Investment-specific shocks and external balances in a small-open economy model

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Abstract

We set up a standard small-open economy business cycle model driven by government spending shocks and two types of productivity shocks. The model is calibrated to quarterly Canadian data and its predicted moments are compared to those in these data. We find that models including either neutral productivity shocks (i.e. shocks to total factor productivity) or investment-specific productivity shocks (in addition to government spending shocks that are included in all models) do not match the data very well. However, the model including both types of productivity shocks matches very well the moments of output, investment and the trade balance.

Keywords: small-open economy, business cycles, investment-specific shocks, Canada

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1. Introduction

A number of papers document the importance of investment-specific technological changes in accounting for growth and business cycle properties of output. Greenwood, Hercowitz and Krusell (1997) find that investment-specific technological changes explain approximately sixty percent of growth in output per man-hour in post-war US data. Greenwood, Hercowitz and Krusell (2000) set up a closed-economy business cycle model where business cycle fluctuations are produced by investment-specific technological changes. They calibrate their model to the US economy and find that these changes explain approximately thirty percent of the variation in output.

Rather than measuring the importance of investment-specific technological changes on output and hours worked using a calibrated model, Fisher (2003) uses vector autoregression models and variance decomposition to identify the share of the variations in hours worked (at business cycle frequencies) explained by investment-specific technological changes and the share explained by neutral technological changes. Neutral technological changes affect symmetrically the productivity of the technology producing consumption goods and the productivity of the technology producing investment goods. For example, in a standard RBC model where consumption and investment goods can always be exchanged one for one, shocks to total factor productivity represent neutral technological changes. Fisher finds that about fifty percent of the fluctuations in hours worked in the US are explained by investment-specific technological changes whereas neutral technological changes account for about six percent of the variation in hours worked.

Boileau (2002) extends the standard two-country RBC model to investigate the effect of introducing trade in capital goods and investment-specific technological changes. He finds that a two-country RBC model with these two extensions is consistent with the cross-country correlation of output and the volatility of the terms of trade (relative
to the volatility of output) for G7 countries. The standard two-country RBC model is unable to match these two features.

We seek to contribute to the literature on investment-specific technological changes/shocks by documenting how including such shocks in a small-open economy model affects its predictions. Contrary to a closed-economy model, a small-open economy model has predictions for the trade balance (and the current account). It is natural to expect that shocks affecting specifically the productivity of newly produced capital (e.g. investment-specific shocks) and shocks affecting simultaneously the productivity of labour, newly produced, and installed capital (e.g. shocks to total factor productivity) will have different effects on investment, output and the trade balance.

We use a general equilibrium small-open economy model to quantify these effects. However, we do not restrict our attention to the properties of output, investment and external balances since general equilibrium effects may well change some properties of consumption and hours worked (for example) when the type of shocks included in the model changes.

While it has been customary to investigate the effects of investment-specific productivity shocks by comparing models with neutral productivity shocks only and models with investment-specific shocks only, we deviate from that trend by also looking at a model where both types of shocks are included. After all, if the two types of disturbances truly hit a given small-open country, a model including both sources of disturbances may be closer to the true data generating process and therefore, may reproduce the moments measured in historical data better.

We specify a general equilibrium small-open economy model that is essentially extending the closed-economy model proposed by Greenwood et al. (2000) to a small-open economy framework. We consider a country inhabited by a large number of identical households and producers. The representative agent in the economy can borrow or
lend on the world market at an exogenously given world interest rate. As in the closed-economy model of Greenwood et al. (2000), production of the final good requires labour as well as services from a stock of machinery and equipment and from a stock of structures. We make the natural assumption that firms can adjust the utilization rate of the stock of machinery of equipment while the utilization rate of the stock of structures is held constant. We assume that there is a government that levies lump sum taxes to finance its expenditures. In order to identify the effects of investment-specific productivity shocks and their interactions with the more traditional neutral productivity shocks, we consider three variants of the model: (1) no investment-specific productivity shocks; (2) no neutral productivity shocks; and (3) neutral and investment-specific shocks.

We calibrate the model to the Canadian economy and compare the statistical moments of consumption, hours, output, investment, trade balance-output ratio and current account-output ratio predicted by the model to those calculated using quarterly detrended Canadian data for the period (1976Q1 to 2003Q4).

We find that a small-open economy model including neutral productivity shocks and investment-specific productivity shocks matches the statistical moments of investment and the trade balance-output ratio much better than variants of this model where only one of the productivity shocks is included. While the model with neutral productivity shocks and investment-specific productivity shocks matches well the moments of output, investment and the trade-balance-output ratio, it fails to match the moments of consumption and hours. Actually, the moments of consumption and hours worked in the model without investment-specific shocks are very similar to those in the model with both neutral and investment-specific shocks. Therefore, adding investment-specific productivity shocks to a standard small-open economy model that already includes neutral shocks improves the model’s ability to match the data since investment-specific shocks improve the model’s predictions for investment.
and the trade balance without worsening the model’s predictions for consumption, hours, output and the current account.

However, the model with both types of shocks suffers from a weakness that is common to very many small-open economy models in that it is not able to match the observed moments of consumption and the trade balance. Accordingly, the search for a small-open economy model that matches well the Canadian economy does not end with the addition of investment specific shock in an otherwise standard model. Additional work is required to find extensions of the model that will improve its predictions, especially for consumption and hours.

The paper is organized as follows. Section 2 presents the model. Section 3 provides details on the numerical solution methods and on the parameter values employed in the simulations. Section 4 compares the statistical moments calculated using our sample of Canadian data to those predicted by three variants of the model described in Section 2. Section 5 concludes.

2. Small-open economy model

2.1 Model with two sources of growth


The small-open economy is populated by a large number of identical households and firms. Therefore, we focus on the problem of a representative household/producer
whose utility depends on consumption of a single good (denoted by $C$) and on hours worked (denoted by $n$).\footnote{We use the following convention regarding notation: uppercase letters denote endogenous variables that are growing along a balanced growth path while lowercase letters denote endogenous variables that are not growing along a balanced growth path (including endogenous variables that have been detrended).}

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ C_t - \mu \Gamma^t n_t^\nu \right]^{1-\alpha}, \quad \alpha > 0, \mu > 0, \nu > 1, \ 0 < \beta < 1, \Gamma > 1
\]  \hspace{1cm} (1)

where $E_0$ denotes the rational expectation operator, $\beta$ is a subjective discount factor and $\Gamma$ is a gross growth rate defined below. We need the term $\Gamma^t$ in the momentary utility function for the model to have a balanced growth path where $n$ is constant. The presence of $\Gamma^t$ in the utility function can be interpreted as representing technological progress in home production activities (see Hercowitz and Sampson, 1991, for more details).

As in Greenwood \textit{et al.} (2000), output is produced using a stock of machinery and equipment ($K_1$), a stock of structures ($K_2$) and hours worked according to the following Cobb-Douglas production function

\[
Y_t = A_t (u_t K_{1t})^{\theta_1} K_{2t}^{\theta_2} (\gamma_x n_t)^{1-\theta_1-\theta_2}, \quad 0 < \theta_1, \theta_2 < 1, \gamma_x > 1
\]  \hspace{1cm} (2)

where $A_t$ is a stationary productivity shock, $u_t$ is the utilization rate of machinery and equipment, and $\gamma_x$ is a labour augmenting factor.

The representative agent in the small-open economy can borrow and lend on world capital markets. We denote holdings of foreign assets at the beginning of period $t$ by $B_t$. In each period, the representative agent must satisfy the budget constraint

\[
C_t + I_{1t} + I_{2t} + G_t + B_{t+1} = Y_t + R_t B_t
\]  \hspace{1cm} (3)

where $I_{1t}$ is investment in machinery and equipment, $I_{2t}$ is investment in structures, $G_t$ is government consumption financed by lump-sum taxes, $Y_t$ is output, and $R_t$ is
the (gross) interest rate faced by the households in the small-open economy. This rate is the sum of a constant world interest rate $R^w$ and of a “risk premium” that depends on aggregate assets ($\bar{B}$) and aggregate output ($\bar{Y}$):\footnote{This risk premium is strictly decreasing and is included to avoid the well-known problem associated with a unit root in asset accumulation. See Schmitt-Grohé and Uribe (2003) for more details about modeling devices targeted at this specific problem.}

$$R_t = R^w + P(\bar{B}_t/\bar{Y}_t) = R^w + \psi_2 \left( e^{\psi_1 - \bar{B}_t/\bar{Y}_t} - 1 \right), \quad \psi_2 > 0. \quad (4)$$

The transition equation for the stock of machinery and equipment is

$$K_{1t+1} = (1 - \delta_1 u^0_1)K_{1t} + \gamma^1_z v_1 I_{1t} - \frac{\phi_1}{2} \left( \frac{\gamma^1_z I_{1t}}{K_{1t}} - (\Gamma - 1 + \delta_1 u^0_1) \right)^2 K_{1t} \quad (5)$$

where $0 < \delta_1 < 1$, $\eta > 1$, $\gamma_z > 1$, $\phi_1 \geq 0$. Note that $\gamma_z$ and $\nu$ denote, respectively, the growth and stochastic components in the productivity of the technology producing new machinery and equipment.

