence of inequity aversion when incentive contracts are implemented, as opposed to conditional reciprocity in Kosfeld and von Siemens (2007). Our model also differs from theirs in that we consider moral hazard, with individual outputs being only imperfectly measurable. Finally, the principal difference is that our organizational structures are endogenous to agents' social preferences, while they are exogenously determined in Kosfeld and von Siemens (2007). We analyse the optimal distribution of payoffs within a group of agents without assuming any *a priori* cooperative situation leading to higher utility for only one particular type of agent. Cooperation emerges here as it is the optimal contract for agents who are sufficiently inequity-averse. We consider a general structure of payments that may be more or less competitive; the optimal structure of payments will depend on agents' preferences. Our model thus allows us to explain why different payment schemes (in terms of the distribution of payoffs) coexist on the market when firms are identical, as well as how these different payment schemes come about.

3 The model

3.1 Heterogeneity: selfish and inequity-averse agents

We differentiate agents according to their social preferences, and in particular their degree of inequity aversion. We model inequity aversion as in Fehr and Schmidt (1999): the utility of inequity-averse agents is negatively affected by the gap between agents' payoffs.[7] We ask whether the joint presence of selfish and inequity-averse agents will affect the efficiency of

[7]Other models of social preferences have been developed. Bolton and Ockenfels (2000) also propose a fairness model based on distributive consequences, but develop an alternative utility function based on the comparison by each agent of his own monetary payoff to the average payoff of the reference group. A separate literature on social preferences concentrates on fairness intentions instead of final distributions. Dufwenberg and Kirchsteiger (2004) analyze these intentions in sequential games, following on from the work of Rabin (1993) for normal games. The contribution of Falk and Fischbacher (2006) lies in the consideration of both intentions and outcome distributions as the determinants of reciprocity. Models based on efficiency (Charness and Rabin, 2002) or maximin preferences produce other kinds of social preferences. Sobel (2005) proposes a survey of this literature.

the incentives provided by performance-based compensation schemes. We assume that the reference group of each agent is the other member of the group. Utility is affected neither by the payoffs of agents belonging to any other group nor by the payoff of the principal. Agents are assumed to be risk-neutral and have the same ability.

As in Fehr and Schmidt, the utility function for player i is as follows:

$$u_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\} \qquad i \neq j \tag{1}$$

where x_i and x_j represent the monetary payoffs of i and j. Inequity-averse individuals dislike both advantageous (represented by the parameter β_i) and disadvantageous (α_i) inequality. Moreover, earning less than the other agent in the group has a greater absolute effect on utility than does earning more: $\alpha_i \geq \beta_i$, with $0 \leq \beta_i < 1$. As $\beta_i < 1$, the utility of agent i always increases with his payoff: he will not be prepared to give up his payoff to reduce inequality, as this would reduce his utility.

We consider an infinite population and two types of agents denoted by $\theta \in \{a, s\}$. A proportion ρ of agents, $0 \leq \rho \leq 1$, are inequity averse with $\alpha \neq 0$ and $\alpha \geq \beta$, $0 \leq \beta < 1$; we call these agents of type $\theta = a$. The remaining $(1 - \rho)$ agents are of type $\theta = s$, and are supposed to be totally selfish, i.e. $\alpha = \beta = 0$. For simplicity, we suppose that all inequity-averse agents have the same degree of inequity aversion.

3.2 The game

There is a continuum of firms described by Lebesgue measure on $[0, 1)$. Each firm can hire groups of two agents. The population of agents is then decomposed into two subpopulations: *first-member* agents and *second-member* agents.[8] There is a continuum of agents in the first-member subpopulation, described by Lebesgue measure on $[1, 2)$, as well as in the second-member subpopulation, described by Lebesgue measure on $[2, 3)$. Introducing the heterogeneity of agents according to their inequity-aversion type, first-member agents described by Lebesgue measure on $[1, 1 + \rho)$ and second-member agents described by Lebesgue measure on $[2, 2 + \rho)$ are inequity averse while first-member agents described by Lebesgue measure on $[1 + \rho, 2)$ and second-member agents described by Lebesgue measure on $[2 + \rho, 3)$ are totally selfish.

[8] Both members in a group have the same role. We define first-members and second-members agents to obtain a clearer presentation of the results.

Without loss of generality and for sake of simplicity, we assume that each firm hires a finite number of agents: each firm engages one group of two agents.[9] As we suppose Lebesgue measure on the intervals of firms, first-mover agents and second-mover agents and as these measures are equal, a coalition with a firm, a first-member agent and a second-member agent is measure-consistent as defined by Kaneko and Wooders (1986). Kaneko and Wooders (1986) expose that measure-consistency requires that "*coalitions should have 'as many' (i.e. the same measure) first members as second members, as many second members as third members, etc.*" (p. 109). Therefore, we define a function ψ from the set of first-member agents to the set of firms and a function φ from the set of second-member agents to the set of firms as measure-preserving isomorphisms in the sense of Kaneko and Wooders (1986) respectively from the set of first-member agents to the set of firms and from the set of second-member agents to the set of firms. The set of coalitions is defined by $\{\{i, \psi(i), \varphi(i)\} : i \in [0, 1]\}$ with i representing the firm, $\psi(i)$, the first-member agent and $\varphi(i)$, the second-member agent.

Each firm can enter the market costlessly. These have identical production and cost functions. Competition between firms works through the compensation contract they offer to agents. The total production of groups is perfectly observable but the principal is unable to identify any individual's output with certainty. The use of performance-based payment schemes by the principal may help to counter this individual moral hazard by setting up appropriate incentives. The game consists of three stages. First, firms decide simultaneously to enter the market by offering a specific payment scheme to agents based on joint group production. In the second stage, assuming a perfectly flexible market, agents simultaneously choose to work for a particular firm under the contract proposed or to take an outside option. Third, agents who choose to work for a firm are matched to another agent who has made the same choice, and both simultaneously choose their effort level.

The joint output is given by the sum of agents' effort levels within the group. Agent i produces output e_i, $e_i \in [0, 1]$, which represents his effort level. Agents who supply positive effort pay a quadratic cost $c(e_i) = e_i^2$. We assume identical agents in terms of ability, so that the production technology and effort cost are the same for all agents.[10] The output produced by agent i is always greater than the cost he pays: $e_i \geq e_i^2$. The net payoff of

[9] Each firm could hire more than one group of two agents. Nevertheless, the definition of results is clearer in this case and results do not differ when there is more than one group hired by each firm.

[10] We make this assumption as we are concerned with agent self-selection depending on their social preferences rather than their ability.

agent i, x_i, is the difference between the gross payment he receives, p_i, which depends on the group effort and the cost of his own effort: $x_i = p_i - e_i^2$. We assume that agents are sensitive to inequality in final net payoffs instead of expected payoffs, which seems closer to the idea of Fehr and Schmidt.[11] Moreover, in real companies, where workers can observe the effort of their co-workers, and link their payoff to this effort, it seems more realistic to assume that they compare their final payoffs after the cost of effort has been deducted.

The contracts offered on the market differ in two dimensions. First, firms decide on the share k, $k \in [0,1]$, of the group output that is given to the agents. Second, and more importantly, firms describe how this group share will be distributed between the agents in the group. Due to the moral hazard problem associated with individual effort, the probability of one agent receiving a specific share of group output depends on the relative performance of both agents in the group. We therefore have a contest structure, where prizes are not lump-sum bonuses but endogenous to joint group output. This gives agents some control over the inequality between payoffs once the distribution of group output between agents has been chosen by the firm. The winner's prize is a share τ, $\tau \in \left[\frac{1}{2}, 1\right]$, of the total output given to agents, $k\,(e_i + e_j)$. This is written as $W(e_i, e_j) = \tau k(e_i + e_j)$. The loser receives $L(e_i, e_j) = (1-\tau)\,k(e_i + e_j)$. The competitiveness of the contest increases with τ, which increases the spread between the share of the group output awarded to the two contestants. Contracts are described by both the share of group output given to the agents, and the distribution of the group output between agents, $\{k, \tau\}$.

A contest with $\tau = 1$ is very competitive, giving the whole agents' share, $k(e_i + e_j)$, to the winner and nothing to the loser. The contest with $\tau = \frac{1}{2}$ corresponds to revenue-sharing, where joint production is divided equally between the group members: winners and losers earn the same prize. This is equivalent to a public-goods game in the sense that group output is equally shared between agents regardless of their personal investment. We will call the contest with $\tau = \frac{1}{2}$ revenue-sharing in the rest of the paper. Firms determine the competitiveness of the contract; agents then decide which option they prefer between

[11] For example, when they apply their model to explain experimental evidence in public-goods games, Fehr and Schmidt (1999) compare agents' net payoffs. In the context of a public-goods game, the payoffs considered are the revenue of the public good minus the individual contribution to the public good. The experimental results from public-goods games are consistent with comparisons between net payoffs (see also Akerlof and Yellen, 1990, for some evidence of the fair wage-effort hypothesis). Nevertheless, we did consider the possibility that agents compare their gross wages. This hypothesis is simpler to implement but leads to less interesting results. Here also, in equilibrium, we have agent segregation between firms with different payment schemes.

the contract(s) offered and an outside option, and their equilibrium effort level, e_i^*, for each contract offered.

The probability of winning the contest increases in the agent's effort, but the agent exerting the greatest effort will not win the prize with certainty. A model of this nature is proposed by Tullock (1980). The probability of winning the prize for each agent depends on the ratio of his own effort to the total effort in the group

$$\Pr\left(p_i = W\right) = \frac{e_i}{e_i + e_j} \tag{2}$$

The probability of winning the prize W is thus increasing in the agent's own effort.[12] The von Neuman-Morgenstern utility of agent i, when matched with agent j, is given by

$$EU_i(e_i, e_j) = \Pr\left(p_i = W\right).u_i(W) + \left(1 - \Pr\left(p_i = W\right)\right).u_i(L) \qquad i \neq j \tag{3}$$

When agent i wins the contest, his utility is:

$$u_i(W) = \begin{cases} \tau k\left(e_i + e_j\right) - e_i^2 & \text{if } \theta = s \\ \tau k\left(e_i + e_j\right) - e_i^2 - \alpha \max\left\{\left(e_i + e_j\right)\left[-k\left(2\tau - 1\right) + \left(e_i - e_j\right)\right], 0\right\} & \\ \qquad -\beta \max\left\{\left(e_i + e_j\right)\left[k\left(2\tau - 1\right) - \left(e_i - e_j\right)\right], 0\right\} & \text{if } \theta = a \end{cases}$$

When agent i is indifferent to the other group member's payoff, his utility when he wins increases with the winner's prize and falls with the cost of effort. The utility of inequity-averse agents falls with the difference between the two group members' net payoffs. The winner's utility is lower due to his advantageous inequity aversion (he feels bad about earning more than his co-worker). Nonetheless, his disadvantageous inequity aversion may reduce his utility when the effort he exerts is substantially greater than that of agent j. This means that agent i has spent too much effort in trying to win the contest, such that his net earnings are less than those of his co-worker, even though he won the contest.

[12] As we do not focus on the evaluation of effort, we assume that the probability that the agent wins the contest is based on a rough costless evaluation by the firm. This assumption is made for the sake of simplicity. Determining the probability of winning the contest in Tullock's model does not make any difference from a situation in which every agent's output is subject to an individual random term (this second method is commonly used in tournament models). In both cases, the probability of winning the contest increases in own effort. As effort levels are chosen from a continuum, the set of probabilities of winning the prize as a function of agents' efforts is also a continuum, although only one agent will actually win.

When agent i loses the contest, his utility is:

$$u_i(L) = \begin{cases} (1-\tau)\,k\,(e_i+e_j) - e_i^2 & \text{if } \theta = s \\ (1-\tau)\,k\,(e_i+e_j) - e_i^2 - \alpha \max\left\{(e_i+e_j)\left[k\,(2\tau-1)+(e_i-e_j)\right],0\right\} & \\ \qquad -\beta \max\left\{(e_i+e_j)\left[-k\,(2\tau-1)-(e_i-e_j)\right],0\right\} & \text{if } \theta = a \end{cases}$$

When they lose, the utility of inequity-averse agents falls with their disadvantageous inequity aversion. The *ex post* inequality between contestants' net payoffs, which rises with τ, is costly for inequity-averse agents whatever their rank in the group.

