Progressive Income Taxes as Built-In Stabilizers*

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December 1, 2004

* The authors would like to thank, without implicating, Cecilia García-Peñalosa, Thomas Piketty and Alain Trannoy for useful comments and suggestions.
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Abstract

This paper studies how progressive income taxes may reduce aggregate volatility by protecting the economy against expectation-driven fluctuations. Our main conclusion is that for sensible parameter values, fully eliminating sunspot and cyclical equilibria requires large progressivity: stabilizing consumption and labor supply movements requires low volatility of after-tax income, which in turn imposes close-to-maximal tax progressivity. We formalize this argument within two benchmark models and show how the power of progressive income taxes as automatic stabilizers depends on the tax base. In the first setting, we show that progressive taxes should be applied to labor income in a heterogeneous agents economy in which wage income mostly finance consumption, so as to rule out local indeterminacy and bifurcations. On the contrary, progressive taxes should be applied to capital income in an OLG economy where consumption comes from savings income. Incidentally, the latter results suggest that capital income taxes may be desirable, when progressive, to make expectation-driven fluctuations less likely.

Keywords: progressive income taxes, business cycles, sunspots, endogenous cycles.


1 Introduction

Following a tradition that dates back, at least, to Musgrave [22], one tends to view redistribution and stabilization as distinct goals of public intervention. However, it is expected that taxes affect both objectives through progressivity (or regressivity), which is a prominent feature of tax schedules prevailing in OECD countries, e.g. in the U.S. or European countries (see, for instance, the Statistical Abstracts of the United States, or Tax and the Economy: A Comparative Assessment of OECD Countries, OECD Tax Policy Studies n. 6, 2002), and in developing countries. Standard arguments (e.g. in textbooks) often stress that taxes may operate as automatic stabilizers. However, linear taxes are expected to have almost no stabilizing effects in dynamic general equilibrium models (this is indeed the case in both models studied in the present paper, as discussed below). Therefore, any impact of taxes on aggregate volatility should be associated with non-linear
features of tax codes, such as progressivity or regressivity.

The purpose of this paper is to study how progressive income taxes may reduce aggregate volatility by protecting the economy against expectation-driven fluctuations (e.g. sunspot or cyclical equilibria). Our main conclusion is that for sensible parameter values, fully eliminating sunspot and cyclical equilibria requires large progressivity. The main mechanism at the heart of our results is the following. When agents have optimistic expectations, they want to raise their consumption and, accordingly, they devote a higher fraction of their time endowment to work so as to increase their income. It follows that stabilizing consumption and labor supply movements requires low volatility of after-tax income, which in turn imposes close-to-maximal tax progressivity.

From this basic observation, one concludes that the power of progressive income taxes as automatic stabilizers should depend on the mechanisms leading to expectation-driven volatility and, consequently, on the tax base. In other words, if consumption is mostly financed by wage (resp. capital) income, then progressive taxes should be applied to wage (resp. capital) income. We formalize this argument within two benchmark models and show how the stabilizing power of progressive income taxes depends on the tax base. We first show that progressive taxes should be deducted from labor income in a heterogeneous agents economy with segmented asset markets in which wage income mostly finances consumption. On the contrary, we demonstrate that progressive taxes should be applied to capital income in an overlapping generations (thereafter OLG) economy where consumption comes from savings income. In both models, progressive income taxes are shown to reduce the range of parameter values that are compatible with local indeterminacy and bifurcations. In particular, the steady state turns out to be locally determinate if income tax progressivity is larger than some threshold value. For sensible parameter values, it turns out that the threshold level of tax progressivity is, in the OLG model, half as high as the corresponding bound in the heterogeneous agents model. Our computations (based on Stephenson [28, Table 1, p. 391]), however, lead us to conclude that tax progressivity in the US is probably smaller than the thresholds predicted by both models.

The two settings that we consider are admittedly two limiting cases, as consumption is financed by labor
income in the first model and capital income in the second one. Obviously, we have in mind the more realistic configuration that lies in between these two extremes but is closer to the former one. Our conclusions clearly predict that tax progressivity should operate mainly through labor income taxes when consumption relies primarily on wage income, as seems to be the case in OECD countries. However, a slightly progressive tax rate on capital income may still be desirable, as long as non-wage income partially finances consumption expenditures, to make expectation-driven business cycles less likely. To our knowledge, this justification for taxing capital income has not been noticed by the literature. Of course, the historical record shows that stabilization concerns were not at the origin of income tax progressivity in advanced countries, about a century ago. However, our analysis indicates that this does not imply that progressive tax schedules do not have, de facto, some stabilizing properties. On the other hand, our analysis also shows that progressive taxes are inefficient to rule out endogenous fluctuations when consumption is financed, even partially, by the returns from financial assets that remain untaxed (e.g. in the monetary economy studied by Benhabib and Laroque [5]).

The rest of the paper is organized as follows. The next section puts into perspective this work by discussing the related literature. Section 3 presents the heterogeneous agents economy with segmented asset markets and discusses how progressive labor income taxes make expectation-driven fluctuations less likely. Section 4 introduces progressive capital income taxes and shows how this feature may rule out cycles and sunspots. Some concluding remarks and directions of future research are gathered in Section 5.

2 Related Literature

This paper attempts to combine two strands of the recent literature. On one hand, Schmitt-Grohe and Uribe [27], Christiano and Harrison [10], Utaka [29], Pintus [23] have studied how given tax policies may protect (resp. expose) the economy against (resp. to) expectation-driven fluctuations (see also a related paper by Aloï, Lloyd-Braga and Whitta-Jacobsen [2] which abstracts from the presence of capital). However, all
these authors consider linear taxation (at the individual’s level). In contrast, Guo and Lansing [14, 15], Guo [12], Guo and Harrison [13] consider non-linear tax rates, although Guo [12] only assumes that capital and labor taxes have different statutory, progressivity features. We emphasize that, in contrast with the latter analysis, our results do not rely on either specifications for preferences or technology, numerical values or simulations. More importantly, our results stress the fact that identifying the relevant tax base is key when assessing the power of progressive taxes as built-in stabilizers. Labor supply movements reacting to waves of optimism or pessimism is here a key mechanism that may lead to sunspots and cycles, as in many papers in the literature (e.g. Guo and Lansing [14, 15], Guo [12], Guo and Harrison [13]). However, what the present paper does, as we have argued in the previous section, is to underline the power of progressive taxes on factor incomes to reduce consumption volatility, which is hardly relevant in representative agent models where total income is supposed to finance consumption. Finally, differently from Guo and Lansing [14, 15], Guo [12], Guo and Harrison [13], all models studied in this paper have constant returns to scale (that is, externalities and imperfect competition are absent), which makes stabilization policies more desirable a priori to improve welfare.

The literature discussed above consider decentralized equilibria that summarize the behavior of a representative agent, thereby ignoring the possible conflicting interests over the role of taxes. In contrast, Judd [16, 17], Kemp et al. [18], Alesina and Rodrik [1], Sarte [26], Lansing [20], Saez [25] study the redistributive and growth effects of taxes. However, differently from these authors, we focus on the stabilizing power of progressive taxation. Our analysis also touches upon the much debated question of capital income taxation, as we show that capital taxes (however small) may be desirable to stabilize the economy. To our knowledge, this argument has not been noticed in the literature and it contrasts with most of the current literature suggesting that capital income taxes should be zero (see Chamley [9], Judd [16]). In contrast with most papers belonging to this strand of the literature, our two settings allow for elastic labor. Moreover, money is present in the first model (originally proposed by Woodford [30]) and it is held by a fraction of agents. These two features are certainly plausible. On one hand, there is a large evidence showing that labor employment
moves at all frequencies in response to changes in real wages (moreover, theory also argues in favor of elastic labor; see, for instance, Boldrin and Horvath [6]). On the other hand, 59% of U.S. households did not hold, in 1989, any interest bearing assets (Mulligan and Sala-i-Martin [21]). Therefore, incorporating elastic labor and money, as a dominated asset, seems to provide relevant extensions of the current literature (see also, Judd [16, 17] on elastic labor). Moreover, the first setting studied in this paper is indeed very close to a commonly used framework in the public finance literature studying redistributive taxation (see, for instance, Judd [16, 17], Kemp et al. [18], Alesina and Rodrik [1], Lansing [20]).

3 Stabilization Through Progressive Labor Income Taxes in a Heterogeneous Agents Economy with Segmented Asset Markets

3.1 Government, Households, and Intertemporal Equilibria

A unique good is produced in the economy by combining labor \( l_t \geq 0 \) and the capital stock \( k_{t-1} \geq 0 \) resulting from the previous period. Production exhibits constant returns to scale, so that output is given by:

\[
F(k, l) \equiv Alf(a),
\]

where \( A \geq 0 \) is a scaling parameter and the latter equality defines the standard production function in intensive form defined upon the capital labor ratio \( a = k/l \). On technology, we shall assume the following.

Assumption 3.1

The intensive production function \( f(a) \) is continuous for \( a \geq 0 \), \( C^r \) for \( a > 0 \) and \( r \) large enough, with \( f'(a) > 0 \) and \( f''(a) < 0 \).
Competitive firms take real rental prices of capital and labor as given and determine their input demands by equating the private marginal productivity of each input to its real price. Accordingly, the real competitive equilibrium wage is:

\[ \omega = \omega(a) \equiv A[f(a) - af'(a)], \]

(2)

while the real competitive gross return on capital is:

\[ R = \rho(a) + 1 - \delta \equiv Af'(a) + 1 - \delta, \]

(3)

where \(0 \leq \delta \leq 1\) is the constant depreciation rate for capital.