The transition equation for the stock of structures is

$$K_{2t+1} = (1 - \delta_2)K_{2t} + I_{2t} - \frac{\phi_2}{2} \left( \frac{I_{2t}}{K_{2t}} - (\Gamma - 1 + \delta_2) \right)^2 K_{2t} \quad (6)$$

where $0 < \delta_2 < 1$ and $\phi_2 \geq 0$.

As is customary in open-economy models, transition equations (5) and (6) includes capital adjustment costs, governed by the parameters $\phi_1$ and $\phi_2$ to slow down investment movements. The adjustment costs functions are set in such a way that the model with adjustment costs has the same steady state as the model without them.\footnote{After we started working on this paper, we became aware of a working paper by M. Mulvaine who is also investigating the properties of a small-open economy with investment specific productivity shocks. While Mulvaine (2004) is related to our paper, there are non-trivial differences in the modeling assumptions made in the two papers. In setting up the household’s preferences, we follow the assumptions commonly made in the literature by using a constant discount factor and...}
The production function (2) and the transition equations (5) and (6) make clear that the stock of structures and the stock of machinery equipment are treated differently in our model. First, due to the more active role played by machinery and equipment in production, we allow for the possibility of varying the utilization rate of the stock of machinery and equipment in response to shocks. Accordingly, we make the depreciation rate of the stock of machinery and equipment depends on the utilization rate of this capital. Second, we include the term $\gamma_t^v$ in transition equation (5) to represent technological changes in the productivity of the technology producing new capital. Looking at the time paths of the relative price of structures and of machinery and equipment shows a dramatic fall in the relative price of machinery and equipment but no obvious trend in the relative price of structures. Figures 1 and 2 plots the relative price of structures and of machinery and equipment for Canada for the period 1976:Q1-2003:Q4. Appendix A contains a description of the Canadian data used in the paper. In Figure 1, the relative prices are calculated by dividing the implicit price deflator for machinery and equipment (or structures) by the implicit price deflator for expenditures on non-durable goods and services. In Figure 2, the relative prices are calculated by dividing the implicit price deflator for machinery and equipment (or structures) by the implicit price deflator for expenditures on non-durable goods. However the relative prices are calculated, the trend properties of the two relative time-separable utility. Mulvaine’s model includes an endogenous discount factor that effectively makes utility time non-separable. Also, our model incorporates government spending shocks while Mulvaine’s do not. As explained above, our model extends the closed-economy model of Greenwood et al. (2000) where there are a stock of structures with a constant utilization rate, a stock of machinery and equipment with an endogenous utilization rate and where only the production of new machinery and equipment is subject to investment-specific productivity shocks. Mulvaine follows the framework proposed by Greenwood et al. (1988) where there is a single capital stock, and where investment-specific shocks affect the productivity of the technology producing the single type of new capital. Finally, as is done in most of the business cycle literature (in closed and open economies), we calibrate our model to quarterly data while Mulvaine calibrate his to annual data.
prices are vastly different. We interpret Figures 1 and 2 as evidence that there are significant technological changes specific to the machinery and equipment sector. Accordingly, we include the factor $\gamma_t^z$ in transition equation (5). The time path of the relative price of machinery and equipment is not a smooth downward trend, i.e. there are variations in the relative price at business cycle frequencies. Figure 3 plots the relative price of machinery and equipment and output per capita where both series have been detrended using the Hodrick-Prescott filter (with smoothing parameter equal to 1600) in order to isolate fluctuations at business cycle frequencies. Clearly, there is a good deal of variation in the relative price of machinery and equipment at business cycle frequencies. Accordingly, we include the investment-specific technology shock $v_t$ in equation (5).

We complete the description of the model by presenting the transition equation for the exogenous variables. Define the growth rate $\Gamma = \gamma_x \gamma_t^z \gamma_z^{\theta_1} \gamma_z^{\theta_2}$ and detrended government expenditures by $g_t \equiv G_t/\Gamma^t$. Neutral technology shocks ($A$), detrended government expenditures ($g$) and investment-specific technology shocks ($v$) evolve according to the following vector autoregressive process (in logs)

$$
\begin{bmatrix}
\ln A_{t+1} \\
\ln g_{t+1} \\
\ln v_{t+1}
\end{bmatrix}
= 
\begin{bmatrix}
\ln A^*(1 - \rho_A) \\
\ln g^*(1 - \rho_g) \\
\ln v^*(1 - \rho_v)
\end{bmatrix}
+ 
\begin{bmatrix}
\rho_A & 0 & 0 \\
0 & \rho_g & 0 \\
0 & 0 & \rho_v
\end{bmatrix}
\begin{bmatrix}
\ln A_t \\
\ln g_t \\
\ln v_t
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{A,t+1} \\
\varepsilon_{g,t+1} \\
\varepsilon_{v,t+1}
\end{bmatrix}
$$

where the unconditional means $A^*$ and $v^*$ are normalized to unity and the unconditional mean $g^*$ will be calibrated below. It is assumed that $\rho_A$, $\rho_g$, and $\rho_v$ lie between 0 and 1 and that the covariance matrix of the vector of innovations $\varepsilon_{t+1}$ is $E[\varepsilon\varepsilon^\top] = \Sigma$ where the off-diagonal elements of $\Sigma$ are not restricted to be zero.

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2.2 Detrended model and optimality conditions

In the model represented by equations (1)-(7), the trends in labour productivity and in the production of new machinery and equipment imply that along a balanced growth path where \( n \) and \( u \) are constant, the variables \( Y, I_1, I_2, C, B, G \) and \( K_2 \) grow at rate \( \Gamma \) while \( K_1 \) grows at rate \( \Gamma \gamma_z \). See Appendix B for a derivation of the growth rates along a balanced growth path. In order to work in a stationary environment, we define the following detrended variables

\[
y_t = Y_t / \Gamma, \quad i_{1t} = I_{1t} / \Gamma, \quad i_{2t} = I_{2t} / \Gamma, \quad c_t = C_t / \Gamma, \quad b_t = B_t / \Gamma, \quad k_{2t} = K_{2t} / \Gamma, \quad k_{1t} = K_{1t} / (\Gamma \gamma_z)^t, \quad (8)
\]

and the adjusted discount factor \( \hat{\beta} = \beta \Gamma^{1-\alpha} \).

In the detrended economy, the representative agent chooses contingency plans for \( \{y_t\}_{t=0}^{\infty}, \{i_{1t}\}_{t=0}^{\infty}, \{i_{2t}\}_{t=0}^{\infty}, \{c_t\}_{t=0}^{\infty}, \{n_t\}_{t=0}^{\infty}, \{u_t\}_{t=0}^{\infty}, \{b_{t+1}\}_{t=0}^{\infty}, \{k_{1t+1}\}_{t=0}^{\infty}, \{k_{2t+1}\}_{t=0}^{\infty} \) to maximize

\[
E_0 \sum_{t=0}^{\infty} \hat{\beta}^t \left[ c_t - \mu n_t^v \right]^{1-\alpha} / (1-\alpha) \quad (9)
\]

subject to

\[
y_t = A_t (u_t k_{1t})^{\theta_1} k_{2t}^{\theta_2} n_t^{1-\theta_1-\theta_2} \quad (10)
\]

\[
c_t + i_{1t} + i_{2t} + g_t + \Gamma b_{t+1} = y_t + R_t b_t \quad (11)
\]

\[
R_t = R^w + \psi_2 \left( e^{v_1 - \bar{b}/\bar{y}} - 1 \right) \quad (12)
\]

\[
\Gamma \gamma_z k_{1t+1} = (1 - \delta_1 u_t^\eta) k_{3t} + v_i i_{1t} - \phi_3 \left( i_{1t} / k_{1t} - (\Gamma \gamma_z - 1 + \delta_1 u_t^\eta) \right)^2 k_{1t} \quad (13)
\]

\[
\Gamma k_{2t+1} = (1 - \delta_2) k_{2t} + i_{2t} - \phi_2 \left( i_{2t} / k_{2t} - (\Gamma - 1 + \delta_2) \right)^2 k_{2t} \quad (14)
\]

and subject to the transition equation for the exogenous stochastic variables (7), and given the initial conditions \( k_{10}, k_{20}, b_0, A_0, g_0 \) and \( v_0 \).