4 Equilibrium and optimal contest structure under complete information

Selfish and inequity-averse agents react differently to the inequality in net payoffs between group members. The former will decide on equilibrium effort without paying attention to the situation of the other group member at the end of the contest; the latter considers the consequences of his behavior on the *ex post* inequality between the agents' net payoffs. In this section we determine the equilibrium of the game described above, after having defined the optimal contest structure for each type of agent when the types are common knowledge. Each firm competes against the other firms to attract agents. This competition is carried out via the performance-based contracts they propose on the labor market. We first solve the third stage of the game by characterising the effort equilibrium in pure strategies, which depends on the contract offered. We then derive the optimal contract for each agent type. Finally, we determine the contract which firms should optimally offer to each agent type, and then the subgame perfect equilibrium of the game.

4.1 Agents' equilibrium behavior

A selfish agent maximizes his expected utility independently of the other group member's payoff. The first-order condition yields:

$$e_s^* = \frac{\tau k}{2} \qquad \forall\, \tau \in \left[\frac{1}{2}, 1\right] \tag{4}$$

A selfish agent thus has a dominant strategy, e_s^*. Equilibrium effort increases in the share of group output awarded to the contest winner whatever the type of the other agent in the group.

The expected utility function of an inequity-averse agent depends on the difference between the group members' net payoffs. The equilibrium effort of an inequity-averse agent, e_a^*, depends on the degree of inequity aversion and τ. As $\tau = \frac{1}{2}$ is a special case, we distinguish equilibrium effort for $\tau \in \left(\frac{1}{2}, 1\right]$ and $\tau = \frac{1}{2}$. When $\tau = \frac{1}{2}$, the contest structure reverts to a revenue-sharing game with public-goods game properties.

For $\tau \in \left(\frac{1}{2}, 1\right]$, the reaction function of inequity-averse agents is continuous and defined by five different linear functions. We first consider equilibrium effort when the two agents in the group are inequity averse. We distinguish two cases according to the degree of inequity aversion: $\alpha + 3\beta < 2$ and $\alpha + 3\beta \geq 2$. We can then determine the equilibrium effort of inequity-averse agents when they are matched with a selfish agent.

If both agents in the group are inequity averse, i.e. the group is homogeneous, there always exists a unique symmetric equilibrium, whatever the values of α, β and τ, defined as follows:

$$\left\{ (e_a^*, e_a^*) : e_a^* = \frac{k\left(\tau\left(1 - 2\beta\right) + \beta\right)}{2 + \alpha - \beta} \right\} \tag{5}$$

The effort of inequity-averse agents at this symmetric equilibrium falls with their degree of disadvantageous inequity aversion. By reducing their effort, these agents minimize the disadvantageous inequality they might face if they lose the competition. Moreover, the symmetric equilibrium effort falls with τ when $\beta > \frac{1}{2}$. An agent who is sufficiently averse to advantageous inequality prefers to reduce the inequality between net payoffs by exerting less effort as the share given to the winner of the contest, τ, rises. In this case, inequity-averse agents, when matched in homogeneous groups, always exert less effort in equilibrium than do selfish agents. This is the unique equilibrium effort level for inequity-averse agents with $\alpha + 3\beta < 2$.

For $\alpha + 3\beta \geq 2$, as well as the symmetric equilibrium, there exist multiple asymmetric equilibria for inequity-averse agents which depend on τ, as follows:

$$\left\{\begin{array}{ll}\left(e_a^*, e_a^*-(2\tau-1)\,k\right): \frac{k(\tau(5-6\beta)+3\beta-2)}{2+\alpha-\beta} \leq e_a^* \leq \frac{k(\tau(1+2\alpha)-\alpha)}{2+\alpha-\beta} & \text{if } \frac{1}{2}<\tau \leq \frac{\alpha+2\beta}{2\alpha+4\beta-1} \\ \left(e_a^*, e_a^*-(2\tau-1)\,k\right): (2\tau-1)\,k \leq e_a^* \leq \frac{k(\tau(1+2\alpha)-\alpha)}{2+\alpha-\beta} & \text{if } \frac{\alpha+2\beta}{2\alpha+4\beta-1}<\tau<\frac{2-\beta}{3-2\beta} \\ \left(e_a^*, 0\right): e_a^*=\frac{k(\tau(1-2\beta)+\beta)}{2(1-\beta)} & \text{if } \frac{2-\beta}{3-2\beta} \leq \tau \leq 1\end{array}\right\} \tag{6}$$

The asymmetric equilibria only exist if agents are sufficiently inequity-averse: $\alpha+3\beta \geq 2$. The difference between the effort of the agents in the group increases in τ and can yield equilibrium effort that is higher than that exerted by selfish agents. As the contract becomes more competitive, one agent will increase effort, while the other will reduce their effort. The existence of asymmetric equilibria is consistent with empirical work showing substantial variation in agents' behaviors in competitive environments (Bull, Schotter, and Weigelt, 1987, and Harbring and Irlenbusch, 2003). In our model, we explain this by the fact that one agent prefers to exert less effort in order to save on effort costs, while the other agent exerts high effort to increase the amount of the reward and his probability of winning the contest. As agents compare their net payoffs, inequality in this sense is reduced in these asymmetric equilibria.

Nonetheless, as competition increases, it becomes increasingly attractive for an agent to exert high effort, with the other agent exerting low effort. The equilibrium expected utility of agents with low effort becomes negative for high values of τ. It would thus seem difficult for agents to coordinate on an asymmetric equilibrium where one of them has only low, or even negative, expected utility.[13]

When an inequity-averse agent is matched with a selfish agent, i.e. the group is heterogeneous, there exists a unique equilibrium which is asymmetric. The equilibrium effort of the inequity-averse agent depends on α, β and τ, as $e_s^*=\frac{\tau k}{2}$ will belong to different intervals, corresponding to the separate parts of the inequity-averse agent's best-response function. The equilibrium is:

$$\left\{\begin{array}{ll}\left(e_a^*, e_s^*\right): e_a^*=\max\left\{0, \frac{k(\tau(2-\alpha-5\beta)+2\beta)}{4(1-\beta)}\right\} & \text{if } \alpha+11\beta \leq 8 \text{ and } \tau>\frac{2(2-3\beta)}{8-\alpha-11\beta} \\ \left(e_a^*, e_s^*\right): e_a^*=\max\left\{0, \frac{k(-3\tau+2)}{2}\right\} & \text{otherwise}\end{array}\right\} \tag{7}$$

[13] In the next section, we will see that these asymmetric equilibria will not actually come about as the contest structure is endogenous to agents' preferences. Due to the competition between firms over the contracts used to attract workers, the only contracts that exist on the market are the optimal contracts for each type of agent. Agents have a different optimal contract depending on their position in the group. The two different behaviors of the asymmetric equilibrium will then not be observed within the same group.

Selfish agents exert high effort in equilibrium and, as a response, inequity-averse agents matched with selfish agents reduce their effort in order to narrow the gap between the payoffs. For some high values of α, β or τ, an inequity-averse agent matched with a selfish agent will exert no effort in equilibrium.[14] The equilibrium effort of an inequity-averse agent is greater when the other agent in the group is also inequity averse, in the symmetric equilibrium, than when he is selfish.

When $\tau = \frac{1}{2}$, both agents in the group receive the same net payoff when they exert the same effort. This payment scheme is analogous to a public-goods game. There is no competition, but the scheme may induce cooperation between agents, depending on agents' social preferences. Only symmetric equilibria exist, which are defined as follows:

$$\left\{(e_a^*, e_a^*) : \frac{k}{4\,(1+\alpha)} \leq e_a^* \leq \frac{k}{4\,(1-\beta)}\right\} \tag{8}$$

More precisely, the equilibrium effort of an inequity-averse agent matched to another inequity-averse agent is:

$$e_a^* \in [\underline{e}_a, \bar{e}_a] \text{ with } \underline{e}_a = \frac{k}{4\,(1+\alpha)} \text{ and } \bar{e}_a = \begin{cases} \frac{k}{4(1-\beta)} & \text{if } \beta \in \left[0, \frac{1}{2}\right) \\ \frac{k}{2} & \text{if } \beta \in \left[\frac{1}{2}, 1\right) \end{cases} \tag{9}$$

If the other agent in the group is selfish, the equilibrium effort of the inequity-averse agent is:

$$e_a^* = \frac{k}{4} \tag{10}$$

The effort of an inequity-averse agent depends on the equilibrium behavior of his co-worker, given by the latter's type. The dominant strategy of a selfish agent in the revenue-sharing game is to free-ride in providing an effort level lower than the Pareto optimum. In the remainder of the paper, we define the free-riding action as the choice of an effort level of $\frac{k}{4}$. This free-riding action is also chosen by inequity-averse agents who are matched to selfish agents.

Nevertheless, when it is common knowledge that all those paid under this scheme are fair-minded, the free-riding equilibrium is not unique. There exist multiple symmetric

[14] As mentioned above, our equilibrium effort findings differ from those in models of inequity aversion in tournaments (Demougin and Fluet, 2003, and Grund and Sliwka, 2005). When prizes are endogenous to agents' effort, some agents who are sensitive to inequity prefer to reduce their effort in order to reduce inequality.

equilibria over a range which widens with inequity aversion: the lower bound of the equilibria set, $\underline{e}_A$, falls in α and the upper bound, $\bar{e}_A$, increases in β. Consequently, the Pareto optimum of the revenue-sharing structure, $e^{OP} = \frac{k}{2}$, will be an equilibrium if it is common knowledge that all participants have a sufficient degree of advantageous inequity aversion ($\beta \geq \frac{1}{2}$).

Cooperation is sustainable at the equilibrium under the revenue-sharing structure if all agents under this payment scheme are sufficiently inequity averse. The equal distribution of group output between agents provides incentives that are better suited to inequity-averse agents. These latter gain under this payment scheme by being able to reach an equilibrium equal, or at least closer, to the Pareto optimum.[15]

Proof. All the proofs can be found in Appendix A. □

Selfish agents respond positively to the incentives provided by competition but are not motivated by revenue-sharing. Their effort rises in the share of group output awarded to the contest winner. Inequity-averse agents behave differently. In the symmetric equilibrium, inequity-averse agents exert lower effort as the share of the group output awarded to the winner rises. Their effort is lower than that chosen by selfish agents, reflecting their desire to limit inequality. On the contrary, under revenue-sharing, in which all agents earn the same prize whatever their contribution, inequity-averse agents may exert greater effort than the free-riding equilibrium, inducing cooperation, if it is common knowledge that both agents in the group are inequity averse. For sufficiently high advantageous inequity-aversion, the Pareto optimum of the revenue-sharing game is an equilibrium.

4.2 Optimal contest structure

Selfish and inequity-averse agents do not react in the same way to the different contest structures. Agents with different social preferences should therefore choose different optimal contracts. We determine the optimal composition of groups according to agent type. The competition between firms to attract agents implies that firms offer the optimal contract of a certain type of agent in order to attract them. If optimal contracts are not offered on the market, a firm can always become better off by entering the market and proposing

[15] The equilibrium effort levels we obtain under revenue-sharing do not contradict the results of Rey-Biel (2008). We find that cooperation may pertain at the equilibrium when agents are inequity averse, without any complementarity between agents' effort levels.

the optimal contract of one agent type. In equilibrium, the optimal contest structure for all agent types necessarily exists on the market.

Under complete information, every agent knows the type of the agent with whom he is matched, and adapts his behavior accordingly. The optimal contract of an agent is defined as that which gives him the greatest expected utility compared to all of the other possibilities. The optimal contract characterizes the optimal distribution of output between agents in the group. This is given by the value of τ which determines the winner's and loser's prizes. We compare the agents' expected utilities for equilibrium effort levels under each contest structure for each group composition. Proposition 1 describes the results.

Proposition 1. *The optimal contest structure is given by*

$$\tau_\theta^* = \begin{cases} 1 & \text{if } \theta = s \\ T & \text{if } \theta = a \text{ with } \alpha < \alpha^*(\beta) \\ \frac{1}{2} & \text{if } \theta = a \text{ with } \alpha \geq \alpha^*(\beta) \end{cases}$$

where $T = \frac{-2+5\beta+3\beta^2-4\beta^3+\alpha^2(4\beta-1)+\alpha(10\beta-3)}{2(2\beta-1)(1+4\alpha+2\alpha^2+2\beta-2\beta^2)}$, $\frac{1}{2} < T < 1$,
and $\alpha^*(\beta)$ *being the solution of* $EU_a(e_a^*, e_a^*)_{\tau=T} - EU_a(e_a^*, e_a^*)_{\tau=\frac{1}{2}} = 0$.