Government is assumed to finance public expenditures that do not affect private decisions by taxing factor incomes. We first assume that the tax rate \(\tau(x)\) on income \(x\) is such that after-tax income is as follows:

\[ [1 - \tau(x)]x \equiv \phi(x). \]

(4)

In this formulation, there are two benchmark cases. When \(\phi(x)\) is proportional to \(x\), then \(\phi\) has unitary elasticity and taxes are linear. Decreasing the elasticity of \(\phi(x)\) from one (when taxes are linear) to zero may be interpreted as increasing tax progressivity. More precisely, one can postulate the following (see for example Lambert [19, chap. 7-8]).

**Assumption 3.2**

*After-tax income \(\phi(x)\) is a continuous, positive function for \(x \geq 0\), with \(\phi(x) \leq x\), \(\phi'(x) > 0\) and \(\phi''(x) \leq 0\), for \(x > 0\). Moreover, the income tax exhibits weak progressivity, that is, \(\phi(x)/x\) is non-increasing for \(x > 0\) or, equivalently, \(\psi(x) \equiv x\phi'(x)/\phi(x) \leq 1\).*

Then \(\pi(x) \equiv 1 - \psi(x)\) is a measure of tax progressivity. In particular, the tax schedule is linear when \(\pi(x) = 0\), or \(\psi(x) = 1\), for \(x > 0\), and the higher \(\pi(x)\), the more progressive the tax schedule.

One can reinterpret the condition \(\psi(x) \leq 1\) as the property that the marginal tax rate \(\tau_m = \partial(\tau x)/\partial x\) is larger than the average tax rate \(\tau\): it is easily shown that \(\tau_m - \tau = \phi(x)/x - \phi'(x)\) so that \(\tau_m \geq \tau\) when
ψ(x) ≤ 1 or π(x) ≥ 0 for all positive x. Moreover, tax progressivity is naturally measured by $π \equiv 1 - ψ$ when one notes that $π = (τ_m - τ)/(1 - τ)$.

A benchmark case of interest to our analysis has been studied by Guo and Lansing [14]: they restrict to the case $τ(x) = 1 - η(x)^{-π}$ where $0 < η < 1$ is a scaling parameter while $π < 1$ is the progressivity parameter which is constant in that case. That is, marginal tax rate is higher (resp. lower) than average tax rate when $π$ is positive (resp. negative). Therefore, this tax system exhibits progressivity when $π$ is positive, and regressivity when $π$ is negative, whereas the tax rate $τ = 1 - η$ is constant when $π = 0$. As in Guo and Lansing [14], we assume that households take into account how the tax rate affects their earnings.

To complete the description of the model, we now characterize the behavior of both classes of agents. A representative worker solves the following utility optimization problem, as derived in Appendix A:

$$\text{maximize } \{V_2(c_{t+1}^w/B) - V_1(l_t)\} \text{ such that } p_{t+1}c_{t+1}^w = p_tφ(ω_t l_t), c_{t+1}^w ≥ 0, l_t ≥ 0,$$

where $B > 0$ is a scaling factor, $c_{t+1}^w$ is next period consumption, $l_t$ is labor supply, $p_{t+1} > 0$ is next period price of output (assumed to be perfectly foreseen), $ω_t > 0$ is real wage, and $φ$ is after-tax wage income, as described in Assumption 3.2. We consider the case such that leisure and consumption are gross substitutes and assume therefore the following:

**Assumption 3.3**

The utility functions $V_1(l)$ and $V_2(c)$ are continuous for $0 ≤ l ≤ l^*$ and $c ≥ 0$, where $l^* > 0$ is the (maybe infinite) workers’ endowment of labor. They are $C^r$ for, respectively, $0 < l < l^*$ and $c > 0$, and $r$ large enough, with $V'_1(l) > 0$, $V''_1(l) > 0$, $\lim_{l \to l^*} V_1'(l) = +∞$, and $V'_2(c) > 0$, $V''_2(c) > 0$, $-cV''_2(c) < V'_2(c)$ (that is, consumption and leisure are gross substitutes).

The first-order condition of the above program (5) gives the optimal labor supply $l_t > 0$ and the next period consumption $c_{t+1}^w > 0$, which can be stated as follows.

$$v_1(l_t) = ψ(ω_t l_t)v_2(c_{t+1}^w) \text{ and } p_{t+1}c_{t+1}^w = p_tφ(ω_t l_t),$$
where \( v_1(l) = lV'(l) \) and \( v_2(c) = cV_2'(c/B)/B \). Assumptions 3.2 and 3.3 implies that \( v_1 \) and \( v_2 \) are increasing while \( v_1 \) is onto \( \mathbb{R}_+ \). Therefore, Assumption 3.3 allows one to define, from Eqs. (6), \( \gamma \equiv v_2^{-1} \circ [v_1/\psi] \) (whose graph is the offer curve), which is a monotonous, increasing function only if the elasticity of \( \psi \) is either negative or not too large when positive.

Capitalists maximize the discounted sum of utilities derived from each period consumption. They consume \( c_t^c \geq 0 \) and save \( k_t \geq 0 \) from their income, which comes exclusively from real gross returns on capital. To fix ideas, we assume, following Woodford [30], that capitalists’ instantaneous utility function is logarithmic. Their optimal choices are then:

\[
c_t^c = (1 - \beta)R_t k_{t-1}, \quad k_t = \beta R_t k_{t-1},
\]

(7)

where \( 0 < \beta < 1 \) is the capitalists’ discount factor and \( R_t > 0 \) is the real gross rate of return on capital.

As usual, equilibrium on capital and labor markets is ensured through Eqs. (2) and (3). Since workers save their wage income in the form of money, the equilibrium money market condition is:

\[
(1 - \tau_t)\omega(a_t)l_t = M_t/p_t,
\]

(8)

where \( M_t \geq 0 \) is money supply and \( p_t \) is current nominal price of output. In this section, we assume that only labor income is taxed (capital taxation is studied in the next section) and that the proceeds are used to produce public goods that do not affect either technology or preferences:

\[
g_t = \tau_t \omega l_t = \omega_l l_t - \phi(\omega_l l_t).
\]

(9)

Finally, Walras’ law accounts for the equilibrium in the good market, that is, \( A(t)(a_t) = c_t^w + c_t^c + k_t - (1 - \delta)k_{t-1} + g_t \). From the equilibrium conditions in Eqs. (2), (3), (6), (7), (8), (9), one easily deduces that the variables \( c_t^{w+1}, c_t^w, l_t, p_{t+1}, p_t, c_t^c \) and \( k_t \) are known once \( (a_t, k_{t-1}) \) are given. This implies that intertemporal equilibria may be summarized by the dynamic behavior of both \( a \) and \( k \).

**Definition 3.1**

An intertemporal perfectly competitive equilibrium with perfect foresight is a sequence \( (a_t, k_{t-1}) \) of \( \mathbb{R}^2_+ \),
\[ t = 1, 2, \ldots, \text{such that} \]
\[
\begin{aligned}
& v_2(\phi(\omega(a_{t+1})k_t/a_{t+1})) = v_1(k_{t-1}/a_t)/\psi(\omega(a_t)k_{t-1}/a_t), \\
& k_t = \beta R(a_t)k_{t-1}.
\end{aligned}
\]

(10)

Note that the distribution of assets does not degenerate in this framework, as different agents hold money and capital (in contrast with Becker [3]).

In view of Eqs. (10) and recalling that \( a = k/l \), the nonautarkic steady states are the solutions \((\pi, \bar{\pi})\) in \( R_{++}^2 \) of \( v_2(\phi(\omega(\pi)))/v_1(1)/\psi(\omega(\pi)) = v_1(1)/\psi(1) \) and \( \beta R(\pi) = 1 \). Equivalently, in view of Eq. (3), the steady states are given by:

\[
\begin{aligned}
& v_2(\phi(\omega(\pi))) = v_1(\bar{\pi})/\psi(\omega(\pi)), \\
& \rho(\pi) + 1 - \delta = 1/\beta.
\end{aligned}
\]

(11)

We shall solve the existence issue by choosing appropriately the scaling parameters \( A \) and \( B \), so as to ensure that one stationary solution coincides with, for instance, \((\pi, \bar{\pi}) = (1, 1)\). For sake of brevity, the proof is given in Appendix B.

**Proposition 3.1 (Existence of a Normalized Steady State)**

Under Assumptions 3.1, 3.2 and 3.3, \( \lim_{c \to 0} cV_2(c) < V_1(1)/\psi(1) < \lim_{c \to +\infty} cV_2(c) \), \((\pi, \bar{\pi}) = (1, 1)\) is a steady state of the dynamical system in Eqs. (10) if and only if \( A = (1/\beta - 1 + \delta)/f'(1) \) and \( B \) is the unique solution of \( \psi(1)\phi(1)V_2(\phi(1))/B = V_1(1) \).

**Proof:** See Appendix B.

### 3.2 Ruling Out Local Indeterminacy Through Tax Progressivity

We now study the dynamics of Eqs. (10) around one of its interior stationary points \((\pi, \bar{\pi})\). These equations define locally a dynamical system of the form \((a_{t+1}, k_t) = G(a_t, k_{t-1})\) if the derivative of \( \omega(a)/a \) with respect
to $a$ does not vanish at the steady state, or equivalently if $\varepsilon_\omega(\overline{a}) - 1 \neq 0$, where the notation $\varepsilon_\omega$ stands for the elasticity of $\omega(a)$ evaluated at the steady state under study. Then, the usual procedure to study the local stability of the steady states is to use the linear map associated to the Jacobian matrix of $G$, evaluated at the fixed point under study.