The dynamic programming problem solved by the representative agent (in the detrended economy) implies the first-order conditions

\[
[c_t - \mu n_t^v]^{-\alpha} = \lambda_{3t} \quad (15)
\]
$$\mu \nu \left[ c_t - \mu n_t^{\nu-1} \right] = (1 - \theta_1 - \theta_2) \lambda_3 y_t n_t$$  \hspace{1cm} (16)

$$\theta_1 \lambda_3 \frac{y_t}{n_t} - \lambda_{1t} \left[ \delta_1 n_t^{\nu-1} k_{1t} - \phi_1 \left( \frac{i_{1t}^{1t}}{k_{1t}^{1t}} - [\Gamma_{\gamma z} - 1 + \delta_1 u_t^{\eta}] \right) \delta_1 n_t^{\eta-1} k_{1t} \right] = 0$$  \hspace{1cm} (17)

$$\lambda_{3t} = \lambda_{1t} \left[ u_t - \phi_1 \left( \frac{i_{1t}^{1t}}{k_{1t}^{1t}} - [\Gamma_{\gamma z} - 1 + \delta_1 u_t^{\eta}] \right) \right]$$  \hspace{1cm} (18)

$$\lambda_{3t} = \lambda_{2t} \left[ 1 - \phi_2 \left( \frac{i_{2t}^{2t}}{k_{2t}^{2t}} - [\Gamma - 1 + \delta_2] \right) \right]$$  \hspace{1cm} (19)

$$\Gamma_{\lambda_{3t}} = \hat{\beta} E_t \lambda_{3t+1} \left[ R^w + \psi_2 \left( e^{\psi_{1-b_t+1}/y_t+1} - 1 \right) \right]$$  \hspace{1cm} (20)

$$\Gamma_{\gamma_{\lambda_{1t}}} = \hat{\beta} E_t \left\{ \theta_1 \lambda_{3t+1} \frac{y_{t+1}}{k_{1t+1}^{1t+1}} + \lambda_{1t+1} \left[ \phi_1 \left( \frac{i_{1t+1}^{1t+1}}{k_{1t+1}^{1t+1}} - (\Gamma_{\gamma z} - 1 + \delta_1 u_{t+1}^{\eta}) \right) \frac{i_{1t+1}^{1t+1}}{k_{1t+1}^{1t+1}} \right] + 1 - \delta_1 u_{t+1}^{\eta} + \frac{\phi_1}{2} \left( \frac{i_{1t+1}^{1t+1}}{k_{1t+1}^{1t+1}} - (\Gamma_{\gamma z} - 1 + \delta_1 u_{t+1}^{\eta}) \right)^2 \right\} \hspace{1cm} (21)

$$\Gamma_{\lambda_{2t}} = \hat{\beta} E_t \left\{ \theta_2 \lambda_{3t+1} \frac{y_{t+1}}{k_{2t+1}^{2t+1}} + \lambda_{2t+1} \left[ \phi_2 \left( \frac{i_{2t+1}^{2t+1}}{k_{2t+1}^{2t+1}} - (\Gamma - 1 + \delta_2) \right) \frac{i_{2t+1}^{2t+1}}{k_{2t+1}^{2t+1}} \right] + 1 - \delta_2 + \frac{\phi_2}{2} \left( \frac{i_{2t+1}^{2t+1}}{k_{2t+1}^{2t+1}} - (\Gamma - 1 + \delta_2) \right)^2 \right\} \hspace{1cm} (22)

where the equilibrium conditions \( \bar{y}_t = y_t \) and \( \bar{b}_t = b_t \) have been imposed. \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) denote the Lagrange multipliers associated with the transition equations (13) and (14) and the constraint obtained by combining (11) and (12).

### 3. Solution method and parameter values

The solution to the model is a set of stochastic processes for the endogenous variables \( n, y, i_1, i_2, u, c, k_1, k_2, b, \lambda_1, \lambda_2 \) and \( \lambda_3 \) that satisfy the relevant transversality conditions as well as equations (10)-(22) where equation (12) has been used to eliminate \( R_t \) from the system. Since the model does not have a closed-form solution for general values of the structural parameters, the system of equations is linearized (around its deterministic steady-state) and solved numerically using the method explained in King, Plosser and Rebelo (2002).
The parameter values used in our quantitative analysis are presented in Table 1. Most values are calibrated or estimated using Canadian quarterly data for the sample 1976Q1 to 2003Q4. We now provide details on how these values were selected.

The value for the parameters $\beta$, $\alpha$, and $\nu$ are taken from Letendre (2004): $\beta = 0.993$, $\alpha = 2$, and $\nu = 1.7$. The value of the remaining parameter appearing in the periodic utility function, $\mu$, is set based on data for hours worked per capita. In our sample of data, the fraction of non-sleeping time spent working by a Canadian is approximately twenty percent. Accordingly, we select a value for $\mu$ to insure that $n$ is equal to 0.20 in steady state. When calibrating their closed-economy RBC model to the US economy King and Rebelo (1999) also choose parameter values is such a way that the fraction of time spent working is twenty percent in steady state.

We set the growth rate $\gamma_z$ equal to 1.0033, the average (quarterly) gross growth rate of the Canadian GDP per capita in our sample. This corresponds to an annual average net growth rate of 1.3%.

We calculate values for $\gamma_z$, $\rho_v$ and $\sigma_v$ by using the fact that our model implies that the inverse of the relative price of machinery and equipment in period $t$ ($p_{me,t}^{-1}$) is given by $\gamma_z^t v_t$. We construct a time series for $p_{me}$ by dividing the implicit price deflator for machinery and equipment by the implicit price deflator for consumption of non-durable goods and services. Then, we estimate the regression

$$\ln p_{me,t}^{-1} = \omega_0 + \omega_1 t + e_t$$

and use the estimate $\hat{\omega}_1$ to calculate $\gamma_z = \exp(\hat{\omega}_1) = 1.0127$. The residuals from the above regression are then regressed on their first lag to find $\rho_v = 0.9802$ and $\sigma_v = 0.0161$.

We calculate values for $g^*$, $\rho_g$ and $\sigma_g$ using data on government expenditures and GDP per capita. We regressed the log of government expenditures per capita on a constant
and a trend and interpret the residuals as estimates of $g_t$. We regress the residuals on their first lag to find $\rho_g = 0.9501$ and $\sigma_g = 0.0092$. We set $g^*$ to insure that the steady-state $G/Y$ ratio is equal to 0.2177, the average of the ratio of government expenditures to output in our sample.

We use data on depreciation and capital stocks to calculate average depreciation rate for the two types of capital. Using data on depreciation and the stock of machinery and equipment, we find an average annual depreciation rate of 0.1606 which corresponds to an average quarterly depreciation rate of 0.0379. Given numerical values for $\alpha$, $\beta$, $\gamma_x$, and $\gamma_z$ the value $\eta = 1.8951$ is selected to insure that the steady-state depreciation rate for machinery and equipment is 0.0379.

For structures, we calculate depreciation and the stock by subtracting machinery and equipment from total capital. We find an average annual depreciation rate of 0.0536 which corresponds to an average quarterly depreciation rate of 0.0131. Accordingly, we set $\delta_2 = 0.0131$. Note that the average annual depreciation rates we calculated using Canadian data are similar to the those reported by Greenwood et al. (2000) for the US economy. They find an annual depreciation rate of 0.056 for structures and 0.124 for equipment.

We use a measure of industrial capacity utilization to proxy for capital utilization. In our sample, the average capacity utilization rate is 81.6 percent. Given numerical values for $\alpha$, $\beta$, $\gamma_x$, $\gamma_z$ and $\eta$, the value $\delta_1 = 0.0557$ is selected to insure that the steady-state utilization rate is 81.6 percent.