Proof. See Appendix B. □

The optimal contest structure according to the type of agents is represented in Figure 1.

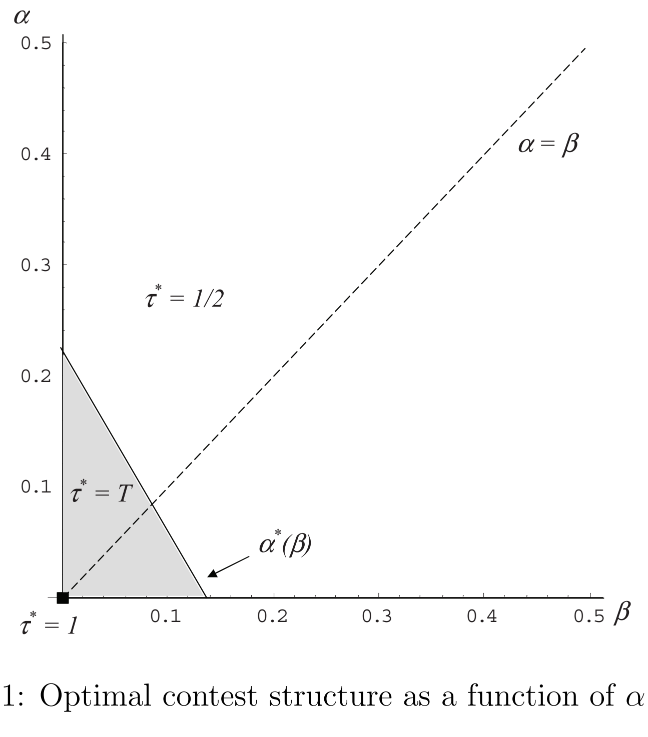

Figure 1: Optimal contest structure as a function of α and β

Proposition 1 shows that the optimal contest structure differs according to agents' inequity-aversion. The graph clearly distinguishes the three optimal structures emphasized in Proposition 1. Selfish agents maximize their expected utility under the most competitive contest structure, $\tau_s^* = 1$ (the black square in the graph), while inequity-averse agents with $\alpha \geq \alpha^* (\beta)$ maximize theirs under revenue-sharing, $\tau_a^* = \frac{1}{2}$ (the white area of the graph). Agents with intermediate degrees of inequity aversion, $\alpha < \alpha^* (\beta)$, prefer competition but with a less unequal distribution of the group output between contestants, $\tau_a^* = T$ with $\frac{1}{2} < T < 1$ (the grey area in the graph).

Corollary. *Group composition is optimal when the agents in the same group have the same preferences.*

As different types of agents are motivated differently by the various payment schemes, homogeneous groups are optimal.[16] Moreover, agents benefit from being matched to someone with similar preferences. An inequity-averse agent derives greater expected utility from being matched to someone with the same preferences under his optimal contest structure, $\tau_a^* = T$ or $\tau_a^* = \frac{1}{2}$, than from being matched to a selfish agent under $\tau_s^* = 1$. By giving the same prize to the winner and the loser, the revenue-sharing structure may induce co-operation between inequity-averse agents if all agents work under their optimal contract. Pareto gains result from the optimal matching between payment schemes and agents' social preferences.

4.3 Subgame-perfect equilibrium

In stage one, every firm decides which contract to offer, $\{k, \tau\}$. As each firm hires one group of agents, the profit of firm h is written as

$$\Pi_h = (e_i + e_j)(1 - k_h) \tag{11}$$

This equals group output minus the share of group output given to the agents to provide incentives. To determine the subgame-perfect equilibrium, we first assume a fixed contest structure, τ, and we calculate the share of the group output firms choose to give to the

[16] This result corroborates the findings of Wooders (1978) and Bennett and Wooders (1979). They show that, for small group sizes, the optimal jurisdictions consist of homogeneous agents in terms of their preferences.

agents, k. The firms' choice of k must satisfy the constraint that agents prefer working for the firm instead of taking the outside option.

As we have shown that every agent, whatever his type, prefers to be matched to an agent with the same preferences, the candidate contracts for the subgame-perfect equilibrium are the optimal contest structures of each type of agent. No other contract will be chosen in the market. Let C_s denote the action of choosing the optimal contract of selfish agents, and C_a that of inequity-averse agents. C_s and C_a define the value of the share of the group output awarded to the contest winner. As firms have complete information over types, each firm decides which contract to offer and to which individual to propose it. In the second stage of the game, every agent chooses between the contract proposed to him and an outside option X. Agents of type $\theta = s$ choose between C_s and X, and agents of type $\theta = a$ choose between C_a and X. In the third stage of the game, agents exert their equilibrium effort levels defined in the previous section.

The outside option consists in staying out of the organizational structures found on the market. This yields utility of $U_0 \geq 0$, which is assumed to be identical for all agents. As agents who take the outside option are not directly connected to other people, they do not compare their payoff to that of any other agent.[17] U_0 is then independent of others' payoffs, even for inequity-averse agents. The participation constraint, $EU_\theta \left(e^*_\theta, e^*_\theta\right)_{\tau=\tau^*_\theta} \geq U_0$ with $\theta = \{a, s\}$, must hold for agents to work for firms in equilibrium. The participation constraint for a selfish agent in equilibrium is given by

$$U_0 \leq \frac{k^2}{4} \quad \text{if } \theta = s \tag{12}$$

In order for selfish agents to always want to work under their optimal contest structure for the maximal share of the group output rewarded to agents, i.e. $k = 1$, we define $U_0 \in \left[0, \frac{1}{4}\right]$. We have shown that the optimal contest structure for inequity-averse agents depends on the intensity of their inequity aversion. Therefore, the participation constraint for inequity-

[17]We could imagine that inequity-averse agents who choose the outside option compare their payoff to the average payoff of agents who choose an organizational structure (Bolton and Ockenfels, 2000). Nevertheless, we hypothesize that agents compare their payoff only to the payoffs of agents who are close to them. Being out of any organization, agents who choose the outside option do not receive information accurate enough for them to compare their payoffs.

averse agents depends on this intensity.

$$U_0 \le \begin{cases} \frac{(1+\alpha-\beta)^2 k^2}{4(1-2\beta)(1+4\alpha+2\alpha^2+2\beta-2\beta^2)} & \text{if } \theta = a \text{ with } \alpha < \alpha^*(\beta) \\ \frac{(3-4\beta)k^2}{16(1-\beta)^2} & \text{if } \theta = a \text{ with } \alpha \ge \alpha^*(\beta) \text{ and } \beta \in \left[0, \frac{1}{2}\right) \\ \frac{k^2}{4} & \text{if } \theta = a \text{ with } \alpha \ge \alpha^*(\beta) \text{ and } \beta \in \left[\frac{1}{2}, 1\right) \end{cases} \tag{13}$$

When U_0 is sufficiently high, inequity-averse agents whose optimal contract is $\tau_a^* = T$ accept to work under this contract and reject the outside option only if α and β are not too high, such that

$$\alpha \le \alpha_0(\beta) \tag{14}$$

with $\alpha_0(\beta) = \frac{-k^2+\beta k^2+8U_0(1-2\beta)-2\sqrt{(k^2-2\beta)^2 U_0(-k^2+8U_0(1-2\beta+2\beta^2))}}{k^2-8U_0(1-2\beta)}$.

Inequity-averse agents whose optimal contract is revenue-sharing, $\tau_a^* = \frac{1}{2}$, will choose this contract only if their advantageous inequity-aversion, β, is sufficiently high. Equilibrium effort has to be close enough to the Pareto optimum to guarantee expected utility higher than that in the outside option. As there are multiple values of symmetric equilibrium effort, we cannot assume that agents coordinate their expectations on any particular equilibrium. The outside option may nonetheless restrict the equilibrium effort set via a forward-induction process.[18] An agent choosing to work under his optimal contract sends a signal to the other agents that he is entering the game with the objective of earning more than U_0, i.e. he wants to reach an equilibrium effort level, e_a^*, from the set of possible equilibria defined in subsection 4.1, such that $EU_a(e_a^*, e_a^*) \ge U_0 \Longleftrightarrow ke_a^* - e_a^{*2} \ge U_0$.

As the upper bound of the set of equilibria, $\bar{e}_a$, depends on β, inequity-averse agents will choose to work under revenue-sharing only if their advantageous inequity-aversion is such that

$$\beta \ge \beta_0 \tag{15}$$

where $\beta_0 = 1 - \frac{k\left(k+\sqrt{k^2-4U_0}\right)}{8U_0}$ (β_0 being defined for $U_0 \ne 0$).

The lower-bound of the equilibria set is then such that $ke_a^* - e_a^{*2} = U_0$, which therefore

[18] The main theoretical literature on forward induction includes Kohlberg and Mertens (1986), van Damme (1989) and Battigalli and Siniscalchi (2002). Asheim and Dufwenberg (2003) follow a very similar intuition to ours, although they do not base worker characteristics on Fehr and Schmidt's model. They show that forward induction permits the Pareto-dominant equilibrium to be selected in a coordination game with discrete strategies (they use a different group-incentive game: "forcing contracts"). According to the definition in Cooper et al. (1992), a coordination game consists of multiple, Pareto-rankable and pure-strategy Nash equilibria. Therefore, revenue-sharing with agents having sufficiently high advantageous inequity-aversion is included in this category of games.

depends on β_0, with

$$\underline{e}_a = \frac{k}{4\left(1-\beta_0\right)} \tag{16}$$

If β_0 is strictly positive, i.e. for $U_0 > \frac{3k^2}{16}$, equilibrium effort will necessarily be higher than that in the free-riding equilibrium, and thus higher than the effort of selfish agents. Cooperation will then be observed under this contract.[19]

Now that participation constraints of agents are defined, we calculate the optimal value of the share of the group output given to the agents, k, depending on the contest structure, τ. We assume first that there are only two firms on the market. In this case, there are 'too few' firms on the market and then each firm hires more than one group of agents. As profits of the firm for each group are identical, we use equation (11) to define profits of firms for a representative group. Identical conclusions are obtained when comparing total profits or profits for a representative group.

Suppose one firm offers $\tau = \tau_s^*$ and another firm offers $\tau = \tau_a^*$. There is no competition between these two firms to attract agents as each one offers the optimal contest structure of one type of agents. Each firm maximizes its profit function subject to the participation constraint in order to determine the optimal share of the group output to give to the agents.

The firm who offers to selfish agents their optimal contest structure, $\tau = \tau_s^* = 1$, will maximize the following profit function under constraint

$$\begin{aligned} &\Pi^*_{\tau=\tau_s^*} = \left(e_s^* + e_s^*\right)\left(1 - k_{\tau=\tau_s^*}\right) \\ &s.t \qquad k^2_{\tau=\tau_s^*} \geq 4U_0 \end{aligned} \tag{17}$$

With $k_{\tau=\tau_s^*}$ being the share of group output given to agents associated with the contest structure $\tau = \tau_s^* = 1$.

The optimal share of the group output this firm chooses to give to the agents is:

$$k^*_{\tau=\tau_s^*} = \begin{cases} 2\sqrt{U_0} & \text{if } U_0 > \frac{1}{16} \\ \frac{1}{2} & \text{otherwise} \end{cases} \tag{18}$$

As both the optimal contest structure and the participation constraint of agents differ

[19] For example, under the assumption of $k = 1$, if $U_0 = 0$, inequity-averse agents with the optimal contract $\tau_a^* = \frac{1}{2}$ will always choose revenue-sharing independently of their value of β. If $U_0 = \frac{1}{5}$, inequity-averse agents will choose revenue-sharing only if their advantageous inequity-aversion is greater than 0.096. If $U_0 = \frac{1}{4}$, the minimum threshold of β for inequity-averse agents to choose this optimal contest is $\frac{1}{2}$.

depending on the inequity aversion degrees of inequity-averse agents, different cases exist for the other firm who offers to inequity-averse agents their optimal contest structure, $\tau = \tau_a^* = T$ if $\alpha < \alpha^*(\beta)$ or $\tau = \tau_a^* = \frac{1}{2}$ if $\alpha \geq \alpha^*(\beta)$. This firm will maximize the following profit function under constraint

$$\begin{array}{lll} \Pi^*_{\tau=\tau_a^*} = \left(e_a^* + e_a^*\right)\left(1 - k_{\tau=\tau_a^*}\right) & & \\ s.t. \quad k^2_{\tau=\tau_a^*} \geq \frac{4(1-2\beta)\left(1+4\alpha+2\alpha^2+2\beta-2\beta^2\right)U_0}{(1+\alpha-\beta)^2} & \text{if } \alpha < \alpha^*(\beta) \\ \qquad k^2_{\tau=\tau_a^*} \geq \frac{16(1-\beta)^2 U_0}{3-4\beta} & \text{if } \alpha \geq \alpha^*(\beta) \text{ and } \beta \in \left[0, \frac{1}{2}\right) \\ \qquad k^2_{\tau=\tau_a^*} \geq 4U_0 & \text{if } \alpha \geq \alpha^*(\beta) \text{ and } \beta \in \left[\frac{1}{2}, 1\right) \end{array} \tag{19}$$

With $k_{\tau=\tau_a^*}$ being the share of group output given to agents associated with the contest structure $\tau = \tau_a^* = T$ if $\alpha < \alpha^*(\beta)$ or $\tau = \tau_a^* = T$ if $\alpha \geq \alpha^*(\beta)$.

