On monetary policy rules for the euro area

Miguel Casares†
Universidad Pública de Navarra

Abstract

In this paper a New Keynesian model is described and partially estimated for the euro area. The model is then used to analyze the stabilizing properties of alternative monetary policy rules for the euro area: instrument (Taylor-type) rules and targeting rules. Our main finding is that if instrument rules are designed optimally they behave similarly to targeting rules both in terms of performance and robustness. This result is obtained for a variety of ECB preferences on stabilizing inflation, the output gap, and the nominal interest rate. Furthermore, the rule quasi-equivalence is also obtained if the ECB objective function is based on welfare maximization.

JEL codes: E0, E4, E5.
Keywords: Instrument rules, targeting rules, euro area.

1 Introduction

Monetary policy rules can be classified into targeting rules and instrument rules as proposed by Lars Svensson (see Svensson (1999, 2002)). Targeting rules are designed by solving a central bank’s optimizing program that defines an objective (loss) function and considers a single model describing the economy. Targeting rules are thus model dependent. Typical policy targets in the central bank objective function are variability of inflation, the output gap, or the nominal interest rate. An implicit reaction function, a monetary policy rule, might be written as the optimal response of the policy instrument (usually the nominal interest rate) to the state variables. Two influential seminal papers on targeting rules are Clarida, Galí, and Gertler (1999), and Svensson (1999).

Other authors prefer to propose simple rules that provide good stabilizing properties in a variety of models, the so-called instrument rules by Lars Svensson. The first instrument rule was the famous constant money growth rule recommended by Friedman (1959). Nowadays, John Taylor and Bennett McCallum probably are the most relevant authors on instrument rules (see McCallum (1988), Taylor (1993), Taylor (1999), and McCallum (1999)).

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††Departamento de Economía, Universidad Pública de Navarra, 31006, Pamplona, Spain. Telephone: +34 948 169336. Fax: +34 948 169721. E-mail: mcasares@unavarra.es.

1In addition, McCallum and Nelson (2003) provide a point-by-point reply to the criticism on instrument rules shown by Svensson (2003).
popular example of an instrument rule for the nominal interest rate is the well-known Taylor rule proposed in Taylor (1983). According to this interest-rate rule, the central bank would raise (cut) the nominal interest rate as a combined response to inflation deviations above (below) its target, and to positive (negative) observations of the output gap. Advocates for instrument rules argue that targeting rules based on optimal control can be misleading due to its strict dependence on the model. In addition, the reaction function implicitly derived from targeting rules becomes “operational” for policy making only within very simple models involving fully observable variables.

The objective of this paper is to investigate the performance of alternative monetary policy rules for the euro area. It may be considered a continuation to the paper by Taylor (1999) where instrument rules were tested in nine different models estimated or calibrated for the euro area. This paper extends that analysis in two ways: the introduction of the New Keynesian model and the use of targeting rules for the evaluation of instrument rules. Regarding the latter, the (in)efficiency of instrument rules can be captured directly by comparing its performance with the optimal targeting rule. As for the New Keynesian framework, it will allow us to carry out robustness exercises to model changes. So, the policy evaluation will also be conducted in terms of the robustness properties of the rules. In addition, the New Keynesian model provides a social welfare function that can be adopted by the central bank as its objective function. It will give rise to a different view to compare targeting and instrument rules (and even stabilizing targeting rules with welfare-theoretic targeting rules). The welfare-theoretic analysis of monetary policy rules is extensively described in the book by Woodford (2003a).

After describing the model in Section 2, it will be partially calibrated and estimated for the euro area in Section 3. In addition, some aspects of the euro area business cycle will be examined as part of a comparison of the model to the data. Targeting rules will be presented in Section 4. Later, the behavior of instrument rules relative to targeting rules will be tested in Section 5 for a variety of ECB policy preferences on stabilizing inflation, the output gap, and the nominal interest rate. In Section 6, the policy analysis will be extended for an ECB objective function based on welfare maximization. Finally, Section 7 concludes the paper with a list of major findings.

### 2 A New Keynesian model

As mentioned in the introduction, this paper employs a New Keynesian model for monetary policy analysis. The New Keynesian methodology has two major characteristics: first, the equations are derived from optimizing behavior and, second, there are nominal rigidities when setting prices and/or wages. Our particular New Keynesian model represents a closed economy with constant capital, monopolistic competition, and nominal rigidities when setting both prices and wages. Comparatively, the model incorporate the same nominal rigidities as in Erceg, Henderson, and Levin (2000) although it also features internal habit formation. It could also be considered a simplified version of the variable-capital model appearing in Smets

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2 Targeting rules directly depend on the model equations, on the technique for calibration/estimation of parameters, and on the definition of the central bank objective function. There are parallel intense debates among researchers on the convenience of using alternative settings for each one of these issues.

3 Some seminal papers on the New Keynesian methodology are Yun (1996), King and Wolman (1996), and Rotemberg and Woodford (1997).
and Wouters (2003), with the additional difference of having internal (not external as theirs) habit formation. Finally, the model is quite similar to the one described in Giannoni and Woodford (2003) where a fraction of prices and wages that could not be set optimally are indexed to lagged inflation. As a difference with respect to their model, it will be assumed here that non-optimal prices and wages are always indexed to the long run (steady-state) rate of inflation.

Now let us briefly describe the model. The economy consists of a continuum of households who are consumers and also producers. The representative $i^{th}$ household has the following instantaneous utility function

$$U(\zeta_t, c_t(i) - h c_{t-1}(i), l_t(i)) = \exp(\zeta_t) \left[ \frac{c_t(i) - h c_{t-1}(i)^{1-\sigma}}{1-\sigma} + \Phi \frac{l_t(i)^{1-\gamma}}{1-\gamma} \right],$$

where $\sigma, \gamma, \Phi > 0$ and $0 \leq h \leq 1$. Utility in period $t$ depends on three arguments: leisure time $l_t(i)$, a partial change in consumption from the previous period $c_t(i) - h c_{t-1}(i)$, and an economy-wide consumption preference shock $\zeta_t$. The latter follows the $AR(1)$ stochastic process $\zeta_t = \rho \zeta_{t-1} + \varepsilon_t^\zeta$ with $\varepsilon_t^\zeta \sim N(0, \sigma_{\zeta}^2)$. Consumption units are given in bundles of differentiated goods as in Dixit and Stiglitz (1977). The presence of lagged consumption features a habit formation effect as advocated by Fuhrer (2000), and now quite extended in the literature.