Greenwood et al. (2000) use the values $\theta_1 = 0.18$ and $\theta_2 = 0.12$ which implies $\theta_1 + \theta_2 = 0.30$. Mendoza (2001) who calibrates his model to the Canadian economy set $\theta_1 + \theta_2 = 0.32$. Therefore, we set $\theta_1 = 0.192$ and $\theta_2 = 0.128$ to accord with $\theta_1 + \theta_2 = 0.32$ as calculated by Mendoza and with the ratios $\theta_i/(\theta_1 + \theta_2)$ for $i = 1, 2$ used by Greenwood, Hercowitz and Krussel.
In our dataset, $SD(i)/SD(y) = 2.91$ and $SD(i_1)/SD(i_2) = 1.60$, where $SD(x)$ denotes the standard deviation of $x$. The capital adjustment costs parameters $\phi_1$ and $\phi_2$ are set to insure that the model matches $SD(i)/SD(y) = 2.91$ and $SD(i_1)/SD(i_2) = 1.60$. The values for $\phi_1$ and $\phi_2$ used in the simulations are reported in the tables of results. To verify the robustness of the results, we also considered imposing $\phi_1 = \phi_2$ and selecting a value for $\phi_1 = \phi_2$ to match $SD(i)/SD(y) = 2.91$. We found that when $\phi_1$ and $\phi_2$ are set this way, the results discussed below are unchanged.

The parameter $\psi_1$ appearing in the equity premium in (4) is equal to the steady-state value of the ratio $B/Y$. Calculating the sample average for this ratio using annual Canadian data on net foreign assets and nominal GDP from 1976 to 2002 (assets are currently available up to 2002 only) yields -0.34. Accordingly, we set $\psi_1 = -0.34$. We follow Letendre (2004) and set $\psi_2 = 0.001$.

Given parameter values for $\theta_1$ and $\theta_2$, we use production function (2) and data on GDP per capita, capital utilization (proxied by industrial capacity utilization), stock of machinery and equipment per capita, stock of structures per capita (calculated as total capital stock less stock of machinery and equipment), and total hours worked to construct a series of “modified Solow residuals”

$$MSR = \ln Y - \theta_1 (\ln u_t + \ln K_{1t}) - \theta_2 \ln K_{2t} - (1 - \theta_1 - \theta_2) \ln n_t.$$  

These modified Solow residuals are detrended by linear regression. The residuals from that regression are regressed on their first lag to calculate $\rho_A = 0.9544$ and $\sigma_A = 0.0065$.

Finally, the sample correlations between the empirical equivalents of $\varepsilon_A$, $\varepsilon_g$, and $\varepsilon_v$ are $\text{cor}(\varepsilon_A, \varepsilon_g) = 0.0482$, $\text{cor}(\varepsilon_v, \varepsilon_g) = 0.0686$, and $\text{cor}(\varepsilon_A, \varepsilon_v) = -0.1286$. We use these correlations in our simulations.
4. Results

4.1 Historical moments

Table 2 reports historical moments for consumption, hours, output, total investment (the sum of investment in structures, machinery, and equipment), the trade balance-output ratio and the current account-output ratio computed using quarterly *per capita* Canadian data covering the sample 1976Q1 to 2003Q4. The trade balance-output ratio, the current account-output ratio and the logs of consumption, hours, output, total investment were detrended using the Hodrick-Prescott filter with its smoothing parameter set to 1,600.

Table 2 and other tables discussed below show standard deviations, standard deviations relative to that of output, correlations with output and first-order autocorrelations of the variables. Moments calculated using simulated data will be compared to the numbers in Table 2. When calculating the moments implied by a theoretical model, we average moments over 1000 replications. For each of these replications, we generate artificial time series of 212 periods in length and we discard the first 100 observations. Therefore, our artificial dataset have 112 observations, just like our sample of quarterly Canadian data. To make historical and artificial data comparable, we apply the HP filter to artificial data for consumption, hours, output, investment (all in percent deviations from their steady-state values) and the trade balance-output ratio and the current account-output ratio (in levels).

To evaluate the fit of a particular statistical moment predicted by a theoretical model, we use the method proposed by Gregory and Smith (1991) to construct confidence intervals for each of the statistical moments shown in the tables discussed below. By simulating the model 1000 times, we get 1000 realizations of the statistical moments.
of interest. For each of these moments, the 1000 realizations are sorted and stored in a vector. Then, the lower (upper) bound of a ninety-five percent confidence interval is given by the twenty-fifth (975th) element of the sorted vector. Similarly, the lower (upper) bound of a ninety-nine percent confidence interval is given by the fifth (995th) element of the sorted vector. In the tables showing the moments predicted by the theoretical models, the superscripts \( \ast \ast \) and \( \ast \) indicate that a moment is not statistically different from its empirical counterpart at the five and one percent levels of significance respectively.

4.2 Model without investment-specific productivity shocks

In order to compare the implications of neutral productivity shocks and investment-specific productivity shocks, we start our analysis by looking at models with only one type of productivity shocks affecting the small-open economy. The first model we consider includes only government spending shocks and neutral productivity shocks. Table 3 reports the moments for this version of the model for two sets of capital adjustment costs parameters. The top panel of Table 3 reports the results computed using the values \( \phi_1 = 0.76 \) and \( \phi_2 = 8.65 \) in the simulations. These values are selected to make the model without investment-specific shocks match \( \text{SD}(i_1 + i_2)/\text{SD}(y) = 2.91 \) and \( \text{SD}(i_1)/\text{SD}(i_2) = 1.60 \). Alternatively, one can use a common value \( \phi_1 = \phi_2 = 2.48 \) selected to make the model match \( \text{SD}(i_1 + i_2)/\text{SD}(y) = 2.91 \). These two sets of values for \( \phi_1 \) and \( \phi_2 \) yield similar results so we report only the moments computed using \( \phi_1 = 0.76 \) and \( \phi_2 = 8.65 \).

The bottom panel of Table 3 reports the results computed using the values \( \phi_1 = 3.38 \) and \( \phi_2 = 8.7 \). These values are selected to make the model with both types of productivity shocks (discussed in section 4.4) match \( \text{SD}(i_1 + i_2)/\text{SD}(y) = 2.91 \) and
SD(i_1)/SD(i_2)=1.60.

We start the discussion of the model without investment-specific shocks by focusing on the statistical moments reported in the top panel of Table 3. The model without investment-specific productivity shocks matches the volatility of output very well. The standard deviation of output is 1.70 in the model and 1.64 in Canadian data (this difference is not statistically different at the five percent level). Although the model reproduces well the volatility of investment, we must remind the reader that \( \phi_1 \) and \( \phi_2 \) are calibrated to control the relative volatility of investment. The model does less well at matching the volatility of the other variables but it is consistent with the empirical fact that consumption, the trade balance-output ratio and the current account-output ratio are less volatile than output and that investment is more volatile than output.

Table 2 shows that consumption, hours and investment are highly but not perfectly correlated with output in Canadian data. It also shows that the trade balance-output ratio as well as the current account-output ratio are moderately countercyclical. The model without investment-specific productivity shocks does poorly at matching the correlation of the variables with output. It predicts perfect correlation of consumption and hours with output, a very large correlation of investment with output and strongly countercyclical trade balance-output and current account-output ratios.

The model also has difficulties generating fluctuations in consumption, hours, output and investment that are as persistent as those observed in Canadian data. Although it should be noted that the autocorrelation of consumption predicted by the model is not statistically different from its empirical counterpart at the one percent level. The model does very well at matching the autocorrelation of the trade balance-output ratio and the current account-output ratio (both moments match their empirical counterparts at the five percent level).
A comparison of the statistical moments reported in the two panels of Table 3 clearly demonstrates the influence of the behaviour of investment on the properties of the trade balance and on the current account. Not surprisingly, increasing investment adjustment costs reduces the volatility of investment. This reduction in the volatility of investment (from 4.92 to 3.24) greatly reduces the volatility of the trade balance-output ratio and current account-output ratio. It also dramatically changes the correlation of these variables with output. For example, Table 3 shows that the standard deviation of \( \frac{tb}{y} \) goes from 0.50 to 0.19 while the correlation of \( \frac{tb}{y} \) with \( y \) goes from -0.57 to 0.12 when \( \phi_1 \) and \( \phi_2 \) are raised. This change in the correlation between \( \frac{tb}{y} \) and \( y \) can be understood by looking at the responses of \( \frac{tb}{y} \) and \( \frac{i}{y} \) to a one standard deviation productivity shock shown in Figure 4. In the figure, \( \frac{tb}{y} \) is expressed in deviation from steady state while \( \frac{i}{y} \) is expressed in percent deviation from steady state. With the larger values for \( \phi_1 \) and \( \phi_2 \), \( \frac{i}{y} \) increases less in response to a productivity shock. Therefore, \( \frac{tb}{y} \) falls less. Actually, as shown in Figure 4, \( \frac{tb}{y} \) increases slightly in response to the shock when using the larger values for \( \phi_1 \) and \( \phi_2 \).