The optimal share of group output this firm gives to the agents is

$$k^*_{\tau=\tau_a^*} = \begin{cases} 1 & \text{if } \alpha < \alpha^*(\beta) \text{ and } U_0 > U_2 \text{ or } \alpha \geq \alpha^*(\beta) \text{ and } U_0 > U_4 \\ K_1 & \text{if } \alpha < \alpha^*(\beta) \text{ and } U_1 < U_0 \leq U_2 \\ K_2 & \text{if } \alpha \geq \alpha^*(\beta) \text{ and } \beta \in \left[0, \frac{1}{2}\right) \text{ and } U_3 < U_0 \leq U_4 \\ K_3 & \text{if } \alpha \geq \alpha^*(\beta) \text{ and } \beta \in \left[\frac{1}{2}, 1\right) \text{ and } U_0 > \frac{1}{16} \\ \frac{1}{2} & \text{otherwise} \end{cases} \tag{20}$$

with $K_1 = \frac{2}{1+\alpha-\beta}\sqrt{(1-2\beta)(1+4\alpha+2\alpha^2+2\beta-2\beta^2)U_0}$, $K_2 = \frac{4(1-\beta)}{3-4\beta}\sqrt{U_0(3-4\beta)}$ and $K_3 = 2\sqrt{U_0}$. We also note $U_1 = \frac{(1+\alpha-\beta)^2}{16(1-2\beta)(1+4\alpha+2\alpha^2+2\beta-2\beta^2)}$, $U_2 = \frac{(1+\alpha-\beta)^2}{4(1-2\beta)(1+4\alpha+2\alpha^2+2\beta-2\beta^2)}$, $U_3 = \frac{3-4\beta}{64(1-\beta)^2}$ and $U_4 = \frac{3-4\beta}{16(1-\beta)^2}$.

The optimal share of group output is the minimum value that verifies the participation constraint, for values higher than $\frac{1}{2}$. Firms make zero profits when $k^*_{\tau=\tau_\theta^*} = 1$. However, when $k^*_{\tau=\tau_\theta^*} < 1$, firms make positive profits. Regarding the values of the optimal share of group output, both firms on the market make either zero profits when the outside option is too high or positive profits.[20] We conclude that when two firms are on the market and each firm offers the optimal contest structure to one agent type, each firm makes positive profits, or zero profits when the outside option is too high.

[20]When the outside option, U_0, is too high, the participation constraint cannot be verified for $k \leq 1$. Therefore, firms cannot hire any group of agents and make zero profits. As firms increase as much as possible their value of k and in order to represent this situation in equation (20), situations where the participation constraint cannot be verified is characterized by an optimal share of group output equal to 1.

To define whether the subgame-perfect equilibrium is separating, i.e. one firm offers the optimal contest structure of selfish agents ($\tau = \tau_s^*$) to selfish agents and the other firm offers the optimal contest structure to inequity averse agents ($\tau = \tau_a^*$), we must control whether at least one of the two firms has an interest to deviate by offering the same contest structure as the other firm on the market.

If both firms offer a contract with the same value of τ, they will enter into competition to attract agents. Agents choose the firm depending on its value of k. Hence, when one firm offers a higher k to agents than the other firm, it will attract all the agents. When both firms offer the same k, agents divide fairly between the two firms. The best response function of firm h to firm l's behavior is as follows:

$$k_{h|\tau=\tau_\theta^*}\left(k_{l|\tau=\tau_\theta^*}\right) = \begin{cases} k^*_{\tau=\tau_\theta^*} & \text{if } k_{l|\tau=\tau_\theta^*} \in \left[0, k^*_{\tau=\tau_\theta^*}\right) \\ \widehat{k}_{\tau=\tau_\theta^*} \in \left[k^*_{h|\tau=\tau_\theta^*} + \varepsilon, 1\right] & \text{if } k_{l|\tau=\tau_\theta^*} = k^*_{\tau=\tau_\theta^*} \\ k_{l|\tau=\tau_\theta^*} + \varepsilon & \text{if } k_{l|\tau=\tau_\theta^*} \in \left(k^*_{\tau=\tau_\theta^*}, 1\right) \\ \widetilde{k}_{\tau=\tau_\theta^*} \in [0,1] & \text{if } k_{l|\tau=\tau_\theta^*} = 1 \end{cases} \tag{21}$$

The best response function shows that $k_{h|\tau=\tau_\theta^*} = 1$ is the best response to $k_{l|\tau=\tau_\theta^*} = 1$ and *vice versa*. Therefore, when two firms offer the same contest structure, either $\tau = \tau_s^*$ or $\tau = \tau_a^*$, they make zero profits due to the competition between the two firms to attract agents. Hence, they have no interest to deviate from the separating situation with one firm offering $\tau = \tau_s^*$ and the other firm offering $\tau = \tau_a^*$. We conclude that when there are two firms on the market, there is a unique equilibrium that is separating.

For already two firms offering the same contest structure the equilibrium share of group output to give to the agents is equal to one, i.e. these two firms make zero profits. Hence, when there is a continuum of firms, firms make also zero profits due to competition. The optimal share of the group output given to the agents k is in this case

$$k^*_{\tau=\tau_\theta^*} = 1 \text{ with } \theta \in \{a, s\}\,. \tag{22}$$

As in the case with only two firms, the separating equilibrium is unique. A positive share of firms offer $\tau = \tau_s^*$ and the others offer $\tau = \tau_a^*$. For a continuum of firms, we suppose that each firm offers a contract to one group of two agents. As firms, first-member agents and second-member agents have the same measure and the share of inequity averse agents in the population is ρ, the share of firms who offer $\tau = \tau_s^*$ is $1-\rho$ while the share of firms who offer $\tau = \tau_a^*$ is ρ.

We have shown that agents prefer to work in a group composed of people with the same preferences, and that the optimal contest structure differs according to the agents' degrees of inequity aversion. Agents of both types are better off when they work for a firm which offers them their optimal contest. Finally, under complete information, as both agents and firms know the types of all agents with certainty, the unique pure-strategy equilibrium is a separating one. Each firm offers one contract to only one agent type, with the proposed contract being the optimal one for that type. Whatever the degree of inequity aversion of fair-minded agents, there is a unique subgame-perfect equilibrium that is. The following proposition describes this equilibrium.

Proposition 2. *Under complete information, there exists a unique and separating subgame-perfect equilibrium. This is described below:*
Firms: A strictly positive proportion of firms, $1-\rho$, *adopt the equilibrium strategy*
$\left\{k^* = k^*_{\tau=\tau_s^*}, \tau^* = \tau_s^* = 1\right\}$,
with the remainder, also strictly positive, ρ, *adopting the equilibrium strategy*

$$\begin{cases} \left\{k^* = k^*_{\tau=\tau_a^*}, \tau^* = \tau_a^* = T\right\} & \text{if } \alpha < \alpha^*(\beta) \\ \left\{k^* = k_{\tau=\tau_a^*}, \tau^* = \tau_a^* = \frac{1}{2}\right\} & \text{if } \alpha \geq \alpha^*(\beta) \end{cases}$$

Agents: The equilibrium strategy of selfish agents is $\{C_s, e_s^*\}$ *and the equilibrium strategy of inequity-averse agents is*

$$\begin{cases} \{C_a, e_a^*\} & \text{if } \alpha \leq \alpha_0(\beta) \text{ when } \alpha < \alpha^*(\beta)\text{, and } \beta \geq \beta_0 \text{ when } \alpha \geq \alpha^*(\beta) \\ \{X\} & \text{if } \alpha > \alpha_0(\beta) \text{ when } \alpha < \alpha^*(\beta)\text{, and } \beta < \beta_0 \text{ when } \alpha \geq \alpha^*(\beta) \end{cases}$$

The set of coalitions in equilibrium is: $\{\{i, \psi(i), \varphi(i)\} : i \in [0,1)\}$ *with* $\psi(i) = i+1$ *and* $\varphi(i) = i+2$.

We represent below an example of agents' equilibrium strategies when there are at least four firms on the market, i.e. when there is competition between firms to attract agents for each contest structure. The relationship between agents' equilibrium strategies and their inequity-aversion degrees is depicted in Figures 2 and 3 for the U_0 values of 0.15 and 0.21 respectively.

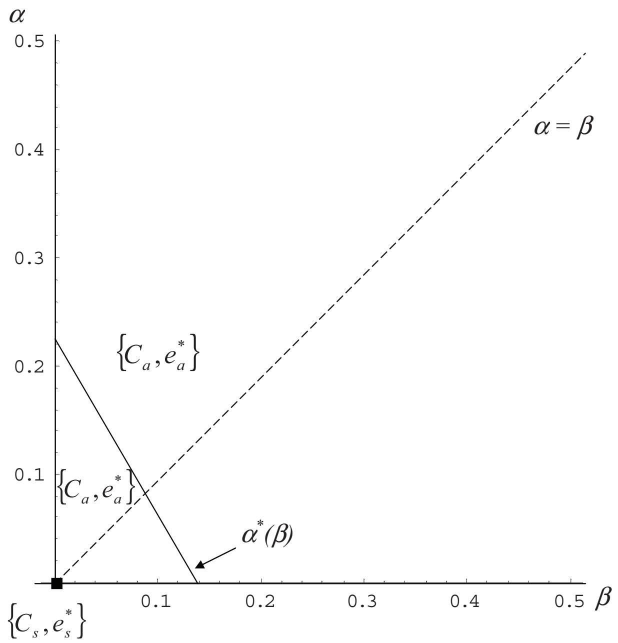

Figure 2: Agents' equilibrium strategies as a function of α and β: $U_0 = 0.15$

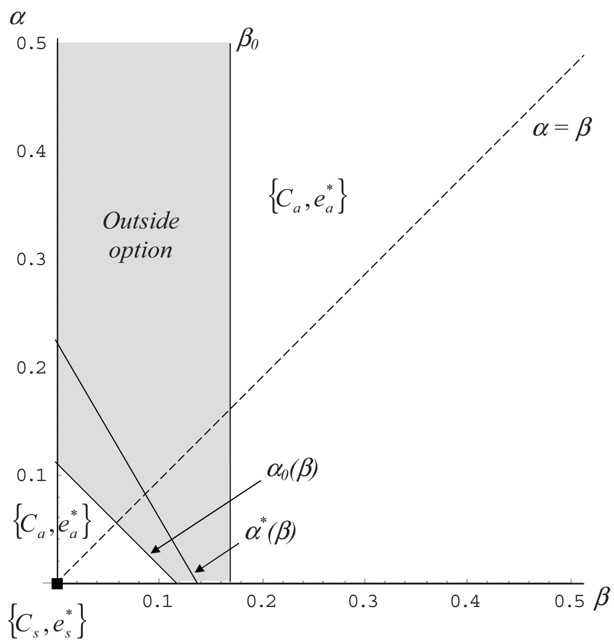

Figure 3: Agents' equilibrium strategies as a function of α and β: $U_0 = 0.21$

These figures clearly represent the separating equilibrium described in Proposition 2. For smaller values of U_0, all agents choose their optimal contest structure and then exert their equilibrium effort. No-one selects the outside option (Figure 2). As the value of U_0 increases, selfish agents choose their optimal contest. For inequity-averse agents, the strategy $\{C_a, e_a^*\}$ is chosen for both lower values of α and β ($\alpha \leq \alpha_0(\beta)$) and higher values of β independently of α ($\beta \geq \beta_0$). This result reflects that the optimal contract of inequity-averse agents with $\alpha < \alpha^*(\beta)$ is different from that of inequity-averse agents with $\alpha \geq \alpha^*(\beta)$: $\tau_a^* = T$ and $\tau_a^* = \frac{1}{2}$ respectively. Inequity-averse agents with $\alpha > \alpha_0(\beta)$ or $\beta < \beta_0$ prefer the outside option (see Figure 3).