Households are not identical for two reasons: they produce differentiated consumption goods, and supply differentiated labor services. In both cases, they have market power to respectively decide the selling price and nominal wage that will be placed in a monopolistic competitive market. Thus, the amount produced by the $i^{th}$ household $y_t(i)$ is linked to her pricing decision $P_t(i)$ through the Dixit-Stiglitz demand equation

$$y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_p} y_t,$$

in which $\theta_p > 0$ is the elasticity of substitution across differentiated goods, and $P_t$ and $y_t$ represent Dixit-Stiglitz aggregate magnitudes of the price level and output. Technology of production is the same across households. Let us assume that the amount produced is given by a Cobb-Douglas function with constant capital (normalized to 1), and a labor-augmenting technology shock. Thus, the production of the $i^{th}$ household would be

$$y_t(i) = (\exp(z_t) n_t^d(i))^{1-\alpha},$$

with $0 \leq \alpha < 1$ as the capital-share coefficient of the Cobb-Douglas technology. The amount produced by the $i^{th}$ household is obtained by employing $n_t^d(i)$ units of the household-specific labor demand whose productivity is influenced by the economy-wide technology shock $z_t$. The technology shock is given by the $AR(1)$ stochastic process $z_t = \rho z_{t-1} + \varepsilon_t^z$ with $\varepsilon_t^z \sim N(0, \sigma_{\zeta}^2)$. In an analogous way to the household-specific output decision, the labor supply of the $i^{th}$ household $n_t^s(i)$ is given by the Dixit-Stiglitz demand equation

$$n_t^s(i) = \left[ \frac{W_t(i)}{W_t} \right]^{-\theta_w} n_t,$$

in which $\theta_w > 0$ is the elasticity of labor supply with respect to the wage $W_t$. The wage is determined by the supply of labor and the demand for labor. Let us assume that the wage $W_t$ is given by the $AR(1)$ stochastic process $W_t = \rho W_{t-1} + \varepsilon_t^w$ with $\varepsilon_t^w \sim N(0, \sigma_{W}^2)$.
where \( W_t(i) \) is the nominal wage set by household \( i \), \( \theta_W \) is the elasticity of substitution across differentiated labor services, and, \( W_t \) and \( n_t \) are the Dixit-Stiglitz aggregated figures for the nominal wage and labor.

The rate of inflation in period \( t \) is defined as \( \pi_t = (P_t/P_{t-1}) - 1 \), and the real wage as \( w_t = W_t/P_t \). In addition, let \( r_t \) denote the real rate of interest that will be reimbursed in period \( t+1 \) for a private bond purchased in period \( t \). As a result from the previous lines, the budget constraint for the representative household in period \( t \) expressed in units of the Dixit-Stiglitz aggregate output is

\[
y_t(P_t(i)/P_t)^{1-\theta_P} = c_t(i) + w_t(n_t^d(i) - n_t(W_t(i)/W_t)^{1-\theta_W}) + (1 + r_t)^{-1}b_{t+1}(i) - b_t(i). \tag{5}
\]

Households raise income from selling production \( y_t(P_t(i)/P_t)^{1-\theta_P} \). Income is spent on bundles of consumption \( c_t(i) \), on paying the market real wage to the excess of labor demand \( w_t(n_t^d(i) - n_t(W_t(i)/W_t)^{1-\theta_W}) \), and on net purchases of private bonds \( (1 + r_t)^{-1}b_{t+1}(i) - b_t(i) \).

Households also face the following time constraint

\[
n_t^s(i) + l_t(i) = 1, \tag{6}
\]

which implies that total time (normalized to 1) is devoted to either labor supply \( n_t^s(i) \) or leisure time \( l_t(i) \).

The optimizing program of the \( i \)-th representative household consists of deciding \( c_t(i), n_t^d(i), b_{t+1}(i), W_t(i), \) and \( P_t(i) \) that maximize

\[
E_t \sum_{j=0}^{\infty} \beta^j U(\zeta_{t+j}, c_{t+j}(i) - h_{c_{t-1+j}}(i), l_{t+j}(i))
\]

subject to constraints (2)-(6) in period \( t \) and analogous expected constraints for all future periods. Note that future utility values are discounted at a constant rate \( \beta < 1.0 \).

The model incorporates nominal rigidities when setting both the selling price and the nominal wage. In particular, we assume that prices and wages can be decided optimally with fixed probability as in Calvo (1983). If either the price or the nominal wage cannot be adjusted optimally they are automatically indexed to the long-run (steady-state) rate of inflation.\(^4\)

A dynamic stochastic general equilibrium (DSGE) model can be obtained from the household’s first order conditions, market-clearing conditions, a monetary policy rule, and definitions of \( \pi_t, w_t, P_t, \) and \( W_t \). There are several examples of the procedure in the literature (see Erceg et al. (2000), Smets and Wouters (2003), and Giannoni and Woodford (2003)). For practical purposes, we will skip most of the technical details on derivation of the whole DSGE model and focus on reducing the model to make it tractable for monetary policy analysis. As a starting point, the dynamic behavior of aggregate output, inflation and the real wage is governed by the following set of equations (after loglinearizing and neglecting

\(^4\)This assumption is necessary to avoid the undesirable steady-state results of the Calvo model with no price indexation (see Casares (2004) for this result).
constant terms)

\[
\dot{y}_t = a_0 \dot{y}_{t-1} + a_1 E_t \ddot{y}_{t+1} - a_2 E_t \ddot{y}_{t+2} - a_3 (R_t - E_t \pi_{t+1}) + a_4 \zeta_t, \quad (7)
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \pi \ddot{y}_t, \quad (8)
\]

\[
\ddot{w}_t = \ddot{w}_t + \frac{\alpha}{1-\alpha} \dot{y}_t - z_t, \quad (9)
\]

\[
\ddot{w}_t = \frac{1}{1+\beta} (\ddot{w}_t - \pi_t) + \frac{\beta}{1+\beta} E_t (\ddot{w}_{t+1} + \pi_{t+1}) + \kappa \pi (\ddot{m}r_s - \ddot{w}_t), \quad (10)
\]

\[
\ddot{m}r_s = \left( \frac{\gamma_{ns}}{(1-\alpha)} + c_1 \right) \dot{y}_t - c_2 \ddot{y}_{t-1} - \beta c_2 E_t \ddot{y}_{t+1} - \frac{\gamma_{ns}}{\alpha} z_t - c_3 \zeta_t \quad (11)
\]

where \( a_0 = \frac{\rho}{1+\rho} \), \( a_1 = \frac{1+\beta+\beta^2}{2} \), \( a_2 = \frac{\beta}{1+\beta} \), \( a_3 = \frac{(1-\beta)(1-\beta)}{1+\beta+\beta^2} \), \( a_4 = \frac{(1-\rho C)(1-\beta \rho_C)(1-\beta)}{1+\beta+\beta^2} \), \( \kappa \pi = \frac{(1-\alpha \rho)(1-\alpha \rho)(1-\alpha)}{\eta W (1-\alpha + \alpha \theta P)} \), \( \kappa \pi = \frac{(1-\alpha \rho W)(1-\alpha \rho W)}{\eta W (1-\beta)(1-\beta + \beta \eta W)} \), \( c_1 = \frac{\sigma (1+\beta)}{(1-\beta)(1-\beta \eta W)} \), \( c_2 = \frac{b \sigma}{1+\beta} \), and \( c_3 = \frac{(1-\beta \rho_C)}{1-\beta} \). Variables topped with a “\( \ddot{\cdot} \)” symbol represent fractional deviations of the original variable from its steady state value; for example, \( \ddot{y}_t = \log(y_t/y^{ss}) \) with \( y^{ss} \) as the steady state level of output. Both the nominal interest rate \( R_t \) and the rate of inflation \( \pi_t \) denote level deviations from its steady-state value. This notation will be maintained throughout the paper.