We assume that, in the neighborhood of the steady state that has been conveniently normalized by the procedure in Proposition 3.1, $\phi$ has a constant elasticity $\psi = 1 - \pi$ with $0 \leq \pi < 1$. Our reason for restricting the analysis to this benchmark is twofold. Most importantly, economic theory does not place strong restrictions on how the elasticity of after-tax income $\psi(x)$ varies with pre-tax income $x$ (see e.g. Lambert [19]). Therefore, we choose to be parcimonious and introduce tax progressivity through a single parameter, that is, $\pi = 1 - \psi$. As it will soon appear, the following analysis could be easily adapted to account for a non-constant elasticity. Straightforward computations yield the following proposition.

**Proposition 3.2 (Linearized Dynamics around a Steady State)**

Under the assumptions of Proposition 3.1, suppose that $\phi$ has constant elasticity in the neighborhood of the steady state $(\overline{a}, \overline{k})$ of the dynamical system in Eqs. (10), i.e. $\psi(x) = 1 - \pi$, with $0 \leq \pi < 1$ measuring labor income tax progressivity. Let $\varepsilon_R, \varepsilon_\omega, \varepsilon_\gamma$ be the elasticities of the functions $R(a), \omega(a), \gamma(l)$, respectively, evaluated at the steady state $(\overline{a}, \overline{k})$ and assume that $\varepsilon_\omega \neq 1$. The linearized dynamics for the deviations $da = a - \overline{a}, dk = k - \overline{k}$ are determined by the linear map:

\[
\begin{cases}
  da_{t+1} = -\frac{\varepsilon_\gamma}{\varepsilon_\omega - 1} da_t + \frac{\varepsilon_R}{\varepsilon_\omega - 1} dk_{t-1}, \\
  dk_t = \frac{\varepsilon_R}{\varepsilon_\omega - 1} da_t + dk_{t-1}.
\end{cases}
\]

The associated Jacobian matrix evaluated at the steady state under study has trace $T$ and determinant $D$,

\[
T = T_1 - \frac{\varepsilon_\gamma - 1}{(1 - \pi)(\varepsilon_\omega - 1)}, \quad \text{with} \quad T_1 = 1 + \frac{|\varepsilon_R| - 1/(1 - \pi)}{\varepsilon_\omega - 1},
\]

\[
D = \varepsilon_\gamma D_1, \quad \text{with} \quad D_1 = \frac{|\varepsilon_R| - 1}{(1 - \pi)(\varepsilon_\omega - 1)}.
\]
Moreover, one has \( T_1 = 1 + D_1 + \Lambda \), where \( \Lambda = \frac{-\pi|\varepsilon_R|}{(1 - \pi)(\varepsilon_\omega - 1)} \).

We shall assume throughout that a steady state exists in the whole range of parameter values that will be considered. To fix ideas, we may assume without loss of generality that the steady state has been normalized at \((\pi, \kappa) = (1, 1)\) (see Proposition 3.1).

Now fix the technology (i.e. \( \varepsilon_R \) and \( \varepsilon_\omega \)), at the steady state, and vary the parameter representing workers’ preferences \( \varepsilon_\gamma > 1 \). In other words, consider the parametrized curve \((T(\varepsilon_\gamma), D(\varepsilon_\gamma))\) when \( \varepsilon_\gamma \) describes \((1, +\infty)\). Direct inspection of the expressions of \( T \) and \( D \) in Proposition 3.2 shows that this locus is a half-line \( \Delta \) that starts close to \((T_1, D_1)\) when \( \varepsilon_\gamma \) is close to 1, and whose slope is \( 1 - |\varepsilon_R| \), as shown in Fig. 1. The value of \( \Lambda = T_1 - 1 - D_1 \), on the other hand, represents the deviation of the generic point \((T_1, D_1)\) from the line \((AC)\) of equation \( D = T - 1 \), in the \((T, D)\) plane.

The main task we now face is locating the half-line \( \Delta \) in the plane \((T, D)\), i.e. its origin \((T_1, D_1)\) and its slope \( 1 - |\varepsilon_R| \), as a function of the parameters of the system. The parameters we shall focus on are the depreciation rate for the capital stock \( 0 \leq \delta \leq 1 \), the capitalist’s discount factor \( 0 < \beta < 1 \), the share of capital in total income \( 0 < s = \pi \rho(\pi)/f(\pi) < 1 \), the elasticity of input substitution \( \sigma = \sigma(\pi) > 0 \), and tax progressivity \( 0 \leq \pi < 1 \), all evaluated at the steady state \((\pi, \kappa)\) under study. In fact, it is not difficult to get the following expressions.

\[
\begin{align*}
D_1 &= (\theta(1 - s) - \sigma)/[(1 - \pi)(s - \sigma)], \quad \Lambda = -\pi\theta(1 - s)/[(1 - \pi)(s - \sigma)], \\
T_1 &= 1 + D_1 + \Lambda, \quad \text{slope}_\Delta = 1 - \theta(1 - s)/\sigma,
\end{align*}
\]

where \( \theta \equiv 1 - \beta(1 - \delta) > 0 \) and all these expressions are evaluated at the steady state under study.
Our aim now is to locate the half-line $\Delta$, i.e. its origin $(T_1, D_1)$ and its slope in the $(T, D)$ plane when the capitalists’ discount rate $\beta$, as well as the technological parameters $\delta, s$, and the level of tax progressivity $\pi$ at the steady state are fixed, whereas the elasticity of factor substitution $\sigma$ is made to vary. One easily show that the benchmark economy with constant tax rate $\pi = 0$ (or $\psi = 1$) is equivalent to the no-taxation case studied by Grandmont, Pintus, and de Vilder [11]: the origin $(T_1, D_1)$ of $\Delta$ is located on the line $(AC)$, i.e. $\Lambda \equiv 0$ (see Fig. 1). The immediate implication of the resulting geometrical representation is that indeterminacy and endogenous fluctuations emerge only for low values of $\sigma (\sigma < \sigma_I$, and indeed for $\sigma$ less than $s$, the share of capital in output) while, on the contrary, local determinacy is bound to prevail for larger values of $\sigma$. One corollary of this is that linear tax rates on wage income do not affect the range of parameter values that are associated with local indeterminacy and bifurcations.

We now show that the more progressive labor income taxes (the higher $\pi$), the less likely the half-line $\Delta$ is to cross the indeterminacy triangle $ABC$, as shown in Figs. 2-4. More precisely, we now show that there exists a minimal level of tax progressivity $\pi_{\text{min}}$ (illustrated in Fig. 5) above which $\Delta$ does not intersect $ABC$: the steady state is either a saddle or a source so that there exists a neighborhood in which no stable endogenous fluctuations occur (see Fig. 6).

The key implication of increasing progressivity $\pi$ from zero can be seen, starting with the benchmark case with linear taxes (see Fig. 1), by focusing on how the two following points vary with $\pi$ (see Figs. 2-5). First, direct inspection of Eqs. (13) shows that $\Lambda$ (the deviation of $(T_1, D_1)$ from $(AC)$) is negative when $\sigma$ is small enough (that is, $T_1 < 1 + D_1$ when $\sigma < s$). In fact, the locus of $(T_1, D_1)$ generated when $\sigma$ increases from zero describes a line $\Delta_1$ which intersects $(AC)$ at point $I$ when $\sigma = +\infty$ (i.e. $\Lambda = 0$). From Eqs. (13), one immediately sees that $D_1(\sigma = +\infty)$ increases with $\pi$, so that point $I$ goes north-east when $\pi$ increases from zero. Second, Eqs. (13) imply that $\Delta_1$ intersects the $T$-axis of equation $D = 0$ when $\sigma = \theta(1 - s)$ (that is, $D_1 = 0$), and that $\Lambda(\sigma = \theta(1 - s))$ decreases, from zero, with $\pi$. An equivalent way of summarizing these two observations is that, when $\pi$ increases from zero, point $I$ (where $\Delta_1$ intersects $(AC)$) goes north-east, along $(AC)$, whereas the slope of $\Delta_1$ decreases from one, so that three different configurations occur in the
(T, D) plane (see Figs. 2-4).

**Proposition 3.3 (Local Stability and Bifurcations of the Steady State)**

Consider a steady state that is assumed to be set at \((\pi, K) = (1, 1)\) through the procedure in Proposition 3.1. If, moreover, \(\theta(1 - s) < s\) and \(\pi < \pi_{\text{min}}\) (that is, tax progressivity is not too large), the following generically holds.

1. If \(0 \leq \pi < 1 - \theta(1 - s)/s\), that is, tax progressivity is small enough (see Fig. 2):

   (a) \(0 < \sigma < \sigma_F\): the steady state is a sink for \(1 < \varepsilon_\gamma < \varepsilon_{\gamma H}\), where \(\varepsilon_{\gamma H}\) is the value of \(\varepsilon_\gamma\) for which \(\Delta\) crosses \([BC]\). Then the steady state undergoes a Hopf bifurcation (the complex characteristic roots cross the unit circle) at \(\varepsilon_\gamma = \varepsilon_{\gamma H}\), and is a source when \(\varepsilon_\gamma > \varepsilon_{\gamma H}\).

   (b) \(\sigma_F < \sigma < \sigma_H\): the steady state is a sink when \(1 < \varepsilon_\gamma < \varepsilon_{\gamma H}\). Then the steady state undergoes a Hopf bifurcation at \(\varepsilon_\gamma = \varepsilon_{\gamma H}\) and is a source when \(\varepsilon_\gamma > \varepsilon_{\gamma H}\).