While it is true that the model with larger capital adjustment costs fits the moment of \( \frac{tb}{y} \) and \( \frac{ca}{y} \) better, it fails to produce realistic investment volatility. So the results displayed in Table 3 show how important the properties of investment are for the properties of the trade balance and the current account. Hence, one should pay close attention to factors that affect the behaviour of investment. We now turn our attention to one such factor, investment-specific productivity shocks.

### 4.3 Model without neutral productivity shocks

In this section we investigate the dynamic properties of a special case of the model described in Section 2 where there are only government spending shocks and investment-specific productivity shocks. In this model, a positive productivity shock in period \( t \)
increases the productivity of the technology producing future machinery and equipment. Not surprisingly, investment in machinery and equipment is extremely volatile in such a model. In order to demonstrate this point, Table 4 reports two sets of statistical moments predicted by the model without neutral shocks. In the top panel, the moments reported are calculated using $\phi_1 = 3.38$ and $\phi_2 = 8.7$ in the simulations. In the bottom panel, the moments reported are calculated using $\phi_1 = 250$ and $\phi_2 = 60$ in the simulations. Even with the very large value for $\phi_1$ and $\phi_2$ that we use to produce the moments reported in the bottom panel of Table 4, the volatility of investment in machinery and equipment relative to the volatility of investment in structures is 2.9 which is larger than 1.60 (the number calculated using Canadian data). If instead we used a common value $\phi_1 = \phi_2 = 85.1$ for the capital adjustment cost parameters to match the volatility of total investment relative to that of output, the results are very similar. Thus, we report only the results computed using $\phi_1 = 250$ and $\phi_2 = 60$.

Comparing the moments in the top panel of Table 4 and those in the bottom panel of Table 3 shows that many of the predictions of the model depends importantly on the type of productivity shocks hitting the small-open economy. Investment-specific shocks generate a great deal of volatility in investment, the trade balance-output ratio and the current account-output ratio but very little volatility in consumption, hours and output. The low volatility of hours and output in the model without neutral shocks is hardly surprising. An investment-specific productivity shock increases directly the productivity of the technology producing new machinery and equipment but does not directly affect the productivity of structures and hours worked. So, as argued by Greenwood et al. (1988), a productivity shock of a given magnitude is actually “smaller” when it is an investment-specific shock (rather than a neutral shock) because it does not affect directly the productivity of all inputs used in the production of the final good. This argument is especially applicable in our set up where we have the investment-specific productivity shocks affecting only a fraction of the capital sector.
Figure 5 shows the responses of output and hours worked (in percent deviations from steady state) to a risk neutral shock (computed using the model discussed in Section 4.2) and to an investment-specific productivity shock (using $\phi_1 = 3.38$ and $\phi_2 = 8.7$ in both cases). The figure shows that the response of hours and output to an investment-specific shock is much smaller than the response to a neutral productivity shock.

The top panel of Table 4 also shows that the model without neutral productivity shocks fails dramatically to reproduce the ranking of standard deviations computed in Canadian data when we set $\phi_1 = 3.38$ and $\phi_2 = 8.7$. The model predicts that consumption, the trade balance-output and the current account-output ratios are more volatile than output while the opposite is true in our sample of Canadian data.

Comparing the autocorrelations reported in the bottom panel of Table 3 to those in the top panel of Table 4, we see that investment-specific shocks are capable to generate more persistence in $c, n$ and $y$. This finding is consistent with the findings of Greenwood et al. (1988) who consider a closed-economy model with investment-specific shocks. Figure 5 shows that the response of output and hours to an investment-specific shock have a hump shape which obviously makes the response of output and hours more persistent than their response to a neutral shock. This hump-shape response of output is not found by Greenwood et al. (2000) in their closed-economy model which is very closely related to our model. When $\phi_1 = 3.38$ and $\phi_2 = 8.7$, we find that the hump in the responses disappears when the parameter $\rho_v$ (first-order autocorrelation in investment-specific shocks) is set below 0.75. When $\phi_1 = 250$ and $\phi_2 = 60$ (as in the bottom panel of Table 4), we find that the hump in the responses disappears when the parameter $\rho_v$ (first-order autocorrelation in investment-specific shocks) is set below 0.95. Since Greenwood, Hercowitz and Krusell calibrate their model to annual data, their estimated $\rho_v$ (0.64) is much lower than the one we estimated (0.9802), which explains why they do not find a hump shape in the impulse
responses they compute.

Increasing the parameters controlling capital adjustment costs to $\phi_1 = 250$ and $\phi_2 = 60$ in order to have more realistic investment volatility yields the moments reported in the bottom panel of Table 4. Except for the volatility of the current account-output ratio that matches the data at the five percent level, the macro aggregates we consider are much less volatile in the model with investment-specific shocks than in Canadian data. Contrary to the model with neutral shocks, the current model does not match the standard deviation of output and of investment (not even at the one percent level).

Looking at the relative standard deviations in the bottom panel of Table 4, we see that the model matches the relative volatility of the trade balance at the five percent level. However, a notable shortcoming of the model (which is robust to the change in capital adjustment cost parameters) is its prediction that consumption is more volatile than output.

The correlations with output and the autocorrelations presented in the bottom panel of Table 4 closely resemble those in the top panel of Table 3. Therefore, abandoning neutral productivity shocks in favor of investment-specific shocks does not change the co-movements between the trade balance (or the current account) and output sufficiently to make it less negatively correlated with output.

The poor performance of the model with investment-specific productivity shocks only at producing realistic volatilities suggests that these shocks are not able, on their own, to capture well the properties of Canadian data when these shocks replace neutral shocks in the structural model we use.
4.4 Model with both types of productivity shocks

An important finding based on the discussion in sections 4.2 and 4.3 is that, in the structural small-open economy considered in this paper, having neutral productivity shocks or investment-specific productivity shocks does not allow the model to match the statistical moments of the trade balance, consumption and hours worked very well. Since both sources of changes in productivity might be relevant to explain business cycle fluctuations (there are evidence of this for the US economy in the paper by Fisher, 2003) and the dynamics of the trade balance (and current account), we now look at the predictions of the model described in section 2. This model incorporates government spending shocks, neutral productivity shocks and investment-specific productivity shocks.

Table 5 reports the moments calculated using our small-open economy model with both types of productivity shocks. The top panel of Table 5 reports the moments for $\phi_1 = 3.38$ and $\phi_2 = 8.7$. Again, these values for the capital adjustment cost parameters were selected to make the model match $\text{SD}(i_1 + i_2)/\text{SD}(y) = 2.91$ and $\text{SD}(i_1)/\text{SD}(i_2) = 1.60$.

As discussed in section 4.3, investment-specific shocks have little effects on consumption, hours worked and output. So it comes at no surprise that the statistical moments for consumption, hours worked and output reported in the top panel of Table 5 are very close to those reported in Table 3. Therefore, adding investment-specific shocks to an otherwise standard small-open economy RBC model fails to improve the predictions of the model regarding consumption, hours worked and output. A positive point is that the addition of investment-specific shocks does not worsen the predictions of the model regarding consumption, hours worked and output.

Given the nature of the investment-specific productivity shocks, they have important
effects on the properties of investment and on the properties of the trade balance and current account since the latters depend on investment. These important effects on $i$, $tb/y$ and $ca/y$ were discussed in section 4.3. When investment-specific shocks are added to a model that already includes neutral productivity shocks, they greatly improve the model’s predicted moments for investment and the trade balance-output ratio. To see this, compare the moments of investment and trade balance-output ratio reported in the bottom panel of Table 3 to the moments reported in the top panel of Table 5. The addition of investment-specific shocks raises the volatility of investment sufficiently to allow the model to match the standard deviation of investment observed in Canadian data.