Equation (7) is an IS curve describing aggregate output fluctuations in an economy with consumption habit formation and constant capital (see Nelson (2002), and Amato and Laubach (2004) for similar derivations). It is obtained by jointly using the first order conditions of consumption and bonds, and the market-clearing condition of aggregate output. The habit formation structure gives rise to the presence of \( N_{t-1} \) and \( N_{t+2} \) in the IS curve. The consumption preference shock \( \zeta_t \) enters the IS curve with a positive effect on output. Actually, our consumption preference shock \( \zeta_t \) plays the role of any expenditure-type shock. Therefore, we will also refer to this shock as a demand shock.\(^5\)

The inflation dynamic behavior is governed by the New Keynesian Phillips curve (8) which was obtained from the selling price first order condition with staggered prices \( \text{à la} \) Calvo (see Sbordone (2001) for a detailed derivation). As shown in (8), inflation \( \pi_t \) depends positively on expected inflation \( E_t \pi_{t+1} \), and the aggregate real marginal cost \( \ddot{m}c_t \). Equation (9) is the loglinearization definition of the real marginal cost as obtained from the Cobb-Douglas production function (3).

As for the real wage equation (10), the papers by Erceg et al. (2000), Sbordone (2002), and Smets and Wouters (2003) describe its derivation and discuss on the implications of introducing staggered wages \( \text{à la} \) Calvo. As displayed in (10), the dynamic evolution of the real wage presents both a backward-looking and forward-looking component. There are also two effects of inflation; the real wage \( \ddot{w}_t \) is affected negatively by current inflation \( \pi_t \) and positively by expected next period inflation \( E_t \pi_{t+1} \). In addition, the gap between the aggregate leisure-consumption marginal rate of substitution and the real wage \( \ddot{m}r_s - \ddot{w}_t \) (originated by the nominal wage rigidities) enters equation (10) with a positive sign. Finally, equation (11) defines \( \ddot{m}r_s \) from the utility function (1) and the production function (3).

The system of equations (7)-(11) can be simplified by substituting the aggregate real marginal cost \( \ddot{m}c_t \) from (9) into the inflation equation (8), and equation (11) which defines the aggregate marginal rate of substitution \( \ddot{m}r_s \) into the real wage equation (10). They lead

\(^5\) Other types of demand-side shocks would be a government expenditure shock like in Rotemberg and Woodford (1999), or an investment shock shock like in Smets and Wouters (2003).
to the three-equation system

\[
\hat{y}_t = a_0 \hat{y}_{t-1} + a_1 E_t \hat{y}_{t+1} - a_2 E_t \hat{y}_{t+2} - a_3 (R_t - E_t \pi_{t+1}) + a_4 \zeta_t, \tag{12}
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \pi \left( \hat{w}_t + \frac{\alpha}{1-\alpha} \hat{y}_t - z_t \right), \tag{13}
\]

\[
\hat{w}_t = \frac{1}{\gamma} (\hat{w}_{t-1} - \pi_t) + \frac{\beta}{1-\beta} E_t (\hat{w}_{t+1} + \pi_{t+1}) + \kappa \left( \frac{\gamma^{n_{ss}}}{1-\alpha} + c_1 \right) \hat{y}_t - c_2 \hat{y}_{t-1} - \beta c_2 E_t \hat{y}_{t+1} - \frac{\gamma^{n_{ss}}}{1-\alpha} z_t - c_3 \zeta_t - \hat{w}_t, \tag{14}
\]

which includes four endogenous variables: \( \hat{y}_t, \pi_t, \hat{w}_t \) and \( R_t \), and two exogenous variables: \( \zeta_t \) and \( z_t \). So, we have four endogenous variables and only three equations. Therefore, a monetary policy rule is required to identify a time path for \( R_t \) and close to the model. Let us introduce initially the following Taylor-type instrument rule with nominal interest rate smoothing

\[
R_t = \mu_1 \pi_t + \mu_2 \hat{y}_t + \mu_3 R_{t-1} \tag{15}
\]

with \( \mu_1, \mu_2, \mu_3 \geq 0 \) excluding those combinations that leave the model undetermined. Notice that \( \hat{y}_t \) is the output gap which we define as the fractional difference between current output \( \hat{y}_t \) and potential output \( \bar{y}_t \),

\[
\hat{y}_t = \bar{y}_t - \bar{y}_t. \tag{16}
\]

The presence of the output gap in (15) involves one more variable entering the model: potential output \( \bar{y}_t \). Following Woodford (2003a), pages 247-249, potential (natural-rate) output is defined as the amount that would prevail in an economy without rigidities. By assuming fully flexible prices and wages, and solving out the model for output, the dynamic equation for fluctuations of potential output is

\[
\left( \frac{\alpha}{1-\alpha} + \frac{\gamma^{n_{ss}}}{1-\alpha} + c_1 \right) \hat{y}_t = c_2 \hat{y}_{t-1} + \beta c_2 E_t \hat{y}_{t+1} + c_3 \zeta_t + \left( 1 + \frac{\gamma^{n_{ss}}}{1-\alpha} \right) z_t. \tag{17}
\]

Potential (natural rate) output depends positively upon realizations of the supply-side shock \( z_t \), and the demand-side shock \( \zeta_t \). Moreover, the habit formation structure makes past and expected future values of the shocks also affect potential output.

In the end, the DSGE model has been reduced to a set of six equations (12)-(17) which fully determine dynamic paths for the six endogenous variables \( \hat{y}_t, \pi_t, \hat{w}_t, R_t, \bar{y}_t, \bar{y}_t \), and \( \bar{y}_t \), provided exogenous processes for \( z_t \) and \( \zeta_t \).

3 Calibration and estimation for the euro area

The parameters of the model will be split into two groups: \( \Omega_1 = \{ \alpha, \beta, \sigma, R, \gamma, \rho, \rho_\zeta \} \) contains those describing the production technology and household preferences, and

\[
\Omega_2 = \{ \mu_1, \mu_2, \mu_3, \kappa, \kappa_w, \sigma_e, \sigma_z, \sigma_\zeta \}
\]

which includes the coefficients of the Taylor-type rule, the elasticities on inflation and real wage equations, and the standard deviations of the shocks. The first group of parameters \( \Omega_1 \) will be calibrated whereas the parameters in \( \Omega_2 \) will be estimated. The partial calibration-estimation strategy was decided because we intentionally wanted to set parameters defining technology and preferences (those collected in \( \Omega_1 \)) in accordance to the standard calibration of business-cycle models similar to the one at hand. In

\footnote{For example, the coefficients should meet the constraint \( \mu_1 + \mu_3 > 1 \) when \( \mu_2 = 0 \). See Rotemberg and Woodford (1999) for a characterization of bubble solutions that lead to model indeterminacy.}
other words, calibration of $\Omega_1$ will consist of setting figures commonly used in the business cycle literature. Thus, an important portion of the economic structure of the model will be consistent with similar models and our findings on monetary policy analysis can be comparable to others. The parameters to be estimated are those more relevant for monetary issues (collected in $\Omega_2$). The estimation procedure is intended to make the model appropriate for the monetary policy analysis in the euro area that will be conducted below.