   (c) \(\sigma_H < \sigma < \sigma_I\): the steady state is a sink when \(1 < \varepsilon_\gamma < \varepsilon_{\gamma F}\). A flip bifurcation occurs at \(\varepsilon_\gamma = \varepsilon_{\gamma F}\) and the steady state is a saddle if \(\varepsilon_\gamma > \varepsilon_{\gamma F}\).

   (d) \(\sigma_I < \sigma < s\) and \(s < \sigma\): the steady state is a saddle when \(\varepsilon_\gamma > 1\).

2. \(1 - \theta(1 - s)/s < \pi < [s - \theta(1 - s)]/[s - \theta(1 - s)/2]\) (see Fig. 3):

   (a) \(0 < \sigma < \sigma_J\): the steady state is a source when \(\varepsilon_\gamma > 1\).

   (b) \(\sigma_J < \sigma < \sigma_F\): the steady state is a sink for \(1 < \varepsilon_\gamma < \varepsilon_{\gamma H}\). Then the steady state undergoes a Hopf bifurcation at \(\varepsilon_\gamma = \varepsilon_{\gamma H}\), and is a source when \(\varepsilon_\gamma > \varepsilon_{\gamma H}\).

   (c) \(\sigma_F < \sigma < \sigma_H\): the steady state is a sink when \(1 < \varepsilon_\gamma < \varepsilon_{\gamma H}\). Then the steady state undergoes a

---

1 The expressions of \(\sigma_F, \sigma_H, \sigma_I, \sigma_J, \varepsilon_{\gamma H}\) and \(\varepsilon_{\gamma F}\) are given in the proof of the proposition in Appendix C.
Hopf bifurcation at $\varepsilon_{\gamma} = \varepsilon_{\gamma \text{H}}$ and is a source when $\varepsilon_{\gamma \text{H}} < \varepsilon_{\gamma} < \varepsilon_{\gamma \text{F}}$. A flip bifurcation occurs at $\varepsilon_{\gamma} = \varepsilon_{\gamma \text{F}}$ and the steady state is a saddle when $\varepsilon_{\gamma} > \varepsilon_{\gamma \text{F}}$.

(d) $\sigma_{\text{H}} < \sigma < \sigma_{\text{I}}$: the steady state is a sink when $1 < \varepsilon_{\gamma} < \varepsilon_{\gamma \text{F}}$. A flip bifurcation occurs at $\varepsilon_{\gamma} = \varepsilon_{\gamma \text{F}}$ and the steady state is a saddle if $\varepsilon_{\gamma} > \varepsilon_{\gamma \text{F}}$.

(e) $\sigma_{\text{I}} < \sigma < s$ and $s < \sigma$: the steady state is a saddle when $\varepsilon_{\gamma} > 1$.

3. $[s - \theta(1 - s)]/[s - \theta(1 - s)/2] < \pi < \pi_{\text{min}}$ (see Fig. 4):

(a) $0 < \sigma < \sigma_{\text{J}}$: the steady state is a source when $\varepsilon_{\gamma} > 1$.

(b) $\sigma_{\text{J}} < \sigma < \sigma_{\text{H}}$: the steady state is a sink when $1 < \varepsilon_{\gamma} < \varepsilon_{\gamma \text{H}}$. Then the steady state undergoes a Hopf bifurcation at $\varepsilon_{\gamma} = \varepsilon_{\gamma \text{H}}$ and is a source when $\varepsilon_{\gamma \text{H}} < \varepsilon_{\gamma} < \varepsilon_{\gamma \text{F}}$. A flip bifurcation occurs at $\varepsilon_{\gamma} = \varepsilon_{\gamma \text{F}}$ and the steady state is a saddle when $\varepsilon_{\gamma} > \varepsilon_{\gamma \text{F}}$.

(c) $\sigma_{\text{H}} < \sigma < \sigma_{\text{I}}$: the steady state is a sink when $1 < \varepsilon_{\gamma} < \varepsilon_{\gamma \text{F}}$. A flip bifurcation occurs at $\varepsilon_{\gamma} = \varepsilon_{\gamma \text{F}}$ and the steady state is a saddle if $\varepsilon_{\gamma} > \varepsilon_{\gamma \text{F}}$.

(d) $\sigma_{\text{I}} < \sigma < s$ and $s < \sigma$: the steady state is a saddle when $\varepsilon_{\gamma} > 1$.

Proof: See Appendix C.

The most important implication of Proposition 3.3 is that the half-line $\Delta$ crosses the indeterminacy region $ABC$ only if $\pi$ is not too large: in other words, there exists a threshold level of labor tax progressivity $\pi_{\text{min}}$ above which the steady state is locally determinate, thereby excluding local sunspots and stable cycles.

Corollary 3.1 (Ruling Out Local Indeterminacy Through Progressive Labor Income Taxes)

Under the assumptions of Proposition 3.3, there exists a threshold level of labor income tax progressivity $\pi_{\text{min}} \equiv 2[\theta(1 - s) - s + \sqrt{s(s - \theta(1 - s))/\theta(1 - s)}]$, such that the steady state is locally determinate (that is, a saddle or a source) when $\pi > \pi_{\text{min}}$ (see Figs. 5-6).
Proof: See Appendix C.

Corollary 3.1 contains an important implication: it is easily shown (by L’Hôpital’s rule) from the expression of $\pi_{\text{min}}$ that it converges, from below, to one when $\theta$ tends to zero. In practice, $\theta = 1 - \beta(1 - \delta)$ is bound to be close to zero when the period is commensurate with business-cycle length, as $\beta \approx 1$ and $\delta \approx 0$. Therefore, for sensible parameter values, fully eliminating expectation-driven fluctuations may require close-to-maximal progressivity. To fix ideas, consider the standard values $\beta = 0.96$, $\delta = 0.1$ (based on annual data) and $s = 1/3$. Then one has that $\pi_{\text{min}} \approx 0.92$ (moreover, $1 - \theta(1 - s)/s \approx 0.73$ and $[s - \theta(1 - s)]/[s - \theta(1 - s)/2] \approx 0.84$). This seems much too high, e.g. with respect to actual US data on average marginal tax rates. For example, our computations from Stephenson [28, Table 1, p. 391] deliver that tax progressivity has ranged in $4\% - 11\%$ over the last sixty years (in accord with some recent evaluations by Bénabou [4, pp. 501-2], Cassou and Lansing [7, p. 11], for example). This suggests that $\pi$ may well be below $\pi_{\text{min}}$ in real economies. Note also that the related contributions by Guo and Lansing [14], Guo [12] require also significant progressivity: with indivisible labor and externalities that are not too large, a straightforward computation from condition (19) in Guo [12, p. 102] reveals that progressivity should be larger than 0.33. Our discussion above shows that in alternative business-cycle models, the required progressivity may be much higher (see also next section, where a similar conclusion is obtained in an OLG model with capital income taxes, in which the threshold level of tax progressivity is lower though).

We should emphasize that Corollary 3.1 does not rule out the case of a steady state being a source (locally) surrounded by a period-two saddle created through a flip bifurcation. However, we do not comment further on this, as it does not appear as a very realistic description of actual business cycles. Moreover, such cycles are not expected to occur in continuous time versions of this type of model.
3.3 Interpreting the Results

Our last step is to provide some intuitive explanation of the mechanisms at work that create some stabilizing power of progressive tax schedules. Roughly speaking, the main effect of progressive tax rates can be interpreted as “taxing away the higher returns from belief-driven labor or investment spurts” (Guo and Lansing [14, p. 482]). However, note that in all models we are focusing on in this paper, returns to scale are constant both at the social level and at the firm level. In particular, the slope of labor demand (as a function of real wage) is negative so that tax progressivity does not reduce the likelihood of indeterminacy by changing the sign of labor demand’s slope (in contrast with Guo and Lansing [14, 15], Guo [12], Guo and Harrison [13]). Therefore, what remains to be elucidated is the sequence of events that make self-fulfilling expectations the driving force of the business cycle even though externalities (or imperfect competition) are absent. More importantly, one would like to understand why large tax progressivity on factor income is required to stabilize the economy. As we know illustrate, key to the results is the fact that the more progressive taxes on labor income, the more stable after-tax wage income and, therefore, the more stable workers’ future consumption.

It is helpful to start with the benchmark case of a linear tax rate (which also covers the case with zero taxes) on labor income. In that case, workers’ decisions are summarized by Eqs. (6) that may be written as follows, as \( \phi \) reduces to the identity function and \( \psi = 1 \):

\[
v_1(l_t) = v_2(p_t \omega_t l_t / p_{t+1}).
\]

(14)

The latter first-order condition shows that when workers expect, in period \( t \), that the price of goods \( p_{t+1} \) will go, say, down tomorrow, they wish to increase their consumption at \( t + 1 \) and, therefore, to work more today (remember that gross substitutability is assumed) so as to save more in the form of money balances to be consumed tomorrow. Moreover, the dynamical system in Eqs. (10) may be written as follows:

\[
\begin{align*}
v_2(\omega_{t+1} l_{t+1}) &= v_1(l_t), \\
k_t &= \beta R(k_{t-1}/l_t) k_{t-1}.
\end{align*}
\]

(15)
A higher labor supply $l_t$ will lead to greater output, larger consumption and a smaller capital-labor ratio $k_{t-1}/l_t$ and, therefore, to a higher return on capital $R_t$, so that, from Eqs. (15), capital demand $k_t$ and investment will increase. Moreover, a larger capital stock $k_t$ tomorrow will tend to increase tomorrow’s real wage $\omega_{t+1}$ which will trigger an increase in tomorrow’s labor supply. However, a higher capital stock will also tend to increase the ratio of capital/labor and, eventually, the effect on capitalists’ savings will turn negative: a higher capital-labor ratio leads to a lower rate of return on capital and, therefore, to lower capital demand and investment. This will lead to lower wage, lower labor supply, etc: the economy will experience a downturn. Note that this intuitive description relies on the presumption that both wage and interest rate are elastic enough to the capital-labor ratio: the elasticity of input substitution $\sigma$ must be small enough.