Also, the addition of investment-specific shocks changes the comovements between investment and output sufficiently to match the correlation between output and investment well. The model implies a correlation of 0.73 while this correlation is 0.65 in Canadian data. As indicated in Table 5, the difference between the correlation implied by the model and that calculated in Canadian data is not statistically significant at the five percent level. The addition of investment-specific shocks however fails to increase the autocorrelation of investment so that is the only statistical moments of investment that the model fails to match.

As shown in Tables 3 and 4, the models with only one type of productivity shocks do not match the statistical moments of the trade balance very well. For example, the model without investment-specific shocks is unable to generate enough volatility in the trade balance-output ratio to match the Canadian data while the model without investment-specific shocks implies a strong negative correlation between output and the trade balance-output ratio. A very interesting features of the model with both types of productivity shocks is that it matches the statistical moments of the trade balance remarkably well. Actually, as indicated in the top panel of Table 5, the model matches all of the statistical moments of $tb/y$ at the five percent level.
The standard deviation of $tb/y$ is 0.86 in the model and 0.88 in the Canadian data. The standard deviation of $tb/y$ relative to the standard deviation of $y$ is 0.52 in the model and 0.54 in the Canadian data. The autocorrelation of $tb/y$ is 0.67 in the model and 0.71 in the Canadian data. Very interestingly, the correlation of $tb/y$ with $y$ is -0.06 in the model and -0.11 in the Canadian data. The slight positive correlation (0.12) between $tb/y$ and $y$ predicted by the model without investment-specific shocks (bottom panel of Table 3) and the large negative correlation (-0.68) predicted by the model without neutral shock (top panel of Table 4) combine just in the right proportion to come very close to match the correlation observed in the data. This explanation also applies to the current account-output ratio.

Given the values $\phi_1 = 3.38$ and $\phi_2 = 8.7$ needed to match $\text{SD}(i_1 + i_2)/\text{SD}(y)=2.91$ and $\text{SD}(i_1)/\text{SD}(i_2)=1.60$, neutral productivity shocks produce a weakly procyclical $tb/y$ while investment-specific productivity shocks produce a strongly countercyclical $tb/y$. Figure 6 shows the response of $tb/y$ (in deviation from its steady-state value) to a one standard deviation neutral productivity shock and to a one standard deviation investment-specific shock. The strong response of investment to an investment-specific shock implies a drop in $tb/y$ in the period of the shock while the weaker response of investment to a neutral shock leaves $tb/y$ almost unchanged in the period of the shock.

As reported in Table 1 and discussed in Section 3, the correlation between the innovations in the neutral productivity shocks ($\varepsilon_A$) and the innovations in the investment-specific shocks ($\varepsilon_v$) is -0.1286. Even though this correlation is not very large, it is certainly not zero. Since the interaction of the two types of productivity shocks is important for the properties of $tb/y$, we verify whether the results are robust to a change in this correlation. Table 5 (bottom panel) reports the moments for the model with both types of productivity shocks and a zero correlation between $\varepsilon_A$ and $\varepsilon_v$. All of the moments matched by the model at the five percent level in the top panel of
Table 5 are also matched at the five percent level in the bottom panel. Therefore, the model’s ability to match the statistical moments of investment and the trade balance-output ratio is robust to changes in the correlation between $\varepsilon_A$ and $\varepsilon_v$.

As can be seen by looking at the results for all models and parametrizations, the moments of the trade balance and current account are extremely similar. The difference between the current account and the trade balance lies in the term $(R_t - 1)B_t$. Therefore, extensions of the model that affect the properties of that term may break the close link that exists between the trade balance and the current account in our model.

5. Conclusion

We set up a small-open economy driven by government spending shocks, neutral (total factor) productivity shocks and/or investment-specific productivity shocks. The model is calibrated to quarterly Canadian data (1976Q1-2003Q4). We find that the model matches the data better when both types of productivity shocks are included. The model with both types of productivity shocks matches very well the historical moments of output and investment and matches remarkably well the historical moments of the trade balance-output ratio. However, the model does not do as well at matching the historical moments of consumption, hours worked and the current account-output ratio.
6. Appendix A: Data

The common period covered by all series is 1976Q1-2003Q4. Thus, the sample used in our empirical work is 1976Q1-2003Q4. All data were downloaded from the CANSIM II web site. CANSIM II labels are indicated in square parentheses. Unless otherwise indicated, the series are calculated using 1997 constant prices (not chained-weighted 1997 prices).

Population: Estimates of population, Canada, both sexes, 18 years and over [V466677]. This series is only available at an annual frequency. It was converted to a quarterly frequency using the procedure interpol in RATS.

Output: Gross domestic product [V1992259].

Consumption: Personal expenditures on non-durable goods [V1992232] + personal expenditures on services [V1992233].

Government expenditure: Government current expenditure on goods and services [V1992235].

Investment: Investment in residential structures [V1992239] + investment in non-residential structures [V1992241]. Investment in machinery and equipment [V1992242].

Exports: Exports of goods and controls [V1992249].

Imports: Imports of goods and controls [V1992253].

Capital: Straight-line end-year net stock of machinery and equipment [V1078485]. The stock of structure is calculated as the difference between the total straight-line end-year net stock of capital [V1078482] and the stock of machinery and equipment. These series are only available at an annual frequency. They were converted to a
quarterly frequency using the procedure \texttt{interpol} in RATS. The corresponding depreciation series are \texttt{[V1078478]} for total capital and \texttt{[V1078481]} for machinery and equipment.

\textit{Utilization:} The rate of capital utilization is proxied using industrial capacity utilization rate constructed by combining \texttt{[V142812]} for the period before 1987 and \texttt{V4331081} for the period after 1986.

\textit{Hours:} Labour force survey estimates, actual hours worked for all industries \texttt{[V4391505]}.

\textit{Current account:} total current account balance at current prices \texttt{[V114421]} deflated using the GDP deflator.

\textit{GDP deflator:} GDP at current prices \texttt{[V498086]} divided by GDP at 1997 constant prices \texttt{[V1992259]}.

\textit{Net foreign assets:} Canada’s international investment position at current prices - year ends - all foreign countries \texttt{[V235422]}. This series is annual and is available up to 2002 at this time. This series and nominal GDP is used to calibrate the parameter $\psi_1$. 

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7. Appendix B — Balanced growth path

7.1 Model

This appendix shows the derivation of the growth rates of the variables along a balanced growth path of a deterministic version of the model. We are interested in the growth rates of the variables along a balanced growth path where hours and capital utilization are constant and where capital adjustment costs are zero. A more detailed description of the model can be found in the body of the paper. In the description of the deterministic version of the model below, the shocks $A_t$, $v_t$, and $g_t$ are replaced by their unconditional means 1, 1, and $g^*$ respectively.

In our small-open economy, the representative household/producer maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[ C_t - \mu \Gamma t \right]^{1-\alpha}$$

subject to

$$Y_t = (u_t K_{1t})^{\theta_1} K_{2t} (\gamma_1 t n_t)^{1-\theta_1-\theta_2}$$  \hspace{1cm} (26)

$$C_t + I_{1t} + I_{2t} + G_t + B_{t+1} = Y_t + R_t B_t$$  \hspace{1cm} (27)

$$R_t = R^w + \psi_2 \left( e^{\psi_1 - B_t / Y_t} - 1 \right)$$  \hspace{1cm} (28)

$$K_{1t+1} = (1 - \delta_1 u_t t) K_{1t} + \gamma_1 I_{1t}$$  \hspace{1cm} (29)

$$K_{2t+1} = (1 - \delta_2) K_{2t} + I_{2t}$$  \hspace{1cm} (30)

$$G_t = g^* \Gamma t$$  \hspace{1cm} (31)

where $\Gamma = \gamma_x \gamma_z \frac{\theta_1}{1-\theta_1-\theta_2}$.
7.2 Equilibrium system of equations