Let us begin with the calibration of the parameters in $\Omega_1$. The Cobb-Douglas production technology is characterized by the capital share coefficient $\alpha$. Here we set $\alpha = 0.36$, as standard in real business cycle models because it implies a realistic ratio of capital to output in steady state. Next, the discount rate is $\beta = 0.995$, implying a rate of intertemporal preference of 0.5% per quarter. It leads to a realistic 2% real interest rate per year in steady state. Regarding the parameters of the utility function, the consumption relative risk aversion coefficient is set at $\sigma = 1.5$, and the habit formation coefficient is $h = 0.85$, slightly higher than $h = 0.8$ as estimated for the US by Fuhrer (2000). The figures decided for $\sigma$ and $h$ are consistent with the estimates calculated for the euro area by Smets and Wouters (2003), and Andrés, López-Salido, and Nelson (2003). With respect to the leisure relative risk aversion coefficient $\gamma$, we decided to assign a value that implies a low labor supply (Frisch) elasticity to the real wage $\left(\frac{lw}{ls} \right)$. That was found in a number of empirical papers that analyze micro evidence (see Altonji (1986), Johnson and Pencavel (1986), Pencavel (1986), and Card (1994)). In turn, we set $\gamma = 16$ so as to imply a Frisch elasticity of labor supply equal to $1/8$ when leisure takes twice the time of labor hours in steady-state, $lw/ls = 2$. The last two calibrated parameters are the coefficients of autocorrelation of the shocks, $\rho_z$ and $\rho_\zeta$. The supply shock will have strong inertia by taking the standard figure from the real business cycle literature $\rho_z = 0.95$, whereas the process originating the demand shock has a moderate serial correlation as we set $\rho_\zeta = 0.6$.

The parameters in $\Omega_2$ were estimated using euro area quarterly data. Our data source for estimation is the area-wide model data base developed by Fagan, Henry, and Mestre (2001). The sample period chosen is 1980.1-2002.4. Detrending techniques were applied to obtain stationary series for output $(\hat{y})$ and the real wage $(\hat{w})$. Thus, the business-cycle component of output is obtained by taking off a linear time trend from the original series; which would imply a constant rate of growth in steady state. For the same argument, the real wage was linearly detrended. Nevertheless, the rate of inflation $(\pi)$ and the nominal interest rate $(R)$ are the original series since these variables do not possess any steady-state growth. Figure 1 contains the plots of the euro area series employed in the estimation over the selected sample period.

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7Smets and Wouters (2003) find quite a lower estimate for $h$ using a utility function with external habit formation. In our utility function specification (1), habit formation depends upon household-specific values of lagged consumption which is referred as an internal habit formation. It may explain why their estimate for the habit formation parameter is a low figure.

8Besides, if the labor supply elasticity is high the impact of a demand-type shock on potential output will be unrealistically large and variability of potential output would be explained mostly from a demand shock.

9Four quarterly series were utilized in the estimation procedure: Real GDP (YER), GDP deflator (YED), Short-Term Interest Rate (STN), and Compensation to Employees (WIN).

10It is clear, however, that both $\pi$ and $R$ feature a downward trend due to the disinflationary episode that characterized the years previous to the launching of the euro. Some authors time detrend these variables. However, we decided to maintain them as the original series so as to be consistent with our model and, therefore, assumed that the observed trend is simply a sample period (non-structural) coincidence.
The estimation procedure is aimed at replicating with the model a number of second-
moment statistics that represent the euro area business cycle. In particular, standard
deviations, coefficients of autocorrelation, and one-to-one coefficients of correlation of the
four observable variables of the model (\( \bar{y}, \pi, R, \) and \( \bar{w} \)). The vector of estimated parameters
\( \Omega_2 \) is the one that minimizes the following sum of squared deviations

\[
[F_d - F_m(\Omega_1, \Omega_2)]'W[F_d - F_m(\Omega_1, \Omega_2)],
\]

where \( F_d \) is the vector of sample data statistics, \( F_m(\Omega_1, \Omega_2) \) is the vector of model statistics,
and \( W \) is the weighting matrix. The entries in the diagonal of \( W \) are the average
sample statistics obtained for each one of the three types of statistics. All the elements
outside the diagonal are zero. Therefore, deviations are weighted taking into account its
relative size across types of statistics.

The estimation results are shown in Table 1 together with the calibrated parameters. The estimated Taylor rule coefficients are \( \mu_1 = 0.47, \mu_2 = 0.13, \) and \( \mu_3 = 0.53 \) in line with
the estimates for individual countries of the euro area reported by Clarida, Galí, and Gertler (1998). However, this estimated rule should not be intended to represent the monetary policy
behavior observed in the euro area countries. There was no single monetary policy in the
current euro area prior to 1999 which is at the very end of the sample period. Moreover, the
policy instrument was in many cases the rate of money growth and not the nominal interest
rate (especially during the 80’s). Therefore, our estimation must be simply understood as a
necessary device to close the model by finding the best estimates for such a proposed linear
relationship between \( R, \pi, \) and \( \bar{y} \).

The estimate of the Phillips curve elasticity is \( \kappa_\pi = 0.028 \). Compared to estimates from
other papers, it is higher than the figure reported in Smets and Wouters (2003) and lower
than the one found by Galí, Gertler, and López-Salido (2001). The estimate of the real wage
elasticity is \( \kappa_{\bar{w}} = 0.003 \) which is lower than the implicit estimate by Smets and Wouters
(2003). These differences might be explained by the fact that in Smets and Wouters (2003)
non-optimal prices and wages are indexed to lagged inflation whereas in our model they are
indexed to steady-state inflation.

Finally, the estimates of the (percent) standard deviations of the shocks are \( \sigma_{\xi z} = 0.95 \)
and \( \sigma_{\xi e} = 5.62 \). There is more expenditure-originated volatility because it favours the
positive correlations between the pairs of variables (\( \bar{y}, \pi \)) and (\( \bar{y}, R \)), which are observed in
the data and cannot be originated from a supply shock. Despite the size of \( \sigma_{\xi e} \) relative to \( \sigma_{\xi z} \), variability of the model is quite evenly coming from both shocks. This issue was
examined by calculating the forecast error variance decomposition at a 100-period time
horizon (25 years). We found that demand-side shocks explain most of the movements in
the output gap (76%), near two thirds of changes in current output (64%), and around half
of fluctuations of the nominal interest rate (53%) and inflation (44%). Meanwhile, shocks
on the supply sector explain the remaining variability of these variables, and most changes
in potential output (85%).

\[\text{References} \]

11 It follows the spirit of the seminal paper by Christiano and Eichenbaum (1992).
12 The average statistics obtained were utilized instead of individual statistics in order to avoid giving a
   extremely large weight to those sample statistics which are very close to zero (particularly the coefficient of
   correlation between \( \pi \) and \( \bar{w} \)).
13 Besides, their estimation technique is different.
14 A supply (technology) shock drives inflation and the nominal interest rate down while output is gradually
   moving up.
Table 2 displays a comparison between the business cycle statistics from the euro area data and those obtained in the calibrated/estimated model. Particularly, the statistics reported are the ones that were used in the estimation procedure: standard deviations (in percent annualized units), coefficients of autocorrelation, and coefficients of correlation.