Now, we would like to shed some light on why progressive tax rates makes the occurrence of self-fulfilling fluctuations less likely. Assume again, for simplicity, that $\phi$ has constant elasticity around the steady state. In that case, Eqs. (6) reduce to:

$$\gamma(l_t) = p_t \phi(\omega_t l_t)/p_{t+1},$$  

(16)

where $\gamma \equiv v_2^{-1} \circ [v_1/\psi]$. When $\pi$ increases from zero to one, the volatility of wage income decreases to zero: eventually, a highly progressive tax rate on labor income (that is, $\pi$ close to one) leads to an almost constant wage bill, which in turn leads to a more stable consumption and, thereby, a smaller reaction of labor supply to optimistic expectations, in comparison to the case of linear taxes. More specifically, Eq. (16) shows that a large progressivity $\pi$ decreases the elasticity of labour supply to expected inflation and real wage. To see this, differentiate Eqs. (16) to get:

$$(\varepsilon \gamma - 1 + \pi) \frac{dl}{l} = -\frac{d\pi^e}{\pi^e} + (1 - \pi) \frac{d\omega}{\omega},$$  

(17)

where $\pi^e$ denotes expected inflation, that is, $\pi^e_{t+1} \equiv p_{t+1}/p_t$. Eq. (17) clearly shows how the higher labor income tax progressivity $\pi$, the less elastic labor supply to both expected inflation and real wage, thereby limiting both the initial impact of expectations movements and their subsequent effect on labor supply. Therefore, optimistic expectations (say, a reduction in $p_{t+1}$) lead to a smaller increase of consumption and
labor when the labor tax rate is highly progressive so that indeterminacy and expectation-driven fluctuations are ruled out.

4 Stabilization Through Progressive Capital Income Taxes

4.1 Taxing Capital Income in the Heterogeneous Agents Economy

Our description of the mechanisms that account for the stabilizing power of progressive labor income taxes also suggest that taxing capital income in a progressive manner is not expected to rule out local indeterminacy and bifurcations: in a nutshell, taxing capital income merely amounts to reducing capitalists’ after-tax revenues, thereby affecting their consumption (which is negligible when the discount factor is close enough to one) and investment demands, as seen from Eqs. (7). This does not alter workers’ consumption and labor supply movements. In other words, only the second equation of the dynamical system (10) is affected. However, our analysis and interpretation in Sections 3.2 and 3.3 have shown that variations in workers’ consumption and labor supply are at the origin of expectation-driven fluctuations so that taxing capitalists’ income (even progressively) is not expected to rule out such business cycles. This is what we now formalize. If we apply our non-linear tax schedule (4) (under Assumption 3.2) to capital income, it is not difficult to derive the dynamical system that now summarizes intertemporal equilibria (the proof is available from the authors upon request):

Definition 4.1

An intertemporal perfectly competitive equilibrium with perfect foresight is a sequence \((a_t, k_{t-1})\) of \(\mathbb{R}^{2+}_+\), \(t = 1, 2, \ldots\), such that

\[
\begin{align*}
    v_2(\omega(a_{t+1})k_t/a_{t+1}) &= v_1(k_{t-1}/a_t), \\
    k_t &= \beta\psi(R(a_t)k_{t-1})\phi(R(a_t)k_{t-1}).
\end{align*}
\]
For sake of brevity, we assume that a normalized steady state exists and, moreover, that $\phi$ has constant elasticity $\psi = 1 - \pi$. One then easily derives:

\begin{equation}
  T = T_1 - \frac{\varepsilon\gamma - 1}{\varepsilon\omega - 1}, \quad \text{with} \quad T_1 = 1 - \pi + \frac{(1 - \pi)|\varepsilon R| - 1}{\varepsilon\omega - 1},
\end{equation}

(19)

\begin{equation}
  D = \varepsilon\gamma D_1, \quad \text{with} \quad D_1 = (1 - \pi)\frac{|\varepsilon R| - 1}{\varepsilon\omega - 1}.
\end{equation}

(20)

Insert Figure 7 here.

The impact of capital income tax progressivity is summarized in Fig. 7. Starting from Fig. 1 (when $\pi = 0$), one can see from Fig. 7 that increasing $\pi$ reduces, here again, the slope of $\Delta_1$ (the locus such that $\varepsilon\gamma = 1$). However, the intersection of $\Delta_1$ with the line $(AC)$ (when $\sigma = +\infty$) now moves south-west along $(AC)$ when $\pi$ increases. The main implication is that the qualitative picture is not much affected by the presence of capital income taxes: more precisely, local indeterminacy still prevails as long as labor supply is not too elastic (that is, if $\varepsilon\gamma$ is not too large) and inputs are not too substitutable (that is, $\sigma < \sigma_I$), see Fig. 7. The expression of $\sigma_I$ (such that $1 + T_1(\sigma) + D_1(\sigma) = 0$) turns out to be as follows.

**Proposition 4.1 (Local Indeterminacy in spite of Progressive Capital Income Taxes)**

*Under Assumptions 3.1, 3.2 and 3.3, a progressive tax on capital income does not rule out local indeterminacy and bifurcations of the steady state of Eqs. (18).* 

*More precisely, the steady state is a sink if labor supply is sufficiently elastic ($\varepsilon\gamma$ not too large) and if the elasticity of capital-labor substitution is small enough, that is, $\sigma < \sigma_I = s/2 + \theta(1 - s)(1 - \pi)/(2 - \pi)$, where $s$ is the share of capital income (see Fig. 7). In particular, $\sigma_I = s/2$ when progressivity is maximal (that is, when $\pi = 1$).*

Again, as argued at the end of Subsection 3.2, $\theta$ is close to zero when the period is short (say, a year or less) so that a progressive capital income tax reduces the scope of local indeterminacy, but only to a negligible extent.
We now show that opposite conclusions are obtained in an OLG economy in which progressive capital income taxes may stabilize consumption and, thereby, immunize the economy against expectation-driven volatility.

### 4.2 Taxing Capital Income in an OLG Economy

In the competitive, non-monetary economy studied in this section (see Reichlin [24]), a unique good is produced, which can be either consumed or saved as investment by a constant population of households living two periods. Agents are identical within each generation, supply labor and save their wage income in the form of capital when young, to be consumed when old. Using the same notation as in Section 3, agents born at time $t$ solve the following program:

$$\max \{ V_2(c_{t+1}/B) - V_1(l_t) \} \text{ such that } k_t = \omega t l_t, c_{t+1} = \phi(R_{t+1}k_t), c_{t+1} \geq 0, l_t \geq 0. $$

(21)

Under Assumptions 3.1, 3.2 and 3.3, it is easily shown that the first-order conditions of the above problem are:

$$v_1(l_t) = \psi(R_{t+1}\omega t l_t) v_2(c_{t+1}) \text{ and } c_{t+1} = \phi(R_{t+1}\omega t l_t).$$

(22)

Therefore, intertemporal equilibria may here again be summarized by the dynamic behavior of both $a$ and $k$, as follows.

**Definition 4.2**

An intertemporal perfectly competitive equilibrium with perfect foresight is a sequence $(a_t, k_{t-1})$ of $\mathbb{R}^{2+}$, $t = 1, 2, \ldots$, such that

$$\begin{cases} v_2(\phi(R(a_{t+1})k_t)) = v_1(k_{t-1}/a_t)/\psi(R(a_t)k_{t-1}), \\ k_t = \omega(a_t)k_{t-1}/a_t. \end{cases}$$

(23)

By comparing Eqs. (10) and (23), one uncovers the correspondence existing between the model with segmented asset markets and the present OLG model: the dynamical system (23) summarizing equilibria in
the OLG economy can be obtained from (10) by changing the distribution of factor incomes. While young agents *save* their wage income and old agents *consume* their capital income, in the OLG setting, the reverse is true in the model of Section 3: workers consume their (real) wage income whereas capitalists save their capital income (entirely if they are extremely patient, that is, \( \beta = 1 \)). This correspondence will obviously help to interpret the results and, in particular, to explain why capital income taxes may be desirable to stabilize the economy. But it does also lead one to derive easily the jacobian matrix of (23) and, therefore, the analog of Proposition 3.2: what is needed is simply to replace \( \varepsilon_\omega - 1 \) by \( \varepsilon_R \) and *vice-versa*. By adapting the procedure in Proposition 3.1, one establishes the existence of a normalized steady state\(^2\) and the following statements hold.