The outcome where selfish and inequity-averse agents work for different firms, depending on the contest structure offered, is stable, whereas a pooling outcome with selfish and inequity-averse agents working for the same firm is not. As all agents maximize their expected utility when social preferences are homogeneous within the group, no firm proposing heterogeneous groups will exist in equilibrium. Two kinds of firms will remain on the market: those offering the optimal contract for selfish agents and those offering the optimal contract for inequity-averse agents. Each firm offers one contest structure to the corresponding agent type, and the subgame-perfect equilibrium is unique and separating. Agents choose to work under their optimal contract only if their participation constraint holds: their expected utility under their optimal contract, when matched with agents with the same social preferences, must be higher than the utility from the outside option.

Average effort is higher in this separating equilibrium, since agents provide greater effort when they work under their optimal contracts. The results thus predict the coexistence on the market of more than one performance-based payment scheme, with this coexistence leading to market efficiency gains. Each specific contest structure is appropriate to one specific agent type. For example, for sufficiently large inequity-aversion, the optimal contract of fair-minded agents is an equal split of group production between the two agents in the group. Cooperation between inequity-averse agents under the revenue-sharing structure may then result. It is then entirely possible that *ex ante* identical firms differ in their degrees of competition and cooperation. We now ask whether the existence of this separating equilibrium is robust to agent types being private information.

5 Self-selection and incomplete information

Agents' social preferences are generally private information. Under incomplete information about agent types, firms cannot offer a contract to a specific agent type (as they cannot identify them). Each firm will propose one type of contract to all agents. In the second stage of the game, therefore, agents self-select one of these contracts. Each agent develops beliefs about the agent type of his partner, and decides equilibrium effort accordingly.

Do we still find a unique and separating equilibrium under asymmetric information? To answer this question, we start by determining the optimal behavior of selfish agents.

The expected utility of a selfish agent depends on the proportion of agents of each type in the whole population. Agents do not know the type of the other group member. If ρ is the fraction of inequity-averse agents in the population, the expected utility of a selfish agent under incomplete information, EU_s^I, is given by

$$EU_s^I(e_s^I, e_j) = \rho EU_s(e_s^I, e_a) + (1-\rho)\, EU_s(e_s^I, e_j) \tag{23}$$

where e_a is the effort of an inequity-averse agent matched with a selfish agent and e_j is the effort of another selfish agent. As there is a continuum of agents, the type of one particular individual does not affect his beliefs regarding his co-worker's type.

Under incomplete information, as under complete information, a selfish agent determines equilibrium effort by maximizing expected utility. The equilibrium effort of a selfish agent, e_s^{I*}, is therefore given by

$$e_s^{I*} = \frac{\tau k}{2} \qquad \forall \tau \in \left[\frac{1}{2}, 1\right] \tag{24}$$

The equilibrium effort level of a selfish agent is his dominant strategy, and increases in both k and τ. Whatever the information regarding agent types, a selfish agent will exert the same equilibrium effort, $e_S^{I*} = e_S^*$. The expected utility of a selfish agent increases with the share of group output given to the winner, τ, whatever the proportion of inequity-averse agents in the population. When $\tau \in \left(\frac{1}{2}, 1\right]$, selfish agents maximize their equilibrium expected utility at $\tau_s^* = 1$.

Proof. See Appendix C. □

Therefore, if a contract with $\tau \in \left(\frac{1}{2}, 1\right]$ exists in equilibrium, the optimal contract for selfish agents, $\tau_s^* = 1$, will also necessarily exist. In this case, an inequity-averse agent knows with certainty that if his optimal contest structure involves $\tau < 1$, he will be matched with another inequity-averse agent. When an inequity-averse agent is matched with someone of the same type, his optimal contest structure is $\tau_a^* = T$ if $\alpha < \alpha^*(\beta)$ and $\tau_a^* = \frac{1}{2}$ if $\alpha \geq \alpha^*(\beta)$. If $\alpha < \alpha^*(\beta)$, neither inequity-averse nor selfish agents will gain from choosing a contract different from their optimal contract. Consequently, there exists a unique and separating equilibrium which is the same as that under complete information.

If $\alpha \geq \alpha^*(\beta)$, the optimal contest structure of inequity-averse agents is revenue-sharing, $\tau_a^* = \frac{1}{2}$. Under this contract, a cooperative situation is an equilibrium when all the participating agents are inequity averse. This is only possible if the equilibrium is separating with agents of each type choosing to work under their optimal contract. Under incomplete information, the revenue-sharing structure may also be attractive to selfish agents who exert low effort (free-riding equilibrium) and take advantage of the greater effort exerted by inequity-averse agents. To guarantee that selfish agents prefer their optimal contract, $\tau_s^* = 1$, their expected utility under $\tau_s^* = 1$ must be greater than that under $\tau = \frac{1}{2}$. We first check whether this condition holds whatever the effort level exerted by inequity-averse agents.

The equilibrium effort of inequity-averse agents, e_a^{I*}, when $\tau = \frac{1}{2}$ is given by

$$e_a^{I*} = \begin{cases} \frac{k}{4} & \text{if } \rho \in \left[0, \frac{\beta}{\alpha+\beta}\right] \\ \left[\frac{k}{4(1-\beta+\rho(\alpha+\beta))}, \frac{k}{4}\right] & \text{if } \rho \in \left(\frac{\beta}{\alpha+\beta}, \frac{\alpha}{\alpha+\beta}\right] \\ \left[\frac{k}{4(1-\beta+\rho(\alpha+\beta))}, \frac{k}{4(1+\alpha-\rho(\alpha+\beta))}\right] & \text{if } \rho \in \left(\frac{\alpha}{\alpha+\beta}, 1\right] \end{cases} \tag{25}$$

The equilibrium effort of inequity-averse agents under $\tau = \frac{1}{2}$ increases in ρ and the situation with separating contracts is characterized by $\rho = 1$ under $\tau = \frac{1}{2}$. If the self-selection condition holds for $\rho = 1$, it will therefore always hold. We have $e_a^{I*} = e_a^*$ for $\rho = 1$. Both types of agents thus self-select into their optimal contract if

$$EU_s\left(e_s^*, e_s^*\right)_{\tau=1} \geq EU_s\left(e_s^*, e_a^*\right)_{\tau=\frac{1}{2}} \forall e_a^* \tag{26}$$

The self-selection condition is not independent of e_a^*, and depends on the values of k associated with the contest structures $\tau_s^* = 1$ and $\tau_a^* = \frac{1}{2}$. To guarantee a separating equilibrium, selfish agents must work under $\tau_s^* = 1$ at the equilibrium. The self-selection condition that holds for any value of e_a^* is thus given by

$$\begin{cases} \frac{k^{*2}_{\tau=\tau_s^*}}{4} \geq \frac{(3-\beta)k^{*2}_{\tau=\tau_a^*}}{16(1-\beta)} & \text{if } \beta \in \left[0, \frac{1}{2}\right) \\ \frac{k^{*2}_{\tau=\tau_s^*}}{4} \geq \frac{5k^{*2}_{\tau=\tau_a^*}}{16} & \text{if } \beta \in \left[\frac{1}{2}, 1\right) \end{cases} \tag{27}$$

We then have the following lemma.

Lemma. *Under incomplete information, inequity-averse agents with $\alpha \geq \alpha^* (\beta)$ can attain equilibrium effort of $e_a^* \in [\underline{e}_a, \bar{e}_a]$, as defined in the previous sections, if and only if the self-selection condition holds:*

$$k^*_{\tau=\tau_a^*} \in \begin{cases} [0,1] & \text{if } \beta \in \left[0, \frac{4k^{*2}_{\tau=\tau_s^*}-3}{4k^{*2}_{\tau=\tau_s^*}-1}\right] \\ \left[0, \frac{2k^*_{\tau=\tau_s^*}\sqrt{(1-\beta)(3-\beta)}}{3-\beta}\right] & \text{if } \beta \in \left(\frac{4k^{*2}_{\tau=\tau_s^*}-3}{4k^{*2}_{\tau=\tau_s^*}-1}, \frac{1}{2}\right) \\ \left[0, \frac{2k^*_{\tau=\tau_s^*}\sqrt{5}}{5}\right] & \text{if } \beta \in \left[\frac{1}{2}, 1\right) \end{cases}$$

When there are only two firms on the market, both firms adjust the share of group output to give to the agents in order to verify the participation constraint and the self-selection condition. Indeed, the two firms make positive profits when the self-selection condition is verified as they do not compete to attract agents.

If $\beta \in \left[0, \frac{4k^{*2}_{\tau=\tau_s^*}-3}{4k^{*2}_{\tau=\tau_s^*}-1}\right]$, the self-selection condition is always verified. Due to the low value of β, equilibrium cooperation of inequity-averse agents in revenue-sharing is low and selfish agents have no incentives to choose the contest structure $\tau = \tau_a^* = \frac{1}{2}$ instead of $\tau = \tau_s^* = 1$. Hence, the optimal share of group output given to the agents of each firm is equal to the optimal share of group output under complete information, which is defined in the previous section. If $\beta \in \left(\frac{4k^{*2}_{\tau=\tau_s^*}-3}{4k^{*2}_{\tau=\tau_s^*}-1}, 1\right)$, a further condition on $k^*_{\tau=\tau_s^*}$ and $k^*_{\tau=\tau_a^*}$ is necessary to guaranty the separating equilibrium. Both the participation and the self-selection constraints hold when $k^*_{\tau=\tau_s^*} \in \left[\frac{2}{3-4\beta}\sqrt{U_0 (3-4\beta)(3-\beta)(1-\beta)}, 1\right]$ if $\beta \in \left[\frac{4k^{*2}_{\tau=\tau_s^*}-3}{4k^{*2}_{\tau=\tau_s^*}-1}, \frac{1}{2}\right)$ and when $k^*_{\tau=\tau_s^*} \in \left[\sqrt{5U_0}, 1\right]$ if $\beta \in \left[\frac{1}{2}, 1\right)$ with $k^*_{\tau=\tau_a^*}$ that verifies the self-selection condition, given the value of $k^*_{\tau=\tau_s^*}$. Therefore, the optimal share of group output to give to the agents of each firm belongs to these intervals. The value of $k^*_{\tau=\tau_s^*}$ and the value of $k^*_{\tau=\tau_a^*}$ depend on the negotiation between the two firms.

When there is a continuum of firms, firms who offer the optimal contract of selfish agents assign the whole of group production, $k^*_{\tau=\tau_s^*} = 1$, to agents in the group due to competition. If they do not, another firm will always have an incentive to enter the market and offer a value of k close to that associated with the contest structure $\tau_s^* = 1$. For the self-selection condition to hold, inequity averse agents with $\alpha \geq \alpha^* (\beta)$ might reduce their equilibrium effort under their optimal contest structure, $\tau_a^* = \frac{1}{2}$. An inequity-averse agent with $\alpha \geq \alpha^* (\beta)$ may prefer to reduce equilibrium effort and be compensated with $k^*_{\tau=\tau_a^*} = 1$

instead of choosing an effort level such as $\bar{e}_a$ and bearing the self-selection condition which implies $k^*_{\tau=\tau^*_a} < 1$.