As Table 2 shows, the standard deviation of output (1.73) is substantially lower than the standard deviation of inflation (2.87) in the euro area data. This difference is captured by the model although the relative volatility of output with respect to inflation is slightly higher than in the data; the standard deviation of output is 2.00 while the standard deviation of inflation is 2.73. The nominal interest rate is the most volatile variable both in the data (3.51) and in the model (3.58). Regarding the real wage, it reports a variability similar to output in the data (1.71) and lower to output in the model (1.30).

The coefficients of autocorrelation obtained from the data are high for inflation (0.88), and even higher for output (0.93), the nominal interest rate (0.96), and the real wage (0.96). The model replicates very closely these figures as the four variables also show long time persistence with coefficients of autocorrelation close to 1.0.

The capacity of the model to match the dynamic relationships between the variables can be evaluated by comparing their coefficients of correlation to output and inflation (see Table 2). Noticeably, all the coefficients of correlation reported in Table 2 are positive both in the model and in the data. Thus, inflation is slightly procyclical with a similar coefficient of correlation to output in the data (0.13) and in the model (0.16). The nominal interest rate is also weakly procyclical in the data (0.28) and in the model (0.19). By contrast, the real wage is more procyclical in the model (0.89) than in the data (0.51). The coefficient of correlation between inflation and the real wage is so close to 0.0 in the data (0.01) that we can say that there is no observed sign in its relationship. The model reports a low positive coefficient (0.16). Finally, there is a strong positive relationship between inflation and the nominal interest rate observed in the data (0.88) which is replicated by the model (0.94). Overall, the model gives a reasonably good matching of the coefficients of correlation. Furthermore, Figure 2 shows that the dynamic correlation functions which incorporate a eight-quarter period of lags and leads. As shown there, the model provides a good approximation to the actual dynamic relationships found in the data with a successful matching of the correlation lags and leads. The only significant difference to be mentioned is the stronger procyclical behavior of the real wage reported by the model (see second cell in Figure 2). The other five cells display great similarities between the model (solid lines) and the euro area data (dotted lines).

In summary, our calibrated/estimated New Keynesian model is able to replicate reasonably well many regularities observed in the euro area business cycle. Consequently, we can select it as an appropriate tool for the monetary policy analysis to be conducted in Section 5 and Section 6.

\[\text{ Remarkably, the model replicates a positive coefficient of correlation between output and the nominal interest rate. This is a standard difficulty of optimizing models with nominal rigidities: the negative slope of the structural IS curve makes it complicated to obtain such a positive correlation.} \]
4 Targeting rules

Now it is time to introduce targeting rules. Let us assume that monetary policy is aimed at targeting three objectives: inflation stability, output gap stability, and nominal interest rate (instrument) stability. The variances of these variables give the measure of their volatility over the business cycle. Since the unconditional expectations of $\pi$, $\tilde{y}$, and $R$ are zero their unconditional variances will be computed taking the squared values.\(^{16}\) Thus, the central bank objective function is represented by the following (generic) stabilizing loss function in period $t$

$$L_t = \pi_t^2 + \lambda_{\tilde{y}}\tilde{y}_t^2 + \lambda_R R_t^2$$  \hspace{1cm} (18)

where $\lambda_{\tilde{y}}$ and $\lambda_R$ are the relative weights on stabilizing the output gap and the nominal interest rate. Note that the weight on stabilizing inflation is normalized to be 1.0. The objective function (18) gives rise to the following optimizing program of the central bank

$$Min \ E_t \sum_{j=0}^{\infty} \beta^j L_{t+j}$$

subject to the following equations representing the economy

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_\pi \hat{w}_t + \frac{\alpha}{1-\beta} \kappa_\pi \tilde{y}_t + \varepsilon_t,$$ \hspace{1cm} (19)

$$\tilde{y}_t = a_0 \tilde{y}_{t-1} + a_1 E_t \tilde{y}_{t+1} - a_2 E_t \tilde{y}_{t+2} - a_3 (R_t - E_t \pi_{t+1}) + \nu_t$$ \hspace{1cm} (20)

$$\hat{w}_t = \frac{1}{1+\gamma}(\hat{w}_{t-1} - \pi_t) + \frac{\beta}{1+\gamma} E_t (\hat{w}_{t+1} + \pi_{t+1}) + \kappa_w \left( \frac{\gamma \pi^{xx}}{(1-\alpha)^{xx}} + c_1 \right) \tilde{y}_t - c_2 \tilde{y}_{t-1} - \beta c_2 E_t \tilde{y}_{t+1} - \hat{w}_t \right) + \chi_t,$$ \hspace{1cm} (21)

in all (past and future) periods.\(^{17}\) Equations (19)-(21) resemble very much the behavioral equations (12)-(14). Actually, they are reached by replacing current output $\tilde{y}_t$ with the sum of the output gap and potential output $\tilde{y}_t + \tilde{y}_t$, as implied by (16). This substitution is also applied to $\tilde{y}_{t-1}$, $\tilde{y}_{t+1}$, and $\tilde{y}_{t+2}$. As a result, the output gap appears now in the behavioral equations (19)-(21) as required for being one targeted variable of the central-bank optimizing program. The three exogenous process $\varepsilon_t$, $\nu_t$, and $\chi_t$, can be expressed as combined functions of the supply and demand shocks.\(^{18}\) In turn, the first order conditions for the optimizing

\(^{16}\)Let us recall that both $\pi$ and $R$ denote deviations from their steady-state values whereas the definition of the output gap (16) implies a zero unconditional expectation.

\(^{17}\)We follow the “timeless perspective” approach introduced in Woodford (1999), page 18.

\(^{18}\)These exogenous processes are:

$$\varepsilon_t = \kappa_\pi \frac{\alpha}{1-\alpha} \tilde{y}_t - \kappa_\pi \hat{y}_t,$$

$$\nu_t = a_0 \zeta_t - \tilde{y}_t + a_0 \tilde{y}_{t-1} + a_1 E_t \tilde{y}_{t+1} - a_2 E_t \tilde{y}_{t+2},$$

$$\chi_t = \kappa_w \left( \frac{\gamma \pi^{xx}}{(1-\alpha)^{xx}} + c_1 \right) \tilde{y}_t - c_2 \tilde{y}_{t-1} - \beta c_2 E_t \tilde{y}_{t+1} - \hat{w}_t \right) + \chi_t,$$

with $\tilde{y}_t$ fully exogenous as defined in (17).
program are (19)-(21), and

\begin{align}
2\pi_t + \varphi^{PH}_t - \varphi^{PH}_{t-1} + \frac{1}{1+\beta} \varphi^w_t - \frac{1}{1+\beta} \varphi^w_{t-1} - a_3 \beta^{-1} \varphi^{IS}_{t-1} &= 0, \\
2\lambda_y \tilde{y}_t - \kappa_\pi \alpha \varphi^{PH}_t - \kappa_w \left( \frac{\sigma}{\Pi - \alpha} \right) \varphi^w_t + \beta \kappa_w c_2 E_t \varphi^w_{t+1} + \kappa_w c_2 \varphi^w_{t-1} + \\
\varphi^{IS}_t - a_0 \beta E_t \varphi^{IS}_{t+1} - a_1 \beta^{-1} \varphi^{IS}_{t-1} + a_2 \beta^{-2} \varphi^{IS}_{t-2} &= 0, \\
2\lambda_R R_t + a_3 \varphi^{IS}_t &= 0, \\
-\kappa_\pi \varphi^{PH}_t + (1 + \kappa_\pi) \varphi^w_t - \beta \frac{1}{1+\beta} E_t \varphi^w_{t+1} - \frac{1}{1+\beta} \varphi^w_{t-1} &= 0,
\end{align}

where \( \varphi^{PH}_t, \varphi^{IS}_t, \) and \( \varphi^w_t \) are the Lagrange multipliers respectively attached to the Phillips curve (19), the IS curve (20), and the real wage equation (21). The optimal control program comprises seven equations (19)-(25) that can be solved for the seven endogenous variables \( \pi_t, \tilde{y}_t, \tilde{w}_t, R_t, \varphi^{PH}_t, \varphi^{IS}_t, \) and \( \varphi^w_t \). The targeting rule would be the solution path for \( R_t \).