**Proposition 4.2 (Linearized Dynamics around a Steady State)**

*Under Assumptions 3.1, 3.2 and 3.3, suppose that \( \phi \) has constant elasticity in the neighborhood of a steady state \((\overline{\pi}, \overline{k})\) of the dynamical system in Eqs. (23) that is assumed to exist, i.e. \( \psi(x) = 1 - \pi \), with \( 0 \leq \pi < 1 \) measuring labor income tax progressivity. Let \( \varepsilon_R, \varepsilon_\omega, \varepsilon_\gamma \) be the elasticities of the functions \( R(a), \omega(a), \gamma(l) \), respectively, evaluated at the steady state \((\overline{\pi}, \overline{k})\). The linearized dynamics for the deviations \( da = a - \overline{\pi}, \ dk = k - \overline{k} \) are determined by the linear map:

\[
\begin{align*}
\frac{da_{t+1}}{dt} &= -\frac{\varepsilon_\gamma - 1 + \varepsilon_\omega - 1}{\varepsilon_R}da_t + \frac{\varepsilon_\omega - 1 + 1/(1 - \pi)}{\varepsilon_R}dk_{t-1}, \\
\frac{dk_t}{dt} &= \frac{\varepsilon_R}{\varepsilon_\omega - 1}da_t + dk_{t-1}.
\end{align*}
\]

(24)

The associated Jacobian matrix evaluated at the steady state under study has trace \( T \) and determinant \( D \), where

\[
T = T_1 + \frac{\varepsilon_\gamma - 1}{(1 - \pi)|\varepsilon_R|}, \quad \text{with} \quad T_1 = 1 + \frac{\varepsilon_\omega - 1 + 1/(1 - \pi)}{|\varepsilon_R|},
\]

\[
D = \varepsilon_\gamma D_1, \quad \text{with} \quad D_1 = \frac{\varepsilon_\omega}{(1 - \pi)|\varepsilon_R|}.
\]

Moreover, one has \( T_1 = 1 + D_1 + \Lambda \), where \( \Lambda = \frac{\pi(1 - \varepsilon_\omega)}{(1 - \pi)|\varepsilon_R|} \).

\(^2\)For sake of brevity, the proof is omitted.
Proposition 4.2 reveals that the geometrical configuration is here qualitatively similar to the one that we have derived in Section 3. In particular, fixing technology and varying the elasticity of the offer curve $\varepsilon_\gamma$ generates, in the $(T,D)$ plane, a half-line $\Delta$ that starts close to $(T_1, D_1)$ when $\varepsilon_\gamma$ is close to one and whose slope is $\varepsilon_\omega = s/\sigma$.

Insert Figures 8-11 here.

It is then straightforward to derive the following expressions which critically depend on the elasticity of input substitution $\sigma \geq 0$ and tax progressivity $0 \leq \pi < 1$.

\[
\begin{align*}
D_1 &= [1 - \delta(1 - s)]/[(1 - \pi)(1 - s)], \quad \Lambda = \pi(\sigma - s)[1 - \delta(1 - s)]/[s(1 - s)(1 - \pi)], \\
T_1 &= 1 + D_1 + \Lambda, \quad \text{slope}_\Delta = s/\sigma.
\end{align*}
\]

(25)

In the benchmark case with linear (or no) taxes (that is, $\pi = 0$ or $\psi = 1$), the situation is as in Fig. 8: the steady state is a sink and looses stability through Hopf bifurcations only when $\sigma < s$. The immediate implication is that endogenous cycles and sunspot equilibria occur only for low values of $\sigma$, that is, only if capital and labor are complementary enough. On the contrary, local determinacy prevails when $\sigma > s$. Just as in our first model of Section 3, it turns out that linear tax rates on capital income do not modify the range of parameter values compatible with local indeterminacy and bifurcations. However, inspection of Eqs. (25) shows that increasing $\pi$ from zero to one increases $D_1$ from a positive value $[1 - \delta(1 - s)]/(1 - s)$ (which is plausibly assumed to be less than one when $\delta$ is close to one; see Fig. 8) to infinity. In other words, there exists a value $\pi_{min}$, illustrated by Fig. 10, above which the steady state is locally determinate (either a saddle or a source). This value is simply defined by the condition that $D_1 = 1$, or equivalently, $\pi_{min} \equiv (\delta(1 - s) - s)/(1 - s)$.

Proposition 4.3 (Local Stability and Bifurcations of the Steady State)

Under the assumptions of Proposition 4.2, the following generically holds when $\delta(1 - s) > s$ and $\pi < \pi_{min} \equiv
(δ(1 − s) − s)/(1 − s) (that is, tax progressivity is not too large); see Fig. 9.

1. 0 < σ < s: the steady state is a sink for \(1 < \varepsilon_\gamma < \varepsilon_H\), where \(\varepsilon_H \equiv (1 − \pi)(1 − s)/[1 − \delta(1 − s)]\) is the value of \(\varepsilon_\gamma\) for which \(\Delta\) crosses \([BC]\). Then the steady state undergoes a Hopf bifurcation (the complex characteristic roots cross the unit circle) at \(\varepsilon_\gamma = \varepsilon_H\), and is a source when \(\varepsilon_\gamma > \varepsilon_H\).

2. s < σ: the steady state is a saddle when \(\varepsilon_\gamma > 1\).

4.3 Capital Income Taxes as Automatic Stabilizers

The most important implication of Proposition 4.3 is that the half-line \(\Delta\) intersects the indeterminacy triangle \(ABC\) only if \(\pi\) is small enough: in other words, there exists a threshold level of capital tax progressivity \(\pi_{\text{min}}\) above which the steady state is locally determinate, thereby excluding local sunspots and stable cycles.

Corollary 4.1 (Ruling Out Local Indeterminacy Through Progressive Capital Income Taxes)

Under the assumptions of Proposition 4.3, there exists a threshold level of capital income tax progressivity \(\pi_{\text{min}} \equiv (\delta(1 − s) − s)/(1 − s)\) such that the steady state is locally determinate (that is, a saddle or a source) when \(\pi > \pi_{\text{min}}\) (see Figs. 10-11).

To fix ideas, adopting the standard values \(s = 1/3\) and \(\delta = 0.96\) (which corresponds to an annual depreciation rate of 0.1 if the period length is 30 years) yields \(\pi_{\text{min}} \approx 0.46\). Although the required level of capital tax progressivity is half as high as the threshold value obtained in subsection 3.2, it is still high in view of actual marginal tax rates, for instance in the US.

Our last step aims at providing an intuitive description of the mechanisms that account for the stabilizing power of progressive taxes on capital income. It is relevant to recall the correspondence noticed above between the two models considered in this paper. Roughly speaking, one goes from one model to the other by
inversing the distribution of factor incomes. This implies that if taxing labor income in a progressive manner rules out local indeterminacy and bifurcations in the heterogenous agents economy, as we have shown, one expects that taxing capital income should have the same effects in the OLG setting. We now go a little further to be more specific about the basic mechanisms at work. As in the previous model (in Section 3), tomorrow’s consumption and today’s labor supply movements following a wave of optimism are necessary to sustain expectation-driven equilibria. Specifically, the first-order conditions in Eqs. (22) can be rewritten as:

\[ \gamma(l_t) = \phi(R_{t+1} \omega_l l_t) \]  

(26)

where \( \gamma \equiv v_2^{-1} \circ [v_1/\psi] \) and \( \phi \) has, for simplicity, constant elasticity around the steady state. Therefore, direct inspection of Eq. (26) reveals that the more progressive the tax rate on capital income, the more stable consumption and labor supply. More precisely, if young agents expect, in period \( t \), the return on capital \( R_{t+1} \) to go, say, up, they wish to increase tomorrow’s consumption and, therefore, to work more today. However, a larger capital stock following the investment boom will eventually depress the return on capital and lower capital demand, turning the economy into a recession because of pessimistic expectations. When \( \pi \) increases from zero to one, the volatility of capital income decreases to zero: eventually, a highly progressive tax rate (that is, \( \pi \) close to one) leads to almost constant capital income and tomorrow’s consumption, which in turn leads to a much smaller reaction of labor supply to optimistic expectations, in comparison to the case of linear taxes. More specifically, with \( \pi \) close to one, \( \phi \) is almost constant and, in that case, Eq. (26) show that a large progressivity \( \pi \) decreases the elasticity of labour supply to expected return on capital and real wage. To see this, differentiate Eqs. (26) to get:

\[ (\varepsilon \gamma - 1 + \pi) \frac{dl}{l} = \frac{dR}{R} + \frac{d\omega}{\omega}. \]  

(27)

Eq. (27) clearly shows how the higher capital income tax progressivity \( \pi \), the less elastic labor supply to both expected capital return and real wage, thereby limiting both the initial impact of expectations movements and their subsequent effect on labor supply. Therefore, optimistic expectations (say, a increase in \( R_{t+1} \)) lead to a smaller increase of consumption and labor when the capital tax rate is highly progressive so that
Indeterminacy and expectation-driven fluctuations are ruled out.

In view of the above discussion, similar results are expected to hold in an extended version of the OLG model in which young agents may consume a fraction of their wage income. More precisely, it is known that the higher the propensity to consume out of wage income, the less likely local indeterminacy and bifurcations (see Cazzavillan and Pintus [8]). Therefore, one expects that taxing (in a progressive way) capital income in such setting would still stabilize old agents’ consumption and make local indeterminacy less likely. However, allowing for consumption in the first period of life would make the analysis heavier.

Moreover, our intuitive discussion also gives a hint on why progressive taxes on labor income do not rule out local indeterminacy: this is simply not helpful to stabilize tomorrow’s consumption and today’s labor supply. In fact, it turns out that this enlarges the range of parameters values compatible with sunspots and cycles, as shown in Fig. 12.

Insert Figure 12 here.