To determine their greatest effort level, $\bar{e}_a$, inequity-averse agents maximize their expected utility under the self-selection condition. When the latter binds, the first-order condition shows that agents' expected utility increases in $\bar{e}_a$ only if $\bar{e}_a < \frac{3}{8}$. The effort level of $\frac{3}{8}$ is the maximum that inequity-averse agents with $\beta = \frac{1}{3}$ can attain in equilibrium. This means that, when the self-selection condition holds, inequity-averse agents with $\beta \in \left(\frac{1}{3}, 1\right)$ always prefer to exert an effort level of $\frac{3}{8}$ or less. Therefore, inequity-averse agents with $\beta \in \left(\frac{1}{3}, 1\right)$ will limit their cooperative behavior and prefer to receive $k^*_{\tau=\tau^*_a} = 1$. Under incomplete information, the equilibrium effort of an inequity-averse agent with $\alpha \geq \alpha^*(\beta)$ is then[21]

$$e_a^{I*} \in \left[\underline{e}_a, \bar{e}_a^I\right] \text{ with } \underline{e}_a = \frac{k}{4(1-\beta_0)} \text{ and } \bar{e}_a^I = \begin{cases} \frac{k}{4(1-\beta)} & \text{if } \beta \in \left[0, \frac{1}{3}\right) \\ \frac{3}{8} & \text{if } \beta \in \left[\frac{1}{3}, 1\right) \end{cases} \tag{28}$$

The set of equilibrium effort levels of inequity-averse agents with $\alpha \geq \alpha^*(\beta)$ is bounded by $e_a^{I*} = \frac{3}{8}$ when $\beta \in \left[\frac{1}{3}, 1\right)$. Under incomplete information, the Pareto optimum of revenue-sharing is not an equilibrium. Therefore, the utility of the outside option must be lower than $\frac{15}{64}$ in order for inequity-averse agents to choose to work under their optimal contract ($\tau^*_a = \frac{1}{2}$) instead of choosing the outside option.

Under incomplete information, as under complete information, agents of each type prefer to work under their optimal contest structure. In equilibrium, the optimal contest structure of each agent type is offered on the market, and the self-selection condition holds. Therefore, there exists again a unique subgame-perfect Bayesian equilibrium that is separating. Proposition 3 describes this equilibrium.

Proposition 3. *Under incomplete information, there exists a unique and separating perfect Bayesian equilibrium with beliefs such that* $b_i(\theta = s \mid C_s) = b_i(\theta = a \mid C_a) = 1$. *It is described below:*

Firms: A strictly positive proportion of firms, $1-\rho$, *adopt the equilibrium strategy*

$\left\{k^* = k^*_{\tau=\tau^*_s}, \tau^* = \tau^*_s = 1\right\}$

with the remainder, also strictly positive, ρ, *adopting the equilibrium strategy*

[21] Although the maximum effort level that inequity averse agents can reach at the equilibrium is $\frac{3}{8}$ when $\tau^*_a = \frac{1}{2}$, i.e. lower than under complete information, the definition of the optimal contest structures is not changed.

$$\begin{cases} \left\{k^* = k^*_{\tau=\tau_a^*}, \tau^* = \tau_a^* = T\right\} & \textit{if } \alpha < \alpha^*(\beta) \\ \left\{k^* = k^*_{\tau=\tau_a^*}, \tau^* = \tau_a^* = \frac{1}{2}\right\} & \textit{if } \alpha \geq \alpha^*(\beta) \end{cases}$$

Agents: The equilibrium strategy of selfish agents is $\{C_s, e_s^*\}$ *and the equilibrium strategy of inequity-averse agents is*

$$\begin{cases} \left\{C_a, e_a^{I*}\right\} & \begin{array}{l} \textit{if } \alpha \leq \alpha_0 \textit{ when } \alpha < \alpha^*(\beta) \\ \textit{and } \beta \geq \beta_0 \textit{ when } \alpha \geq \alpha^*(\beta) \textit{ and } U_0 \in \left[0, \frac{15}{64}\right] \end{array} \\ \{X\} & \begin{array}{l} \textit{if } \alpha > \alpha_0 \textit{ when } \alpha < \alpha^*(\beta) \\ \textit{and } \beta < \beta_0 \textit{ when } \alpha \geq \alpha^*(\beta) \textit{ or } U_0 \in \left(\frac{15}{64}, \frac{1}{4}\right] \end{array} \end{cases}$$

The set of coalitions in equilibrium is: $\{\{i, \psi(i), \varphi(i)\} : i \in [0,1)\}$ *with* $\psi(i) = i + 1$ *and* $\varphi(i) = i + 2$.

When the agents' type is private information, the unique perfect Bayesian equilibrium is a separating one. The two contest structures proposed on the market at the equilibrium are each optimal for one agent type. Some firms offer the optimal contract of selfish agents and others that of inequity-averse agents. All firms distribute all of group production between agents, due to competition. In the second stage, agents of each type decide for which firm to work. Each agent chooses to work under the contract designed for him if the self-selection and participation constraints bind. Agents self-select optimally and the unique equilibrium is a separating one. Consequently, agents believe with certainty that the other agent in their group has the same preferences as themselves. Homogeneous groups are formed endogenously, and efficiency gains result from the optimal self-selection of agents.

The unique equilibrium under incomplete information is then a separating equilibrium, as was the case under complete information. The two equilibrium contest structures are the two optimal ones and everyone works under their optimal contract. The conditions on α and β defining the optimal contest structure for inequity-averse agents are the same as those under complete information. The optimal contest structures of inequity-averse agents with $\alpha < \alpha^*(\beta)$ and $\alpha \geq \alpha^*(\beta)$ are respectively $\tau_a^* = T$ and $\tau_a^* = \frac{1}{2}$. The optimal contest structure of selfish agents is $\tau_s^* = 1$, the same as that under complete information.

Nevertheless, an additional condition on the range of U_0 is required to observe inequity-averse agents choosing their optimal contract instead of the outside option when $\alpha \geq \alpha^*(\beta)$, which is in this case $\tau_a^* = \frac{1}{2}$. For a separating equilibrium with all agents working under their optimal contest structure to pertain, U_0 must be lower than $\frac{15}{64}$, whereas we only required U_0 to be lower than $\frac{1}{4}$ under complete information. Indeed, the equilibrium effort of fair-minded agents with inequity aversion such that $\alpha \geq \alpha^*(\beta)$ is upper-bounded by $\frac{3}{8}$ when agent types are private information.

The cost of information asymmetry is reflected in reduced cooperative behavior by inequity-averse agents under $\tau_a^* = \frac{1}{2}$, in order to maintain the self-selection of both types of agents. Due to the attractiveness of this contract for selfish agents, inequity-averse agents cannot attain the same level of cooperative behavior as under complete information. The potential equilibrium expected utility of inequity-averse agents then falls. Nevertheless, although agent types are not observable, all agents are optimally matched under their optimal contract, and inequity-averse agents may still behave cooperatively under revenue-sharing.

Proposition 4. *The separating equilibrium with all agents working under their optimal contract is robust to incomplete information.*
When inequity-averse agents have preferences such that $\alpha \geq \alpha^*(\beta)$, the cost of incomplete information is reduced cooperative behavior at the equilibrium compared to that under complete information.

When agents are heterogeneous in terms of social preferences, there exists a unique and separating perfect Bayesian equilibrium under incomplete information. All agents work under the contest structure which is optimal for them, and homogeneous groups result. The self-selection of agents leads to efficiency gains via greater average effort than that in which agents are not allowed to freely choose their firm. The type of agents is unknown *ex ante* by other agents and firms, but once agents have chosen the contract under which they want to work, firms can identify the agents' type, and agents know that they will be matched with an agent of the same type: as they have the same preferences, they have made the same choice.

We therefore do not need to determine agents' types directly. They identify themselves via their choice of one of the contracts proposed by the different firms on the market. Self-selection drives the separating nature of the equilibrium. Under both incomplete and complete information, different organizational forms exist, even when firms are *ex ante* identical. Some of these forms emphasize competition between agents working in the same group, while others encourage cooperation. Firms offering very unequal payment schemes coexist in a free-entry market with firms offering more egalitarian contracts.

6 Conclusion

Our main result is that a unique group incentive mechanism cannot be efficient when the working population is heterogeneous in terms of social preferences such as inequity aversion. In real company settings, multiple performance-based payment schemes are required to ensure appropriate incentives for different types of agents. Under labor-market competition, we have shown that the co-existence of various organizational structures based on the degree of competition or cooperation is optimal, even though firms are *ex ante* identical. The unique equilibrium is separating in the sense that all agents work under their optimal contract. Within each firm, where firms are differentiated by the payment contract they offer, agents exhibit the same degree of inequity aversion, so that groups are homogeneous. This result is robust to agent types being private information.

Agents who are only motivated by their own payoff are strongly positively affected by competitive environments. However, inequity-averse agents who dislike *ex post* inequality between net payoffs exert less effort in equilibrium under this kind of payment scheme. They respond better to less-competitive schemes. For sufficiently high inequity aversion, fair-minded agents are better off under a revenue-sharing scheme which shares joint output equally between the agents in the group. Cooperation may result in equilibrium if it is common knowledge that the advantageous inequity-aversion of all agents is sufficiently high.

Under the assumption of no mobility costs, we show that there exists a unique and separating equilibrium with each firm proposing the optimal contest structure of one type of agent. When it is possible to identify types, agents of each type work under their optimal contract with certainty, and this is common knowledge. The necessary and sufficient condition for this outcome to be stable under incomplete information is that selfish agents do not prefer the optimal contest structure of inequity-averse agents. This always holds when inequity-averse agents have sufficiently low advantageous inequity-aversion. However, when the optimal contract of inequity-averse agents is revenue-sharing, agents are optimally matched only if the self-selection condition binds. This constraint implies a fall in the potential cooperative behavior of inequity-averse agents compared to the complete-information case. No pooling equilibria exist. Efficiency gains emerge when the agents work under their optimal compensation scheme.

The conclusions of the analysis, especially under incomplete information, highlight the importance of agent self-selection based on social preferences when agents can costlessly choose the firm for which they want to work. One unique performance-based payment scheme is not efficient for the whole working population. In a business environment where individuals can select their payment scheme, self-selection leads to homogeneous groups and implies the *ex post* identification of personality types. Appropriate incentives are therefore chosen by agents who appreciate them, which leads to an average effort level higher than that when agent self-selection is not possible. On a labor market with competition between *ex ante* identical firms, these results explain the joint existence of corporate cultures based on different degrees of competition and cooperation within groups.

One extension of this work would be to differentiate agents by their ability. Agents' ability likely affects their effort decisions and their choice of payment scheme. High-skill agents may have a stronger preference for competition than low-skill agents. It would be interesting to see whether the optimum here consists of homogeneous groups in terms of both ability and social preferences. Agents' risk aversion is another characteristic of interest. Further, complementarity between agents' effort within the same group may shed some light on competition and cooperation depending on the complexity of the task. When diverse personality types exist, agents' self-selection leads to more efficient exploitation of the different incentives provided by various variable payment schemes.

Appendices

Appendix A - Calculation of equilibrium effort under complete information

- *Selfish agents*

The equilibrium effort of a selfish agent, e_s^*, is determined by maximizing the agent's expected utility, which depends on the effort of the other group member, e_j

$$e_s^* \in \arg\max_{e_s} EU_s(e_s, e_j) = \tau k e_s + (1-\tau)\, k e_j - e_s^2$$

$$\frac{\partial EU_s(e_s, e_j)}{\partial e_s} = 0 \Longleftrightarrow e_s^* = \frac{\tau k}{2}$$

Whatever the type and behavior of his co-worker, a selfish agent has a dominant strategy,

e_s^*.

- *Inequity-averse agents*

The expected utility of an inequity-averse agent a is written differently according to the agent's own effort and that of the agent to whom he is matched, agent j.

If $0 \leq e_a \leq e_j - (2\tau - 1)\,k$, the expected utility of agent a is:

$$\begin{aligned} EU_a(e_a, e_j)_1 &= e_a \left[\tau k - \beta\left((2\tau - 1)\,k - (e_a - e_j)\right)\right] \\ &+ e_j \left[(1 - \tau)\,k - \beta\left(-\,(2\tau - 1)\,k - (e_a - e_j)\right)\right] - e_a^2 \end{aligned}$$

If $e_j - (2\tau - 1)\,k < e_a < e_j + (2\tau - 1)\,k$, the expected utility of agent a is:

$$\begin{aligned} EU_a(e_a, e_j)_2 &= e_a \left[\tau k - \beta\left((2\tau - 1)\,k - (e_a - e_j)\right)\right] \\ &+ e_j \left[(1 - \tau)\,k - \alpha\left((2\tau - 1)\,k + (e_a - e_j)\right)\right] - e_a^2 \end{aligned}$$

If $e_j + (2\tau - 1)\,k \leq e_a \leq 1$, the expected utility of agent a is:

$$\begin{aligned} EU_a(e_a, e_j)_3 &= e_a \left[\tau k - \alpha\left(-\,(2\tau - 1)\,k + (e_a - e_j)\right)\right] \\ &+ e_j \left[(1 - \tau)\,k - \alpha\left((2\tau - 1)\,k + (e_a - e_j)\right)\right] - e_a^2 \end{aligned}$$

The utility function of an inequity-averse agent a is then defined as follows, over the total range, $e_a \in [0, 1]$,

$$EU_a(e_a, e_j) = \begin{cases} EU_a(e_a, e_j)_1 & \text{if } 0 \leq e_a \leq e_j - (2\tau - 1)\,k \\ EU_a(e_a, e_j)_2 & \text{if } e_j - (2\tau - 1)\,k < e_a < e_j + (2\tau - 1)\,k \\ EU_a(e_a, e_j)_3 & \text{if } e_j + (2\tau - 1)\,k \leq e_a \leq 1 \end{cases}$$

The equilibrium effort of an inequity-averse agent, e_a^*, is determined by maximizing the agent's expected utility, which depends on the effort of the other group member, e_j

$$e_a^* \in \arg\max_{e_a} EU_a(e_a, e_j)$$

We then construct the best-response function of an inequity-averse agent a.