Can we express it in an operational way? In spite of the rather simple New Keynesian (see (24)) which is known by the central bank.

wage and the IS-related Lagrange multipliers, and the Lagrange multipliers might help the main objective. Besides, “output gap stability” or “interest rate stability” these figures should be because stabilizing either the output gap or the nominal interest rate which would imply close-to-zero figures for \( \xi_t, v_t, \) and \( \chi_t \).

As a practical consequence, the targeting rule would be difficult to apply due to the large numbers of variables involved. Furthermore, some of the state variables are not observable by the monetary authorities.\(^19\)

According to these two arguments (too many variables and presence of unobservable variables), we will try to approximate the behavior of a targeting rule with much simpler instrument rules. It will serve to exemplify how instrument rules may replicate quite closely the optimal policy behavior of targeting rules without undue practical complexity.

\section{Evaluating alternative monetary policy rules}

This section is divided into two parts. In the first part attention will be focused on the performance of instrument rules and targeting rules (efficiency). Particularly, we will measure how much instrument rules deviate from the optimal targeting rule. In the second part of the section, rules will be compared according to their robustness to model changes. In other words, we will examine how poorly they perform if the model that represents the economy is not correct in some aspect.

As a preliminary issue, some discussion on the values assigned to the loss function weights (\( \lambda_\tilde{y} \) and \( \lambda_R \)) needs to be addressed. The ECB preferences with respect to the policy targets might play a key role in the design of an euro area monetary policy rule (either targeting or instrument rule). The main objective declared by the ECB is “to maintain price stability”,\(^20\) which would imply close-to-zero figures for \( \lambda_\tilde{y} \) and \( \lambda_R \). However, it is not clear how small these figures should be because stabilizing either the output gap or the nominal interest rate might help the main objective. Besides, “output gap stability” or “interest rate stability”

\[^{19}\]Particularly the unobservable variables are the three exogenous terms (shocks), the lagged output gap, and the Lagrange multipliers \( \varphi^{PH}_{t-1}, \varphi^w_{t-1} \). Put differently, the only observable variables are the lagged real wage and the IS-related Lagrange multipliers, \( \varphi^{IS}_{t-1} \) and \( \varphi^{IS}_{t-2} \), that depend only on the nominal interest rate (see (24)) which is known by the central bank.

\[^{20}\]See European Central Bank (2003).
may be considered secondary objectives for the ECB. In any case, our analysis will cover a variety of policy preferences. Since the weight on stabilizing inflation is always 1.0, the rules will be tested under different parameterization for $\lambda_{\pi}$ and $\lambda_R$. The proposed grids of three different values for $\lambda_{\pi}$ and $\lambda_R$ are

$$\lambda_{\pi} = [0, 0.01, 0.1] \text{ and } \lambda_R = [0, 0.1, 0.5],$$

which, by making combinations, give rise to a total number of 9 pairs of policy parameters $(\lambda_{\pi}, \lambda_R)$.

### 5.1 Efficiency

The targeting rule is completely efficient since it is the rule obtained from the central-bank optimizing program.\(^{21}\) In turn, the level of efficiency of any instrument rule can be obtained by examining how much it deviates from the targeting rule. As in Levin and Williams (2003), this deviation can be measured by the percent change in the unconditional expectation of the instantaneous loss function (18), denoted $%\Delta L$. However, this measure does not have any intuitive significance because it represents loss units. One alternative way of showing differences is by comparing the (annualized) standard deviations of inflation $\sigma_{\pi}$, the output gap $\sigma_{\pi}$, and the nominal interest rate $\sigma_R$, as components of the objective function (18). Both ways will be used to compare rules performance.

As a preliminary exercise, let us examine the Taylor-type instrument rule (15) with the estimated coefficients $\mu_1 = 0.47$, $\mu_2 = 0.13$, and $\mu_3 = 0.53$. Table 3 shows a poor stabilizing capacity of the estimated rule in comparison to targeting rules for any pair of ECB weights $(\lambda_{\pi}, \lambda_R)$. The percent change of the loss function (%$\Delta L$) is always higher than 700%, and extremely high when $\lambda_R = 0.0$. In many cases, the annualized standard deviations obtained with the targeting rule are substantially lower than the ones obtained with the estimated Taylor rule (compare figures of $\sigma_{\pi}$, $\sigma_{\pi}$, and $\sigma_R$ across columns of Table 3). Hence, the stabilizing performance of the estimated Taylor rule is far from being satisfactory for the ECB stabilizing preferences considered here. Alternatively, we searched for the policy weights of a targeting rule that best replicated the volatilities observed in the data. There is one added row at the bottom of Table 3 that shows our findings. The best matching is obtained with $\lambda_{\pi} = 2.37$ and $\lambda_R = 1.83$. Nevertheless, the approximation is still very poor since the value of the loss function with the estimated Taylor-type rules is 387% higher. Roughly speaking, the standard deviations with the targeting rule are half of the values provided with the estimated rule. As a result, there is no parameterization of a stabilizing targeting rule able to approximate well the variabilities observed in the data.

Does the previous paragraph imply that Taylor-type rules are very inefficient in the euro area? Not necessarily. We still need to analyze the performance of the best Taylor-type rule according to our euro area model and the ECB stabilizing preferences. In other words, the Taylor-type instrument rule (15) can be optimized by setting the values of the coefficients of the rule that best approximate the targeting rule. This exercise was done by finding the triplet of coefficients $(\mu_1^*, \mu_2^*, \mu_3^*)$ that give a minimal value in the loss function (18) for each pair of policy weights $(\lambda_{\pi}, \lambda_R)$. Results of this policy exercise are shown in Table 4. The

\(^{21}\)Nevertheless, Jensen and McCallum (2002) argue that the targeting rule obtained under timeless perspective commitment is slightly suboptimal according to the criterion of minimizing the unconditional expectation of the central bank loss function. Woodford (2003b) responds to that criticism.
overall picture looks very different from Table 3. Let us start by discussing the results on the strict inflation targeting monetary policy ($\lambda \equiv \lambda_R = 0.0$). This policy rule deserves a special attention as representative of the ECB first objective, say, price stability. As Table 4 displays, the strict inflation targeting can be fully achieved by setting a Taylor-type instrument rule with $\mu_1 = \infty$, and $\mu_2 = \mu_3 = 0.0^{22}$ If so, there would be constant inflation ($\sigma_\pi = 0.0$), and a extremely high interest rate volatility ($\sigma_R = 244.5$). This policy is fully efficient in the sense of keeping inflation “on target” but it requires a huge instrument volatility. Indeed, variability of the nominal interest rate is so high that this policy cannot be considered desirable from a realistic policy perspective.