In that case, one can show (computations are available from the authors upon request) that the steady state is a sink (locally indeterminate) when $\varepsilon_{\gamma}$ is not too large and $\sigma < \sigma_I \equiv s(1 - s)/[1 - \delta(1 - s)]$ (see Fig. 12, which covers the case $\pi > \pi_{\min}$). For instance, $\sigma_I = 1 - s > s$ when, plausibly, $\delta = 1$ and $s < 1/2$. Therefore, in comparison with the case with linear taxes or progressive taxes on capital income, opposite results are obtained, as the range of $\sigma$’s for which local indeterminacy and bifurcations occur is enlarged by the introduction of progressive labor income taxes. In view of our intuitive discussion above, this is straightforward to explain. In that case, the first-order conditions of young agents’ problem are:

$$\gamma(l_t) = R_{t+1}\phi(\omega_l l_t),$$

(28)
as only wage income is taxed. Eq. (28) clearly shows that wage income is almost stable when the tax rate on labor income is highly progressive. However, this is not enough to stabilize tomorrow’s consumption that depends also on movements of the interest rate. Therefore, consumption and labor supply will still be subject
to endogenous volatility, however progressive the tax on labor income is. Now, to explain that progressive labor income taxes make local indeterminacy more likely requires recalling that, in this model, cyclical paths (be they deterministic or stochastic) arise because of two conflicting effects on savings that operate through wage and interest rate (see Cazzavillan and Pintus [8]). More precisely, when the capital stock increases (say, from its steady state value), this triggers an increase in wage and savings that will be, eventually, reversed by a lower interest rate that, on the contrary, depresses savings. For a given deviation of the capital stock, the (after-tax) wage effect will be smaller when progressive taxes on labor income are introduced so that the effect of the interest rate will be more likely to reverse it.

5 Conclusion

The main result of this paper is that income taxes immunize the economy against expectation-driven fluctuations only if the level of progressivity is high enough and, most importantly, if progressive taxation is applied to the relevant tax base so as to stabilize consumption and labor supply movements. Incidentally, our results suggest that capital income taxes may be desirable, when progressive, to reduce the likelihood of expectation-driven volatility. We have argued that progressivity is, in the real world, probably lower than the threshold values predicted by both models, above which local determinacy of the steady state prevails. However, our analysis suggests that less-than-maximal progressivity may still be helpful to reduce the range of parameter values that are compatible with sunspots and cycles.

Some directions for future research naturally follow. In the OLG setting, it seems relevant to introduce consumption/savings choices in the first period of life. In that context, one expects that a smaller level of progressivity on both incomes could rule out local indeterminacy and bifurcations by stabilizing both young and old agents’ consumptions. It remains to be seen if this is more in line with actual marginal tax rates. It is also expected that progressive taxes are inefficient to rule out endogenous fluctuations when consumption is financed, even partially, by the returns from financial assets that remain untaxed (e.g. in the monetary
A Progressive Labor Income Taxes and Workers’ Choices

In this section, we show how workers’ decisions can be reduced to a two-period problem. Workers solve the following problem:

\[
\max \sum_{t=1}^{+\infty} \beta^{t-1} V_2^w(c_t^w/B) - \beta^t V_1(l_t),
\]

subject to

\[
p_t c_t^w + p_t k_t^w + M_{t-1}^w \leq M_t^w + (r_t + (1-\delta)p_t)k_{t-1}^w + (1-\tau_t)w_t l_t,
\]

(30)

\[
p_t c_t^w + p_t k_t^w \leq M_{t-1}^w + (r_t + (1-\delta)p_t)k_{t-1}^w,
\]

(31)

where \(B > 0\) is a scaling parameter, \(0 < \beta_w < 1\) is the discount factor, \(c_t^w \geq 0\) is consumption, \(l_t \geq 0\) is labor supply. On the other hand, \(M_{t-1}^w \geq 0\) and \(k_{t-1}^w \geq 0\) are respectively money demand and capital holdings at the beginning of period \(t\), \(p_t > 0\) is the price of consumption goods, \(w_t > 0\) is nominal wage, \(r_t > 0\) is nominal return on capital and \(0 < \delta \leq 1\) is capital depreciation, while \(\tau_t\) is the non-linear tax rate on wage income.

As in Section 3 (see eq. (4)), we assume that \(\tau_t \equiv \tau(\omega_t l_t)\), with:

\[
[1 - \tau(\omega l)]w l/p \equiv \phi(\omega l),
\]

(32)

where \(\omega = w/p\) defines real wage.

Define \(\lambda_t \geq 0\) and \(\epsilon_t \geq 0\) the Lagrange multiplier associated, respectively, to (30) and (31) at date \(t\). Necessary conditions are then the following.

\[
\beta^{t-1} V_2^w(c_t^w/B) - (\lambda_t + \epsilon_t)p_t \leq 0, \quad = 0 \text{ if } c_t^w > 0,
\]

(33)

\[-(\lambda_t + \epsilon_t)p_t + (\lambda_{t+1} + \epsilon_{t+1}) (r_{t+1} + (1-\delta)p_{t+1}) \leq 0, \quad = 0 \text{ if } k_t^w > 0,
\]

\[-\lambda_t + \lambda_{t+1} + \epsilon_{t+1} \leq 0, \quad = 0 \text{ if } M_t^w > 0,
\]

\[-\beta^t V_1'(l_t) + \lambda_t p_t \omega_t \phi'(\omega_t l_t) \leq 0, \quad = 0 \text{ if } l_t > 0.
\]
Therefore, capital holdings are zero at all dates \((k_t^w = 0)\) if the second inequality of (33) is not binding, that is, if:

\[
V_2'(c_t^w / B) > \beta_w (r_{t+1}/p_{t+1} + 1 - \delta) V_2'(c_{t+1}^w / B),
\] (34)

if one assumes that \(c_t^w > 0\) for all \(t\) (we will show that this is the case around the steady state). Condition (34) implies that workers choose not to hold capital, and it depends on workers’ preferences because of the financial constraint (31).

Moreover, the financial constraint (31) is binding if \(\epsilon_t > 0\), that is, if:

\[
\omega_t \phi' (\omega_t l_t) V_2'(c_t^w / B) / B > \beta_w V_1'(l_t),
\] (35)

if one assumes that \(l_t > 0\) (again, we will show that this is the case around the steady state). Condition (35) therefore implies that (31) is binding.

Under conditions (34) and (35), workers spend their money holdings, i.e. \(p_t c_t^w = M_{t-1}^w\), and save their wage income in the form of money, i.e. \(M_t^w = (1 - \tau_t) w_t l_t\), so as to consume it tomorrow, i.e. \(p_{t+1} c_{t+1}^w = M_t^w\). Therefore, workers choose \(l_t \geq 0\) and \(c_{t+1} \geq 0\) as solutions to:

\[
\max \{ V_2(c_{t+1}^w / B) - V_1(l_t) \} \quad \text{s.c.} \quad p_{t+1} c_{t+1}^w = p_t \phi(\omega_t l_t).
\] (36)

The solutions to (36) are unique under Assumption 3.2 and 3.3 and characterized by the following first-order condition, as (6) in the main text:

\[
v_1(l_t) = \psi(\omega_t l_t) v_2(c_{t+1}^w), \quad p_{t+1} c_{t+1}^w = p_t \phi(\omega_t l_t),
\] (37)

where \(v_2(c) \equiv c V_2'(c / B) / B, \quad v_1(l) \equiv l V_1'(l)\).

Finally, it is straightforward to show that, under the assumptions that capitalists discount future less heavily than workers (that is, \(\beta_w < \beta\)) and that \(\beta_w < 1\), conditions (34) and (35) are met at the steady state under study defined in Proposition 3.1. \(\square\)
B Proof of Proposition 3.1

In view of Eqs. (10) and recalling that \( a = k/l \), the nonautarkic steady states are the solutions \((\pi, \bar{I})\) in \( \mathbb{R}^2_{++} \) of \( v_2(\phi(\omega(\pi)\bar{I})) = v_1(\bar{I})/\psi(\omega(\pi)\bar{I})) \) and \( \beta R(\pi) = 1 \). Equivalently, in view of Eq. (3), the steady states are given by:

\[
\begin{align*}
    v_2(\phi(\omega(\pi)\bar{I})) &= v_1(\bar{I})/\psi(\omega(\pi)\bar{I})), \\
    \rho(\pi) + 1 - \delta &= 1/\beta.
\end{align*}
\] (38)

We shall solve the existence issue by setting appropriately the scaling parameters \( A \) and \( B \), so as to ensure that one stationary solution coincides with, for instance, \((\pi, \bar{I}) = (1, 1)\). The second equality of Eqs. (38) is achieved by scaling the parameter \( A \), while the first is achieved by scaling the parameter \( B \). That is, we set \( A = (1/\beta - 1 + \delta)/f'(1) \) to ensure that \( \pi = 1 \). On the other hand, \( \psi(\omega(\pi)\bar{I})v_2(\pi) = v_1(\bar{I}) \) is then equivalent to

\[
\psi(\omega(1)) \frac{\phi(\omega(1))}{B} V_2'(\frac{\phi(\omega(1))}{B}) = V_1'(1).
\] (39)

From Assumption 3.3, \( v_2 \) is decreasing in \( B \) so the latter condition is satisfied for some unique \( B \) if and only if:

\[
\lim_{c \to 0} c V_2'(c) < \frac{V_1'(1)}{\psi(\omega(1))} < \lim_{c \to +\infty} c V_2'(c).
\] (40)