Performing the maximization of the representative household/producer’s problem yields 8 first-order conditions

\begin{align*}
(C_t - \mu \Gamma_t^w n_t^w)^{-\alpha} &= \Lambda_{3t} \quad (32) \\
(C_t - \mu \Gamma_t^w n_t^w)^{-\alpha} (\mu \Gamma_t^w n_t^{w-1}) &= (1 - \theta_1 - \theta_2) \Lambda_{3t} \frac{Y_t}{n_t} \quad (33) \\
\Lambda_{3t} \theta_1 \frac{Y_t}{u_t} &= \Lambda_{1t} \eta \delta_1 u_t^{w-1} K_{1t} \quad (34) \\
\Lambda_{3t} &= \gamma_z \Lambda_{1t} \quad (35) \\
\Lambda_{3t} &= \Lambda_{2t} \quad (36) \\
\Lambda_{3t} &= \beta \{ \Lambda_{3t+1} [R^w + \psi_2 (e^{\psi_1 - \tilde{B}_{t+1}/Y_{t+1} - 1})] \} \quad (37) \\
\Lambda_{1t} &= \beta \left[ \Lambda_{3t+1} \theta_1 \frac{Y_{t+1}}{K_{1t+1}} + \Lambda_{1t+1} (1 - \delta_1 u_{t+1}^{\eta}) \right] \quad (38) \\
\Lambda_{2t} &= \beta \left[ \Lambda_{3t+1} \theta_2 \frac{Y_{t+1}}{K_{2t+1}} + \Lambda_{2t+1} (1 - \delta_2) \right] \quad (39)
\end{align*}

where \( \Lambda_1, \Lambda_2 \) and \( \Lambda_3 \) denote the Lagrange multipliers associated with the transition equations (29)-(30) and the constraint obtained by combining (27) and (28), respectively. Together with the constraints (26), (27), (29) and (30), the first-order conditions above form the system of equations to solve.

7.3 Derivation of growth rates

We denote the gross growth rate of variable \( X \) by \( \gamma_X \).

Combining equations (32) and (33) to eliminate \( \Lambda_{3t} \) yields

\[
\mu \nu \Gamma_t^w n_t^{w} = (1 - \theta_1 - \theta_2) Y_t.
\]
Since $n_t$ is constant along the balanced growth path, dividing the above equation by itself lagged once implies $\gamma_Y = \Gamma$.

Updating equation (35) by one period in the future and then substituting it into (38) yields

$$\beta \gamma_{A_1} \left[ \theta_1 \gamma_z \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta_1 u_{t+1}^\eta \right] = 1.$$  

Since $u_{t+1}$ is constant along the balanced growth path, $\gamma_z \frac{Y_{t+1}}{K_{t+1}}$ must be constant along the BGP. That implies $\gamma_{K_1} = \gamma_z \gamma_Y = \gamma_z \Gamma$.

Similarly, combining equations (36) and (39) yields

$$\beta \gamma_{A_3} \left[ \theta_2 \frac{Y_{t+1}}{K_{2t+1}} + 1 - \delta_2 \right] = 1.$$  

Therefore, $\frac{Y_{t+1}}{K_{2t+1}}$ must be constant and then we have $\gamma_{K_2} = \gamma_Y = \Gamma$.

Imposing equilibrium conditions $\bar{B}_{t+1} = B_{t+1}$ and $\bar{Y}_{t+1} = Y_{t+1}$ and dividing both sides of equation (37) by $\Lambda_3$ yields

$$\beta \gamma_{A_3} \left[ R^w + \psi_2 (e^{\psi_1} \bar{B}_{t+1}/Y_{t+1} - 1) \right] = 1.$$  

This implies that $B_{t+1}/Y_{t+1}$ must be constant along the BGP, thus, $\gamma_B = \gamma_Y = \Gamma$.

Substituting equation (35) into equation (34) and rearranging yields

$$\theta_1 \gamma_z \frac{Y_t}{K_{1t}} = \eta \delta_1 u_t^\eta.$$  

Since $u_t$ is constant along the BGP, $\frac{Y_t}{K_{1t}}$ must be constant along the BGP. Confirming that $\gamma_{K_1} = \gamma_z \gamma_Y = \gamma_z \Gamma$.

Dividing both sides of the accumulation equation for the stock of machinery and equipment (29) by $K_{1t}$ yields

$$\gamma_{K_1} = (1 - \delta_1 u_t^\eta) + \gamma_z \frac{I_{1t}}{K_{1t}}.$$  

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Since \( u_t \) is constant along the BGP, \( \gamma_z \frac{I_{1t}}{K_{1t}} \) must be constant. This implies that \( \gamma_{K_1} = \gamma_z \gamma_{I_1} \). We already have \( \gamma_{K_1} = \gamma_z \Gamma \), therefore, \( \gamma_{I_1} = \Gamma \).

Similarly, dividing both sides of the accumulation equation for the stock of structures (30) by \( K_{2t} \) yields

\[
\gamma_{K_2} = (1 - \delta_2) + \frac{I_{2t}}{K_{2t}}.
\]

Therefore, \( \frac{I_{2t}}{K_{2t}} \) must be constant along the BGP, and \( \gamma_{I_2} = \gamma_{K_2} = \Gamma \).

Dividing both sides of the representative household/producer’s budget constraint (27) by \( \Gamma^t \) yields

\[
\frac{C_t}{\Gamma^t} + \frac{I_{1t}}{\Gamma^t} + \frac{I_{2t}}{\Gamma^t} + \frac{G_t}{\Gamma^t} + \frac{B_{t+1}}{\Gamma^t} = \frac{Y_t}{\Gamma^t} + \frac{R_t B_t}{\Gamma^t}.
\]

We have proved above that the variables \( I_{1t}, I_{2t}, B_t \) and \( Y_t \) grow at rate \( \Gamma \) along the BGP. In addition, \( \frac{G_t}{\Gamma^t} = g^* \) and \( R_t \) equals world interest rate \( R^w \). Therefore, in the equation above, \( \frac{I_{1t}}{\Gamma^t}, \frac{I_{2t}}{\Gamma^t}, \frac{G_t}{\Gamma^t}, \frac{B_{t+1}}{\Gamma^t}, \frac{Y_t}{\Gamma^t} \) and \( \frac{R_t B_t}{\Gamma^t} \) are all constant. This implies that \( \frac{C_t}{\Gamma^t} \) must be constant too. Thus, \( \gamma_C = \Gamma \).

Dividing both sides of equation (32) by \( \Gamma^t \) and rearranging yields

\[
\frac{\Lambda_{3t}}{(\Gamma^{-\alpha})^t} = (\frac{C_t}{\Gamma^t} - \mu n_t^\epsilon)^{-\alpha}.
\]

We know that \( n_t \) and \( \frac{G_t}{\Gamma^t} \) are constant along the BGP, therefore, \( \Lambda_{3t}/(\Gamma^{-\alpha})^t \) must be constant, and then \( \gamma_{\Lambda_3} = \Gamma^{-\alpha} \).

From equation (35), it is straightforward to get

\[
\gamma_{\Lambda_3} = \gamma_z \gamma_{\Lambda_1}.
\]

Therefore, \( \gamma_{\Lambda_1} = \gamma_z^{-1} \gamma_{\Lambda_3} = \gamma_z^{-1} \Gamma^{-\alpha} \).

Finally, equation (36) implies \( \Lambda_2 \) and \( \Lambda_3 \) grow at the same rate, i.e., \( \gamma_{\Lambda_2} = \gamma_{\Lambda_3} = \Gamma^{-\alpha} \).
8. References


Table 1: Parameter values

Preferences:
\[ \beta = 0.993, \alpha = 2, \nu = 1.7, \mu = 2 \]

Production function:
\[ \theta_1 = 0.192, \theta_2 = 0.128 \]

Depreciation and utilization:
\[ \delta_1 = 0.0557, \delta_2 = 0.0131, \eta = 1.8951 \]

Adjustment costs and risk premium:
\[ \psi_1 = -0.34, \psi_2 = 0.001 \]
Values for \( \phi_1 \) and \( \phi_2 \) are reported in tables of results below.