Under other ECB policy weights, the targeting rule always performs slightly better than the optimized instrument rule. Particularly, the loss function values obtained with the instrument rule are between 4% and 35% higher than the ones obtained with the targeting rule. Regarding the standard deviations, values of $\sigma_y$, $\sigma_\bar{y}$, and $\sigma_R$ obtained under the Taylor-type instrument rule are always very close to the ones obtained with the targeting rule. In fact, there are a number of cases in which the standard deviation of one targeted variable (especially $\sigma_\bar{y}$) is lower with the optimized instrument rule. Therefore, we find that an optimized instrument rule à la Taylor is nearly as efficient as the targeting rule. Confirming this result, Figure 3 plots impulse response functions for the particular ECB preference setting $\lambda \equiv \lambda_R = 0.1$ and $\lambda_R = 0.5$. It can be observed that responses to either supply or demand shocks are indeed very similar with the targeting rule (solid line) and the optimized Taylor-type rule (dotted line) for the three targeted variables $\pi$, $\bar{y}$, and $R$. Actually, it is quite difficult to distinguish them because they almost overlap (especially in the responses of $\pi$). By contrast, responses are always much larger with the estimated Taylor-type rule (dash-dotted line). Therefore, the variability induced by the estimated Taylor-type rule is much bigger than the one observed when optimizing that instrument rule.

The design of optimized Taylor-type rules leads to three interesting conclusions on the optimal coefficients $(\mu_1^*, \mu_2^*, \mu_3^*)$ which also appear in Table 4. First, the optimal inflation coefficient $\mu_1^*$ is very sensitive to the ECB-preference parameters $\lambda \equiv \lambda_R = 0.0$ and $\lambda_R$. Thus, if the ECB pursues strict inflation targeting ($\lambda \equiv \lambda_R = 0.0$) the optimal inflation coefficient $\mu_1^*$ rises asymptotically to $\infty$. The value of $\mu_1^*$ remains high when $\lambda \equiv \lambda_R$ and $\lambda_R$ are low. As either the weight of output gap volatility $\lambda \equiv \lambda_R$ or, especially, the weight of interest rate volatility $\lambda_R$ goes up the optimal inflation coefficient $\mu_1^*$ goes substantially down (check this result in Table 4). The second result states that the optimal output gap coefficient $\mu_2^*$ is generally low.23 Actually, $\mu_2^*$ is only substantially higher than zero with a positive $\lambda \equiv \lambda_R$ and $\lambda_R = 0.0$. Our third conclusion is that the interest rate smoothing coefficient $\mu_3^*$ is slightly higher than one for all the proposed combinations of $(\lambda \equiv \lambda_R)$, except in the limiting case of strict inflation targeting ($\lambda \equiv \lambda_R = 0.0$) when it becomes zero. Moreover, the value of $\mu_3^*$ moves in a very narrow range always above the unit coefficient in all the former cases. This high policy inertia (superinertia) on Taylor-type rules was also recommended in Rotemberg and Woodford (1999).24

It is frequently argued that an instrument rule should be fully operational in the sense

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22 The optimized Taylor-type rule contains an infinite coefficient ($\mu_1^* = \infty$) which is represented in practice by $\mu_1^* = 10^6$.

23 A low value for $\mu_2^*$ in Taylor-type rules was already suggested by Rotemberg and Woodford (1999) to approximate targeting rules that maximize social welfare from a sticky price optimizing model.

24 See Woodford (2003b) for further discussions on monetary policy inertia.
that the variables involved are perfectly observable by the central bank (e.g., see McCallum (1999)). The Taylor-type instrument rule (15) includes the output gap \( \hat{y}_t \) defined in (16) as the fractional difference between fluctuations in current output (\( \hat{y}_t \), observable) and fluctuations in potential output (\( \hat{y}_t \), unobservable). Thus, if \( \hat{y}_t \) cannot be identified, the inclusion of \( \hat{y}_t \) in (15) would no longer maintain the rule operational. We tested two alternative solutions for the identification problem:

i) Substitute the unobservable output gap \( \hat{y}_t \) for the observable current output \( \hat{y}_t \). Results are shown in Table 5. Overall, the optimized instrument rule with \( \hat{y}_t \) maintains most of the good performance of the original optimized Taylor-type rule for all the cases analyzed (compare Table 5 with Table 4). There is an increase in the loss function in seven cases, whereas the losses remain the same in the two cases that had no output gap response in the instrument rule. Anyway, differences are quantitatively very small and the triplets of standard deviations are almost identical to the ones provided in Table 4. Also, it is worth noticing that the optimal values of \( \mu_2^* \) are now very low (always below 0.10) or even equal to 0.0 in a number of cases. Accordingly, an operational instrument rule could also be designed assuming that there is no output-related response as argued in our second proposal.

ii) Set the output gap coefficient at \( \mu_2^* = 0.0 \). The absence of an output gap term does not lead to any significant worsening in the stabilizing properties of optimized instrument rules (see Table 6). The increments of losses relative to the targeting rule are still quite small and only slightly higher than the ones obtained with a positive output gap response (compare Table 6 with Table 4). In addition, the standard deviations obtained with \( \mu_2^* = 0.0 \) are also very similar to the ones obtained with a positive \( \mu_2^* \). Consequently, the optimized instrument rule (15) constrained to \( \mu_2^* = 0.0 \) is nearly as efficient as the unconstrained optimized rule.

After examining the performance of the rules, we can conclude by saying that instrument rules à la Taylor can be designed to yield stabilizing properties that closely replicate those obtained with targeting rules. Moreover, two proposals for operational instrument rules which involve only observable variables were also evaluated and still maintained good performance relative to targeting rules.

5.2 Robustness

Exercises on robustness are conducted by testing how the monetary policy rules obtained under one model specification perform in different model settings (see Levin and Williams (2003) as one reference paper). Implicitly, we are supposing that the monetary authorities take a modeled representation of the economy which is “wrong” in some particular aspect. Thus, the policy rule is designed according to one model which is not the “right” model. Here we will consider three changes in the baseline model to evaluate robustness: i) eliminate price rigidities (assume flexible prices), ii) eliminate wage rigidities (assume flexible wages), and iii) set household’s preferences without habit formation.

i) Robustness to price flexibility.

If prices are fully flexible, the first order condition of the goods price implies that the real marginal cost is always constant, i.e. \( \hat{m}_c = 0 \). Using this condition in the real marginal cost definition (9) yields

\[
\hat{y}_t = \left( \frac{1-\alpha}{\alpha} \right) \hat{w}_t + \left( \frac{1-\alpha}{\alpha} \right) z_t, \quad (13')
\]
which would replace the New Keynesian Phillips curve (13) in the six-equation system described in Section 2. The robustness analysis is carried out by solving the model with flexible prices (including (13')) where the monetary policy rule is obtained supposing that the economy featured price rigidities (i.e., using a “wrong” model by including (13) instead of (13')). Table 7 informs about the results by showing the standard deviations obtained with the wrongly-defined targeting and instrument rules to be compared with the ones obtained with the correct targeting rule. It also reports their increments in the loss function ($\% \Delta L$) relative to the loss value obtained with the correct targeting rule.