C Proof of Proposition 3.3 and Corollary 3.1

C.1 Proposition 3.3: Geometrical Configuration

To prove formally the occurrence of three configurations, depending on \( \pi \), our first task is to show that the point \((T_1(\sigma), D_1(\sigma))\), as a function of \( \sigma \), indeed describes part of a line \( \Delta_1 \). From the fact that \( T_1(\sigma) = \)
1 + D_1(\sigma) + \Lambda(\sigma) and D_1(\sigma) are fractions of first degree polynomials in \sigma with the same denominator (see Eq. (13)), we conclude that the ratio of their derivatives \( \frac{D_1'(\sigma)}{T_1'(\sigma)} \), or \( \frac{D_1'(\sigma)}{D_1'(\sigma) + \Lambda'(\sigma)} \), is independent of \( \sigma \). Straightforward computations show that the slope of \( \Delta_1 \) is:

\[
slope_{\Delta_1} = \frac{D_1'(\sigma)}{T_1'(\sigma)} = \frac{s - \theta(1-s)}{s - \theta(1-s) + \pi\theta(1-s)}. \tag{41}
\]

From Eq. (13), we conclude that \( \Lambda(\sigma) \) vanishes when \( \sigma \) goes to infinity. It follows that \( \Delta_1 \) intersects the line (AC) at a point \( I \) of coordinates \( (T_1(+\infty), D_1(+\infty)) \), where \( D_1(+\infty) = 1/(1 - \pi) > 0 \) (see Fig. 2-5). We shall focus throughout on the configuration presented in Figs. where \( D_1(+\infty) \geq 1 \) and the slope of \( \Delta_1 \) is smaller than 1 (that is, \( \pi \geq 0 \)). We shall ensure the latter condition by imposing, as in the case of linear (or of no) taxes, that \( \theta(1-s) < s \) (that is, the share of capital is large enough). This condition is not very restrictive when \( \theta = 1 - \beta(1 - \delta) \) is small, which is bound to be the case when the period is short since \( \beta \) is then close to one and \( \delta \) is close to zero. Note that the geometrical method can be applied as well when these conditions are not met.

Then it follows that both \( \Lambda(\sigma) \) and \( D_1(\sigma) \) are decreasing functions (see Eq. (13)), so that \( T_1(\sigma) \) is also a decreasing function, i.e. \( T_1'(\sigma) = D_1'(\sigma) + \Lambda'(\sigma) < 0 \). Accordingly, the slope of \( \Delta_1 \) is smaller than one. From the above assumptions, one also gets all the necessary information to appraise the variations of \((T_1(\sigma), D_1(\sigma))\) as well as of the slope of \( \Delta \), when \( \sigma \) moves from 0 to \(+\infty\). In particular, \( T_1(0) \) and \( D_1(0) = \theta(1-s)/[s(1-\pi)] \) are positive and the corresponding point is above \( I \) on the line \( \Delta_1 \) when \( \pi > 0 \) (see Fig. 2-5). As \( \sigma \) increases from 0, \( T_1(\sigma) \) and \( D_1(\sigma) \) are decreasing and tend to \(-\infty\) when \( \sigma \) tends to \( s \) from below. When \( \sigma = s \), the function \( \omega(a)/a \) of \( a \) has a critical point, i.e. its derivative with respect to \( a \) vanishes, and the dynamical system derived from Eqs. (10) is not defined. When \( \sigma \) increases from \( s \) to \(+\infty\), \( T_1(\sigma) \) and \( D_1(\sigma) \) are still both decreasing, from \(+\infty\) to \((T_1(+\infty), D_1(+\infty))\), which is represented by the point \( I \) in Figs. 2-5. On the other hand, the intersection of \( \Delta_1 \) with \([BC]\) is characterized by \( D_1(\sigma) = 1 \) which leads, in view of Eq. (13), to \( \sigma_J = [\theta(1-s) - s(1-\pi)]/\pi \). In addition, the slope of \( \Delta \) as a function of \( \sigma \) increases monotonically from \(-\infty\) to 1 as \( \sigma \) moves from 0 to \(+\infty\), and vanishes when \( D_1(\sigma) = 0 \). Moreover, the half-line \( \Delta \) is above \( \Delta_1 \) when \( \sigma < s \), and below it when \( \sigma > s \).
Therefore, three configurations arise when $\pi$ increases from zero. When $\pi < 1 - \theta(1 - s)/s$ (case 1) then $D_1(0) < 1$: the geometric picture is as in Fig. 2 and it is not qualitatively different from the case of linear (or no) taxes (compare Figs. 1 and 2). Second, $D_1(0) > 1$ when $\pi < 1 - \theta(1 - s)/s$, and two cases arise depending on whether $\pi$ is smaller or larger than $[s - \theta(1 - s)]/[s - \theta(1 - s)/2]$. More precisely, the slope of $\Delta$ at $\sigma = \sigma_J$ (that is, when $D_1(\sigma_J) = 1$) is smaller than $-1$ when $\pi < [s - \theta(1 - s)]/[s - \theta(1 - s)/2]$ (case 2). In that case, the steady state is a source when $\sigma$ is close enough to zero ($0 < \sigma < \sigma_J$) and it undergoes the same sequences of bifurcations as in case 1 when $\sigma > \sigma_J$. Finally, case 3 occurs when the slope of $\Delta$ at $\sigma = \sigma_J$ is larger than $-1$, that is, when $\pi > [s - \theta(1 - s)]/[s - \theta(1 - s)/2]$ and $\pi < \pi_{\text{min}}$.

**C.2 Proposition 3.3: Local Bifurcation Values**

In this subsection, we derive all bifurcation values as functions of the structural parameters. We define $\theta \overset{\text{def}}{=} 1 - \beta(1 - \delta)$, and $s_{\Delta}(\sigma) \overset{\text{def}}{=} 1 - \theta(1 - s)/\sigma$ as the slope of the half-line $\Delta$.

An eigenvalue of $-1$: the flip bifurcation.

The equality $s_{\Delta}(\sigma) = -1$ allows one to derive $\sigma_F = \theta(1 - s)/2$, so that $s_{\Delta}(\sigma) \leq -1$ when $\sigma \leq \sigma_F$.

Equation $1 + T(\varepsilon_\gamma) + D(\varepsilon_\gamma) = 0$ yields $\varepsilon_\gamma_F = (1 - \pi)(2s + \theta(1 - s) - 2\sigma)/\{1 - \alpha/(1 - s)[2\sigma - \theta(1 - s)]\}$.

The condition that $1 + T_1(\sigma) + D_1(\sigma) = 0$ or, equivalently $\varepsilon_\gamma_F = 1$, gives the last flip bifurcation value $\sigma_F = [\theta(1 - s)(2 - \pi) + 2s(1 - \pi)]/[2(2 - \pi)]$ so that $\varepsilon_\gamma_F > 1$ when $\sigma < \sigma_J$.

A pair of eigenvalues of modulus $1$: the Hopf bifurcation.

The condition that $T(\varepsilon_\gamma_H) = -2$ when $D = 1$, i.e. when $\varepsilon_\gamma = \varepsilon_\gamma_H = 1/D_1$, is rewritten as $Q_H(\sigma) \overset{\text{def}}{=} a\sigma^2 + b\sigma + c$, the roots of which contain the bifurcation value $\sigma_H$. The coefficients of $Q_H(\sigma)$ are:

$$a = 4,$$

$$b = -4[s + \theta(1 - s)],$$

$$c = \theta(1 - s)[\theta(1 - s) + 3s].$$
It is easily shown that there must exist two distinct real roots, and that $\sigma_H = s[1+\theta(1-s)/s-\sqrt{1-\theta(1-s)/s}]/2$ is the lowest.

The condition $D_1(\sigma) = 1$ yields, in view of Eqs. (13), $\sigma_J = [\theta(1-s) - s(1-\pi)]/\pi$. Moreover, the bifurcation value $\varepsilon_H = (1-\pi)(s-\sigma)/[\theta(1-s)-\sigma]$ follows from $D = \varepsilon D_1 = 1$, i.e. $\varepsilon_H = 1/D_1$. □

C.3 Proof of Corollary 3.1

Proving Corollary 3.1 relies on the geometrical configuration and local bifurcation values of Proposition 3.3, as exposed in Subsections C.1 and C.2. In particular, there exists a critical value $\pi_{min}$ such that $\Delta_1$ goes through point $B$ (see Fig. 5). The expression $\pi_{min}$ is the value of $\pi$ that solves $\sigma_I = \sigma_J$, that is, $\pi \equiv 2[\theta(1-s) - s + \sqrt{s(s-\theta(1-s))}]/[\theta(1-s)]$. This implies that $\Delta$ does not intersect the $ABC$ triangle when $\pi > \pi_{min}$. Therefore, the steady state is either a saddle or a source for all $\varepsilon > 1$ (see Fig. 6).

References


Figure 1
The heterogeneous agents model with linear taxes

Figure 2
Progressive labor-income taxes in the heterogeneous agents model – Case 1
Figure 3
Progressive labor-income taxes in the heterogeneous agents model – Case 2

Figure 4
Progressive labor-income taxes in the heterogeneous agents model – Case 3
Figure 5
Threshold level of progressive labor-income taxes $\pi_{\text{min}}$ in the heterogeneous agents model

Figure 6
Ruling out local indeterminacy through progressive labor-income taxes in the heterogeneous agents model
Figure 7
Progressive capital-income taxes in the heterogeneous agents model
Figure 8
The OLG model with linear taxes

Figure 9
Progressive capital-income taxes in the OLG model
Figure 10
Threshold level of progressive capital-income taxes $\pi_{\text{min}}$ in the OLG model

Figure 11
Ruling out local indeterminacy through progressive capital-income taxes in the OLG model
Figure 12
Progressive labor-income taxes in the OLG model