Exogenous processes:
\[ \rho_A = 0.9544, \sigma_A = 0.0065, \rho_v = 0.9802, \sigma_v = 0.0161, \]
\[ g^* = 0.0686, \rho_g = 0.9501, \sigma_g = 0.0092, \]
\[ \text{cor}(\varepsilon_A, \varepsilon_g) = 0.0482, \text{cor}(\varepsilon_v, \varepsilon_g) = 0.0686, \text{cor}(\varepsilon_A, \varepsilon_v) = -0.1286, \]
\[ \gamma_x = 1.0033, \gamma_z = 1.0127 \]
Table 2: Moments from Canadian data (1976Q1-2003Q4)

<table>
<thead>
<tr>
<th></th>
<th>SD</th>
<th>SD/SD(y)</th>
<th>Cor. with y</th>
<th>Autocor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>0.92</td>
<td>0.56</td>
<td>0.74</td>
<td>0.85</td>
</tr>
<tr>
<td>n</td>
<td>1.74</td>
<td>1.06</td>
<td>0.89</td>
<td>0.86</td>
</tr>
<tr>
<td>y</td>
<td>1.64</td>
<td>1</td>
<td>1</td>
<td>0.90</td>
</tr>
<tr>
<td>i</td>
<td>4.76</td>
<td>2.91</td>
<td>0.65</td>
<td>0.86</td>
</tr>
<tr>
<td>tb/y</td>
<td>0.88</td>
<td>0.54</td>
<td>-0.11</td>
<td>0.71</td>
</tr>
<tr>
<td>ca/y</td>
<td>0.26</td>
<td>0.16</td>
<td>-0.22</td>
<td>0.66</td>
</tr>
</tbody>
</table>

SD stands for standard deviation. $c$ denotes consumption. $n$ denotes the fraction of time spent working. $y$ denotes output. $i$ is the sum of investment in structures, machinery and equipment. $tb/y$ denotes the trade balance-output ratio. $ca/y$ denotes the current account-output ratio. All data are per capita. Except for the trade balance-output ratio and the current account-output ratio, all data are in logs. All data were detrended using the HP filter with smoothing parameter 1600. Appendix A provides more details on the data used to compute the moments in Table 2.
Table 3: Moments from model without investment-specific shocks

<table>
<thead>
<tr>
<th>Variable</th>
<th>SD</th>
<th>SD/SD(y)</th>
<th>Cor. with y</th>
<th>Autocor.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>φ₁ = 0.76</td>
<td>φ₂ = 8.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>1.63</td>
<td>0.96</td>
<td>1.00</td>
<td>0.70*</td>
</tr>
<tr>
<td>n</td>
<td>1.00</td>
<td>0.59</td>
<td>1.00</td>
<td>0.71</td>
</tr>
<tr>
<td>y</td>
<td>1.70**</td>
<td>1.00</td>
<td>1.00</td>
<td>0.71</td>
</tr>
<tr>
<td>i</td>
<td>4.92**</td>
<td>2.91**</td>
<td>0.94</td>
<td>0.63</td>
</tr>
<tr>
<td>tb/y</td>
<td>0.50</td>
<td>0.29</td>
<td>-0.57</td>
<td>0.65**</td>
</tr>
<tr>
<td>ca/y</td>
<td>0.48</td>
<td>0.29</td>
<td>-0.59</td>
<td>0.66**</td>
</tr>
</tbody>
</table>

|          | φ₁ = 3.38   | φ₂ = 8.7   |              |          |
| c        | 1.60  | 0.97     | 1.00        | 0.69     |
| n        | 0.97  | 0.59     | 1.00        | 0.69     |
| y        | 1.66** | 1.00     | 1.00        | 0.69     |
| i        | 3.24  | 1.96     | 0.99        | 0.68     |
| tb/y     | 0.19  | 0.12     | 0.12**      | 0.74**   |
| ca/y     | 0.19* | 0.12     | 0.15*       | 0.74**   |

Moments from the model are averages of 1,000 replications of length 112 periods. They were computed using HP filtered percent deviations from steady state. For symmetry with Canadian data, artificial data on the trade balance-output and current account-output ratios are not expressed in percent deviation from steady state. The superscripts ** and * indicate that a moment is not statistically different from its empirical counterpart at the five and one percent levels of significance respectively.
Table 4: Moments from model without neutral productivity shocks

<table>
<thead>
<tr>
<th>Variable</th>
<th>SD</th>
<th>SD/SD(y)</th>
<th>Cor. with y</th>
<th>Autocor.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>φ₁ = 3.38</td>
<td>φ₂ = 8.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>0.57</td>
<td>1.18</td>
<td>0.97</td>
<td>0.74**</td>
</tr>
<tr>
<td>n</td>
<td>0.29</td>
<td>0.59</td>
<td>1.00</td>
<td>0.79**</td>
</tr>
<tr>
<td>y</td>
<td>0.49</td>
<td>1.00</td>
<td>1.00</td>
<td>0.79*</td>
</tr>
<tr>
<td>i</td>
<td>4.03**</td>
<td>8.31</td>
<td>0.80</td>
<td>0.67</td>
</tr>
<tr>
<td>tb/y</td>
<td>0.85**</td>
<td>1.76</td>
<td>-0.68</td>
<td>0.67**</td>
</tr>
<tr>
<td>ca/y</td>
<td>0.84</td>
<td>1.74</td>
<td>-0.72</td>
<td>0.67**</td>
</tr>
<tr>
<td></td>
<td>φ₁ = 250</td>
<td>φ₂ = 60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>0.52</td>
<td>1.27</td>
<td>0.97</td>
<td>0.70</td>
</tr>
<tr>
<td>n</td>
<td>0.24</td>
<td>0.59</td>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>y</td>
<td>0.41</td>
<td>1.00</td>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>i</td>
<td>1.20</td>
<td>2.91**</td>
<td>0.99</td>
<td>0.69</td>
</tr>
<tr>
<td>tb/y</td>
<td>0.25</td>
<td>0.61**</td>
<td>-0.64</td>
<td>0.69**</td>
</tr>
<tr>
<td>ca/y</td>
<td>0.24**</td>
<td>0.60</td>
<td>-0.64</td>
<td>0.69**</td>
</tr>
</tbody>
</table>

Moments from the model are averages of 1,000 replications of length 112 periods. They were computed using HP filtered percent deviations from steady state. For symmetry with Canadian data, artificial data on the trade balance-output and current account-output ratios are not expressed in percent deviation from steady state. The superscripts ** and * indicate that a moment is not statistically different from its empirical counterpart at the five and one percent levels of significance respectively.
Table 5: Moments from model with both types of productivity shocks

<table>
<thead>
<tr>
<th>Variable</th>
<th>SD</th>
<th>SD/SD(y)</th>
<th>Cor. with y</th>
<th>Autocor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1 = 3.38$</td>
<td>$\phi_2 = 8.7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>1.64</td>
<td>0.98</td>
<td>0.99</td>
<td>0.70</td>
</tr>
<tr>
<td>$n$</td>
<td>0.99</td>
<td>0.59</td>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>$y$</td>
<td>1.68**</td>
<td>1.00</td>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>$i$</td>
<td>4.85**</td>
<td>2.91**</td>
<td>0.73**</td>
<td>0.68</td>
</tr>
<tr>
<td>$tb/y$</td>
<td>0.86**</td>
<td>0.52**</td>
<td>-0.06**</td>
<td>0.67**</td>
</tr>
<tr>
<td>$ca/y$</td>
<td>0.85</td>
<td>0.51</td>
<td>-0.06**</td>
<td>0.67**</td>
</tr>
</tbody>
</table>

| $\phi_1 = 3.38$ | $\phi_2 = 8.7$ | corr($\epsilon_A, \epsilon_v$) = 0 |          |           |
| $c$       | 1.70 | 0.98     | 0.99        | 0.70*    |
| $n$       | 1.02 | 0.59     | 1.00        | 0.70      |
| $y$       | 1.73**| 1.00     | 1.00        | 0.70      |
| $i$       | 5.18**| 3.01**   | 0.77**      | 0.68      |
| $tb/y$    | 0.86**| 0.50**   | -0.17**     | 0.67**    |
| $ca/y$    | 0.85 | 0.50     | -0.17**     | 0.67**    |

Moments from the model are averages of 1,000 replications of length 112 periods. They were computed using HP filtered percent deviations from steady state. For symmetry with Canadian data, artificial data on the trade balance-output and current account-output ratios are not expressed in percent deviation from steady state. The superscripts ** and * indicate that a moment is not statistically different from its empirical counterpart at the five and one percent levels of significance respectively.
Figure 1: Relative prices of components of investment

Prices relative to price of non-durable goods and services
Figure 2: Relative prices of components of investment

Prices relative to price of non-durable goods
Figure 3: Cycles in output and relative price of machinery and equipment
Figure 4: Impulse response function

Trade balance/output and investment/output

Periods

$tb/y$, $i/y$ in deviation from steady state

- $tb/y$-low costs
- $i/y$-low costs
- $tb/y$-high costs
- $i/y$-high costs
Figure 6: Impulse response function
Trade balance over output

- Deviations from steady state

- Periods

- Neutral productivity shock
- Investment-specific productivity shock