As the expected utility function of agent a is defined by three different functions, to determine the best-response function of this agent over the whole range, i.e. for $e_a \in [0, 1]$ and $e_j \in [0, 1]$, we determine his optimal effort under each function that makes up his expected utility. In addition, we determine the agent's optimal effort at every point where the function changes, i.e. when $EU_a(e_a, e_j)_1$ changes to $EU_a(e_a, e_j)_2$, and when $EU_a(e_a, e_j)_2$ changes to $EU_a(e_a, e_j)_3$.

We first introduce some notation.

$$A = \tfrac{k(\tau(1+2\alpha)-\alpha)}{2(1+\alpha)} \quad B = \tfrac{k(\tau(-3-2\alpha)+2+\alpha)}{2(1+\alpha)} \quad C = \tfrac{k(\tau(2\beta-3)-\beta+2)}{2+\alpha-\beta}$$
$$D = \tfrac{k(\tau(5-6\beta)+3\beta-2)}{2+\alpha-\beta} \quad E = \tfrac{k(\tau(5-6\beta)+3\beta-2)}{2(1-\beta)} \quad F = \tfrac{k(\tau(1-2\beta)+\beta)}{2(1-\beta)}$$

The calculations produce the following best-response function for agent a.

$$e_a^*(e_j) = \begin{cases} A & \text{if } 0 \leq e_j \leq B \\ e_j + (2\tau - 1)k & \text{if } B < e_j < C \\ \max\left\{0, \frac{k(\tau(1-2\beta)+\beta)-e_j(\alpha+\beta)}{2(1-\beta)}\right\} & \text{if } C \leq e_j \leq D \\ \max\{0, e_j - (2\tau-1)k\} & \text{if } D < e_j < E \\ F & \text{if } E \leq e_j \leq 1 \end{cases}$$

We check that the best-response function for an inequity-averse agent is continuous over the range. Consequently, the best-response function consists of five different linear functions. When $e_j = B$, then $e_a^* = A$; when $e_j = C$, then $e_a^* = \frac{k(\tau(1+2\alpha)-\alpha)}{2+\alpha-\beta}$; when $e_j = D$, then $e_a^* = \frac{k(\tau(1-2\alpha-4\beta)+\alpha+2\beta)}{2+\alpha-\beta}$, and when $e_j = E$, then $e_a^* = F$.[22] We introduce two more functions.

$$G = \tfrac{k(\tau(1+2\alpha)-\alpha)}{2+\alpha-\beta} \quad H = \tfrac{k(\tau(1-2\alpha-4\beta)+\alpha+2\beta)}{2+\alpha-\beta}$$

There is one particular case when $\tau = \frac{1}{2}$. At this value we have $A = B$, $C = D$ and $E = F$. We therefore distinguish two cases: $\tau \in \left(\frac{1}{2}, 1\right]$ and $\tau = \frac{1}{2}$.

As inequity-averse agents are identical in their degree of inequity aversion, the best-response function of an inequity-averse agent j is identical to that of agent a. If both group members are inequity averse, equilibrium effort is determined by the intersection of the two best-response functions, $e_a^*(e_j)$ and $e_j^*(e_a)$. If one agent is inequity averse and the other selfish, equilibrium effort is determined by the intersection of the best-response function of the inequity-averse agent, $e_a^*(e_j)$, and that of the selfish agent, $e_s^* = \frac{\tau k}{2}$. As the

[22]Certain values of α, β and τ produce special cases. For $\tau \in \left[\frac{2+\alpha}{3+2\alpha}, \frac{2-\beta}{3-2\beta}\right)$, we have $B < 0$: e_a^* is not equal to A but rather $e_j + (2\tau-1)k$ if $0 \leq e_j < C$. For $\tau \in \left[\frac{2-\beta}{3-2\beta}, 1\right]$, we have $B < 0$ and $C < 0$: $e_a^* = \frac{k(\tau(1-2\beta)+\beta)-e_j(\alpha+\beta)}{2(1-\beta)}$ if $0 \leq e_j \leq D$. Moreover, when $\tau > \frac{\alpha+2\beta}{2\alpha+4\beta-1}$ and $2\alpha+4\beta > 1$, then $e_a^* = 0$ if $\frac{k(\tau(1-2\beta+\beta))}{\alpha+\beta} \leq e_j \leq (2\tau-1)k$. This last case exists only if $\tau \in \left[\frac{2+\alpha}{3+2\alpha}, 1\right]$.

reaction function of inequity-averse agents is composed of five separate linear functions, once the intersection points are defined, we check whether the intersection point in question does indeed belong to the effort-level interval that was supposed at the beginning of the calculation. Only intersection points which satisfy this criterion are equilibria.

When $\tau \in \left(\frac{1}{2}, 1\right]$, we determine the equilibrium strategy of an inequity-averse agent who is first matched to another inequity-averse agent and then to a selfish agent.

There exists a symmetric equilibrium when the group is homogeneous, which is defined as follows:

$$(e_a^*, e_a^*) = \left(\frac{k\left(\tau\left(1-2\beta\right)+\beta\right)}{2+\alpha-\beta}, \frac{k\left(\tau\left(1-2\beta\right)+\beta\right)}{2+\alpha-\beta}\right)$$

This equilibrium exists for all values of the parameters α, β and τ, and is unique if $\alpha+3\beta<2$. For $\alpha+3\beta \geq 2$, this is still an equilibrium but other equilibria exist. They are defined such as

$$\left(e_a^*, e_j^*\right) = \begin{cases} \left(e_a^*, e_a^* - \left(2\tau-1\right)k\right) \text{ with } D \leq e_a^* \leq G & \text{if } \frac{1}{2} < \tau \leq \frac{\alpha+2\beta}{2\alpha+4\beta-1} \\ \left(e_a^*, e_a^* - \left(2\tau-1\right)k\right) \text{ with } \left(2\tau-1\right)k \leq e_a^* \leq G & \text{if } \frac{\alpha+2\beta}{2\alpha+4\beta-1} < \tau < \frac{2-\beta}{3-2\beta} \\ \left(\frac{k(\tau(1-2\beta)+\beta)}{2(1-\beta)}, 0\right) & \text{if } \frac{2-\beta}{3-2\beta} \leq \tau \leq 1 \end{cases}$$

To illustrate, Figure 4 depicts the figures of the best-response functions of two inequity-averse agents with $\alpha+3\beta<2$, i.e. when the group is homogeneous and only a symmetric equilibrium exists. We know that $A > B$, $E > F$ and $G > H$ when $\tau \in \left(\frac{1}{2}, 1\right]$. As $\alpha+3\beta<2$, we further have $D > G$ and $H > C$. Moreover, we assume $C > A$.

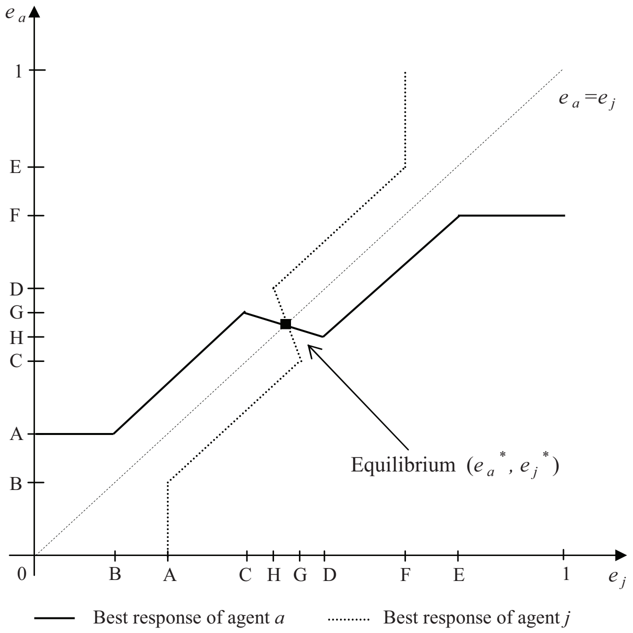

Figure 4: Best-response functions of two inequity-averse agents when $\tau \in \left(\frac{1}{2}, 1\right]$ and $\alpha + 3\beta < 2$

In a heterogeneous group, when the inequity-averse agent is matched to a selfish agent, the equilibrium is always unique because the selfish agent always has a dominant strategy: $e_s^* = \frac{\tau k}{2}$. This equilibrium is asymmetric for $\tau \in \left(\frac{1}{2}, 1\right]$. Depending on the values of α, β and τ, we have either $e_s^* \in [C, D]$ or $e_s^* \in (D, E]$. We have $e_s^* \in [C, D]$ if $\alpha + 11\beta \leq 8$ and $\tau > \frac{2(2-3\beta)}{8-\alpha-\beta}$, and $e_s^* \in (D, E]$ otherwise. This yields the following equilibrium when the group is heterogeneous.

$$(e_a^*, e_s^*) = \begin{cases} \left(\max\left\{0, \frac{k(\tau(2-\alpha-5\beta)+2\beta)}{4(1-\beta)}\right\}, \frac{\tau k}{2}\right) & \text{if } \alpha + 11\beta \leq 8 \text{ and } \tau > \frac{2(2-3\beta)}{8-\alpha-11\beta} \\ \left(\max\left\{0, \frac{k(-3\tau+2)}{2}\right\}, \frac{\tau k}{2}\right) & \text{otherwise} \end{cases}$$

Note that, whatever the values of α, β and τ, the equilibrium effort of an inequity-averse agent, e_a^*, is always less than that of a selfish agent, e_s^*. Moreover, the equilibrium effort of the inequity-averse agent is lower when $e_s^* \in (D, E]$ than when $e_s^* \in [C, D]$: effort is lower for higher values of e_s^*.

When $\tau = \frac{1}{2}$, we have $A = B = \frac{k}{4(1+\alpha)}$ and $E = F = \frac{k}{4(1-\beta)}$. The reaction function of an inequity-averse agent a in this case can easily be determined

$$e_a^*(e_j) = \begin{cases} \frac{k}{4(1+\alpha)} & \text{if } 0 \leq e_j \leq \frac{k}{4(1+\alpha)} \\ e_j & \text{if } \frac{k}{4(1+\alpha)} < e_j < \frac{k}{4(1-\beta)} \\ \frac{k}{4(1-\beta)} & \text{if } \frac{k}{4(1-\beta)} \leq e_j \leq 1 \end{cases}$$

When both group members are inequity averse, only multiple symmetric equilibria exist, defined as follows:

$$(e_a^*, e_a^*) \text{ with } \frac{k}{4(1+\alpha)} \leq e_a^* \leq \frac{k}{4(1-\beta)}$$

When it is common knowledge that the group is composed of inequity-averse agents, both inequity-averse agents exert the same effort level.

Figure 5 represents the best-response functions of two inequity-averse agents when $\tau = \frac{1}{2}$. The shape of these functions is always the same because we always have $A = B < C + D < E = F$ when $\tau = \frac{1}{2}$.

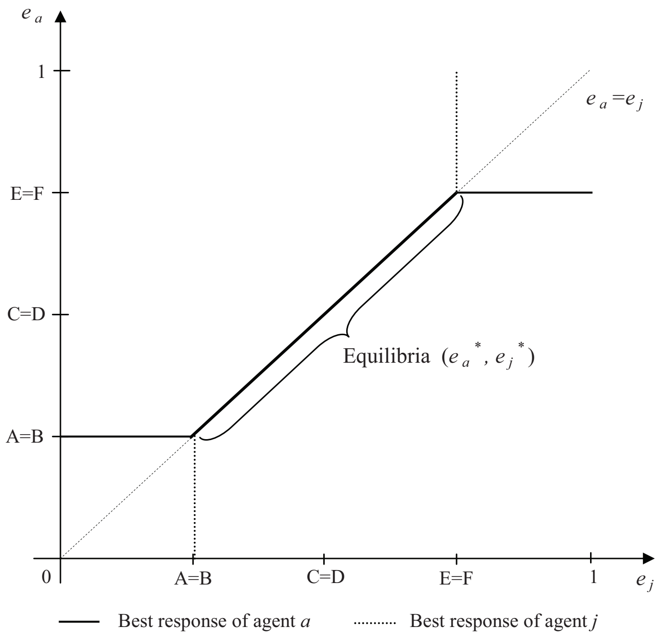

Figure 5: Best-response functions of two inequity-averse agents when $\tau = \frac{1}{2}$

We should make clear that the upper bound of the set of equilibria is not always equal to $\frac{k}{4(1-\beta)}$. In fact, the Pareto optimum of the revenue-sharing scheme is such that

$$\frac{\partial EU_\theta(e^{OP}, e^{OP})}{\partial e^{OP}} = 0 \Longleftrightarrow e^{OP} = \frac{k}{2} \qquad \text{with } \theta = \{a, s\}$$

An inequity-averse agent will then never provide effort greater than e^{OP} because, whatever the strategy of j, the agent's expected utility is decreasing in effort beyond the Pareto optimum (even without taking into account any reduction in utility due to inequity). The equilibrium effort level is upper bounded by $e_a^* = \frac{k}{2}$ when $\beta \in \left[\frac{1}{2}, 1\right)$. The set of symmetric equilibria (e_a^*, e_a^*) is then defined with e_a^* as

$$e_A^* \in \begin{cases} \left[\frac{k}{4(1+\alpha)}, \frac{k}{4(1-\beta)}\right] & \text{if } \beta \in \left[0, \frac{1}{2}\right) \\ \left[\frac{k}{4(1+\alpha)}, \frac{k}{2}\right] & \text{if } \beta \in \left[\frac{1}{2}, 1\right) \end{cases}$$

When the inequity-averse agent is matched with a selfish agent, there exists a unique equilibrium. A selfish agent has a dominant strategy, $e_s^* = \frac{\tau k}{2}$, that becomes $e_s^* = \frac{k}{4}$ when $\tau = \frac{1}{2}$. The unique equilibrium is therefore defined as follows:

$$(e_a^*, e_s^*) = \left(\frac{k}{4}, \frac{k}{4}\right)$$

The unique equilibrium then corresponds to free-riding behavior by both agents in the group.

Appendix B - Proof of Proposition 1

To establish the optimal contest structure of each agent type, we first determine the optimal τ over the range $\left(\frac{1}{2}, 1\right]$ and compare agents' expected utility under this optimal τ to their expected utility under $\tau = \frac{1}{2}$.

- *Selfish agents*

The expected utility of a selfish agent matched to another selfish agent is greater than that when he is matched to an inequity-averse agent.

$$EU_s\left(e_s^*, e_s^*\right) \geq EU_s\left(e_s^*, e_a^*\right) \qquad \forall \tau \in \left[\frac{1}{2}, 1\right]$$

Selfish agents then prefer to be matched to other selfish agents. A specific case occurs for $\tau = 1$, for which the expected utility of a selfish agent only depends on his own effort: in this case he is indifferent to the type of the other group member.

To determine the optimal τ of a selfish agent, we calculate the derivative of his expected utility, when both agents in the group are selfish, depending on τ.

$$\frac{\partial EU_s\left(e_s^*, e_s^*\right)}{\partial \tau} > 0 \qquad \forall \tau \in \left[\frac{1}{2}, 1\right]$$

The optimal value of τ for selfish agents is therefore $\tau_s^* = 1$.

The prize awarded to the contest winner is based on group output, and so increases in agents' effort. As the equilibrium effort exerted by a selfish agent is greatest when $\tau_s^* = 1$, this is the optimal structure for agents of this type.

- *Inequity-averse agents*

We know that $\tau = 1$ if an inequity-averse agent is matched to a selfish agent, as selfish agents only work under contracts with $\tau = 1$. We therefore look for the optimal τ, $\tau \in \left(\frac{1}{2}, 1\right]$, for an inequity-averse agent who is matched to someone similar. It immediately results that only the symmetric effort equilibrium can be considered here: the two inequity-averse agents at any asymmetric equilibrium will have diverse optimal contracts. The expected utility of the inequity-averse agent who exerts greater effort level in equilibrium increases in competition, i.e. in τ, while the expected utility of the low-effort agent falls in τ. The two inequity-averse agents therefore have a conflict of interest over the choice of τ. As firms compete via the payment scheme they offer to workers, these agents will never choose the same contract, so that the asymmetric equilibria cannot come about. We thus consider the optimal τ of an inequity-averse agent who exerts the effort of the symmetric equilibrium (e_a^*, e_a^*), with $e_a^* = \frac{k(\tau(1-2\beta)+\beta)}{2+\alpha-\beta}$.

$\frac{\partial EU_a(e_a^*, e_a^*)}{\partial \tau} = 0$ corresponds to a maximum and yields the optimal value of τ assuming $\tau \in \left(\frac{1}{2}, 1\right]$:

$$\tau = \frac{-2 + 5\beta + 3\beta^2 - 4\beta^3 + \alpha^2\left(4\beta - 1\right) + \alpha\left(10\beta - 3\right)}{2\left(2\beta - 1\right)\left(1 + 4\alpha + 2\alpha^2 + 2\beta - 2\beta^2\right)}$$

$\tau = \frac{-2+5\beta+3\beta^2-4\beta^3+\alpha^2(4\beta-1)+\alpha(10\beta-3)}{2(2\beta-1)(1+4\alpha+2\alpha^2+2\beta-2\beta^2)}$ exists for $0 \leq \beta < \frac{1}{6}$ and $\alpha < \frac{1}{2}\sqrt{5 - 16\beta\left(1 - \beta\right)} - \frac{1}{2} - \beta$. Hence inequity-averse agents with $\beta \geq \frac{1}{6}$ or $\alpha \geq \frac{1}{2}\sqrt{5 - 16\beta\left(1 - \beta\right)} - \frac{1}{2} - \beta$ maximize their expected utility under the contract $\tau = \frac{1}{2}$.

For simplicity, we denote $T = \frac{-2+5\beta+3\beta^2-4\beta^3+\alpha^2(4\beta-1)+\alpha(10\beta-3)}{2(2\beta-1)(1+4\alpha+2\alpha^2+2\beta-2\beta^2)}$.

We now determine whether agents with $0 \leq \beta < \frac{1}{6}$ and $\alpha < \frac{1}{2}\sqrt{5 - 16\beta\left(1 - \beta\right)} - \frac{1}{2} - \beta$ prefer to work under $\tau = T$ or $\tau = \frac{1}{2}$. As there are multiple equilibria under $\tau = \frac{1}{2}$, we consider whether agents' expected utility is higher under $\tau = T$ or $\tau = \frac{1}{2}$ for the maximum equilibrium effort that they can attain.[23]

$EU_a\left(e_a^*, e_a^*\right)_{\tau=T} > EU_a\left(e_a^*, e_a^*\right)_{\tau=\frac{1}{2}}$ for α and β not too high. The optimal contract of inequity averse agents is therefore

$$\tau_a^* = \begin{cases} T & \text{if } \alpha < \alpha^*\left(\beta\right) \\ \frac{1}{2} & \text{if } \alpha \geq \alpha^*\left(\beta\right) \end{cases}$$

with $\alpha^*\left(\beta\right)$ being the solution of $EU_a\left(e_a^*, e_a^*\right)_{\tau=T} - EU_a\left(e_a^*, e_a^*\right)_{\tau=\frac{1}{2}} = 0$. The values of $\alpha^*\left(\beta\right)$ are indicated in the following table to give a clear idea of them due to the complexity of the solution; $\alpha^*\left(\beta\right) = \frac{2\left(-1+4\beta-2\beta^2-2\beta^3\right)+\sqrt{2(-1+2\beta)^5(-3+4\beta)}}{2(1-6\beta+6\beta^2)}$.

β	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.087	0.09
$\alpha^*\left(\beta\right)$	0.225	0.209	0.193	0.177	0.161	0.145	0.130	0.114	0.098	0.087	-

The last step is to determine whether inequity-averse agents prefer to work under their optimal contract, τ_a^*, with an agent who has the same inequity aversion, or under $\tau_s^* = 1$ with a selfish agent.

We find that $EU_a\left(e_a^*, e_a^*\right)_{\tau=T} > EU_a\left(e_a^*, e_s^*\right)_{\tau=1}$ always holds. This means that an inequity-averse agent with $\alpha < \alpha^*\left(\beta\right)$ always prefers to work with an agent with the same preferences rather than with a selfish agent.

$EU_a\left(e_a^*, e_a^*\right)_{\tau=\frac{1}{2}} > EU_a\left(e_a^*, e_s^*\right)_{\tau=1}$ always holds for some values of e_A^*. Therefore, via forward induction, inequity-averse agents with $\alpha \geq \alpha^*\left(\beta\right)$ always prefer to be matched with someone who is also inequity averse.

[23]This assumption is made via forward induction. Suppose there exists an agent who has greater expected utility under $\tau = \frac{1}{2}$ than under $\tau = T$ only for sufficiently high equilibrium effort under $\tau = \frac{1}{2}$. Knowing that this agent maximizes his expected utility, he signals to other agents that he can sustain a sufficiently high equilibrium by choosing to work under $\tau = \frac{1}{2}$ instead of $\tau = T$. Therefore, an agent prefers to work under $\tau = \frac{1}{2}$ as long as his expected utility for his maximum equilibrium effort is higher under $\tau = \frac{1}{2}$ than under $\tau = T$.

Appendix C - Optimal contest structure under incomplete information

To determine the optimal contract of selfish agents under incomplete information, we analyze how the selfish agents' expected utility varies with τ in equilibrium.

$$\frac{\partial EU_s^I\left(e_s^{I*}, e_a^{I*}\right)}{\partial \tau} > 0 \qquad \forall \tau \in \left(\frac{1}{2}, 1\right]$$

We have analytically solved this inequality, but due to the complexity of the solution we only present here the graphs depicting the equilibrium expected utility of selfish agents as a function of τ and ρ for two different degrees of inequity aversion (Figures 6 and 7). It is clear that selfish agents maximize their equilibrium expected utility for $\tau = 1$ when $\tau \in \left(\frac{1}{2}, 1\right]$, whatever the share of inequity-averse agents in the population. The optimal contract for selfish agents is thus $\tau_s^* = 1$.

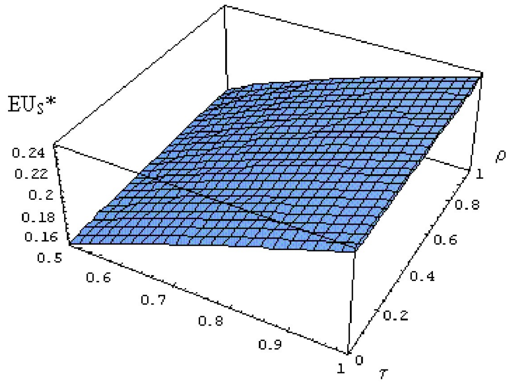

Figure 6: The expected utility of selfish agents when $\alpha = 0.5$ and $\beta = 0.1$

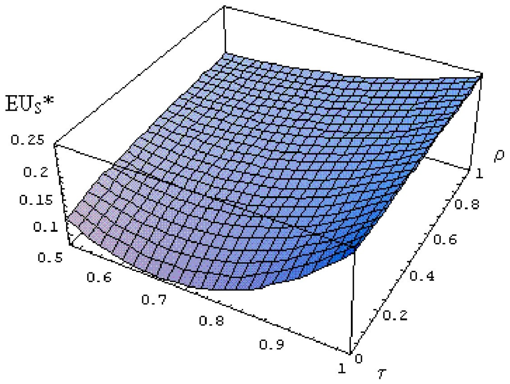

Figure 7: The expected utility of selfish agents when $\alpha = 1$ and $\beta = 0.6$

We therefore know that if a contract with $\tau \in \left(\frac{1}{2}, 1\right]$ exists at the equilibrium, at least one equilibrium contract will be the optimal contract for selfish agents, which implies a very competitive contest structure, $\tau_s^* = 1$.

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