Three major conclusions arise after observing Table 7. First, both targeting rules and optimized Taylor-type rules are fully robust to price stability if the ECB pursues strict inflation targeting ($\lambda_y = \lambda_R = 0.0$). In that case both rules deliver the optimal monetary policy even though they were designed assuming price stickiness. Second, both rules are moderately robust to price flexibility in all cases except when the ECB stabilizing preferences include a positive output gap targeting ($\lambda_y \geq 0.0$) and no interest rate targeting ($\lambda_R = 0.0$). In these two cases, the loss function value and the standard deviations of the targeted variables are clearly higher than the ones obtained under the “right” targeting rule. In the six cases with $\lambda_R > 0.0$, deviations from the optimal rule are small in terms of loss function changes and standard deviations (see Table 7). So, rules are especially robust to price flexibility when there is nominal interest rate targeting. Third, the rules report a similar level of robustness for any pair of policy weights. We cannot conclude that the optimized Taylor-type rule is more robust to price flexibility than the targeting rule (or vice versa).

Comparing the $\% \Delta L$ columns of Table 7, the instrument rule is more robust (lower $\% \Delta L$) in 4 cases, the targeting rule in other 4 cases, and they both are fully robust in the limiting case of strict inflation targeting.

\textit{ii) Robustness to wage flexibility.}

If there were no rigidities on setting wages, the nominal wage first order condition would give rise to a constant ratio of the consumption-leisure marginal rate of substitution over the real wage. It implies (in loglinearized terms) $\tilde{\text{m}}\tilde{s}_t = \tilde{w}_t$. By using the definition of $\tilde{m}\tilde{s}_t$ given in (11) we obtain the following real wage equation

$$\tilde{w}_t = \left( \frac{c_1}{(1-\alpha)\tau_x} + c_1 \right) \tilde{y}_t - c_2 \tilde{y}_{t-1} - \beta c_2 E_t \tilde{y}_{t+1} - \frac{c_2}{\tau_x} z_t - c_3 z_t,$$

which would replace equation (14) in the six-equation system of Section 2.

How do monetary policy rules behave in a flexible-wage economy when they were designed assuming wage rigidities? Are they robust to wage flexibility? Table 8 displays a comparison relative to the optimal targeting rule with the right model. Results are in line with the robustness to price flexibility. Both instrument rules and targeting rules are perfectly robust to wage flexibility if the ECB pursues strict inflation targeting ($\lambda_y = \lambda_R = 0.0$) because they completely stabilize inflation. If there is output gap targeting ($\lambda_y > 0.0$) and no interest rate targeting ($\lambda_R = 0.0$), both the inflation and output gap stabilization targets can be simultaneously attained with a targeting rule (see Erceg et al. (2000) for more details on this result). Remarkably, the incorrect targeting rule still delivers full stabilization of inflation and the output gap. In these two cases of Table 8 (second and third row), the “wrong” instrument rule reports an infinite increment in its loss function while the application of the
the following parameterization at zero \((2000)\). This is readily included in the model by setting the habit formation coefficient \(\eta^c\) obtained is only slightly higher with the instrument rule relative to the targeting rule. These similarities are even more clear when there is nominal interest rate targeting \((\lambda_R > 0)\). The Taylor-type rule is then more robust to flexible wages than the targeting rule although figures of increments in the loss function and standard deviations are quite alike. In any case, deviations from the performance of the “right” targeting rule are always moderate and the effects of model mis-specification are smaller for higher values of \(\lambda^c\) and \(\lambda_R\). The wrong policy making tend to increase volatility of inflation and reduce volatility of the nominal interest rate. The volatility of the output gap is generally low in all cases since one source of nominal frictions has been eliminated.

Summarizing the previous paragraph, “wrong” Taylor-type instrument rules are fully robust to wage flexibility in one case \((\lambda^c = \lambda_R = 0.0)\), moderately robust in 6 cases \((\lambda_R > 0.0)\), and no robust at all in 2 cases \((\lambda^c > 0, \lambda_R = 0.0)\). As for the “wrong” targeting rules, they are fully robust in 3 cases \((\lambda_R = 0.0)\) and moderately robust in the other 6 cases \((\lambda_R > 0.0)\). When comparing across types of rules, Taylor-type rules are more robust to wage flexibility than targeting rules except in 2 cases (when \(\lambda^c > 0\) and \(\lambda_R = 0.0\)) that the opposite result is found.

\[\text{iii) Robustness to preferences without habit formation.}\]

Finally, we will study the robustness of policy rules to some change in households’ preferences. In particular, the utility function will no longer feature habit formation as in Erceg \textit{et al.} (2000). This is readily included in the model by setting the habit formation coefficient at zero \((h = 0.0)\). In turn, the equations of the model (12)-(17) can be maintained with the following parameterization \(a_0 = 0.0, a_1 = 1.0, a_2 = 0.0, a_3 = \frac{1}{\beta}, a_4 = \frac{(1-\rho_c)}{\sigma}, c_1 = \sigma, c_2 = 0.0\) and \(c_3 = 1.0\), that affect the IS curve, the real wage equation, and the potential output equation as follows

\[
\begin{align*}
\hat{y}_t &= E_t \hat{y}_{t+1} - \frac{1}{\sigma} (R_t - E_t \hat{\pi}_{t+1}) + \frac{(1-\rho_c)}{\sigma} \zeta_t, \\
\hat{w}_t &= \frac{1}{1-\beta} (\hat{w}_{t-1} - \hat{\pi}_t) + \frac{\beta}{1-\beta} E_t (\hat{w}_{t+1} + \hat{\pi}_{t+1}) + \kappa_w \left( \left( \frac{\gamma_{\pi}^{**}}{(1-\alpha)\lambda^c} + \sigma \right) \hat{y}_t - \frac{\gamma_{\pi}^{**}}{\lambda^c} z_t - \zeta_t - \hat{w}_t \right), \\
\left( \frac{\alpha}{1-\alpha} + \frac{\gamma_{\pi}^{**}}{(1-\alpha)\lambda^c} + \sigma \right) \hat{w}_t &= \zeta_t + \left( 1 + \frac{\gamma_{\pi}^{**}}{\lambda^c} \right) z_t.
\end{align*}
\]

Now the robustness exercise consists of evaluating the stabilizing properties of rules wrongly obtained by assuming habit formation \((h = 0.85)\) in this economy without habit formation \((h = 0.0)\). The results on the robustness analysis are shown in Table 9. Once again both optimized Taylor-type rules and targeting rules are fully robust to a change in household preferences if the ECB only targets inflation volatility \((\lambda^c = \lambda_R = 0.0)\). In the other 8 combinations of the ECB policy weights \((\lambda^c, \lambda_R)\), results are mixed being each rule more robust in half of the cases. Targeting rules are clearly more robust when \(\lambda^c > 0.0\) and \(\lambda_R = 0.0\) because instrument rules fail to reduce output gap volatility \((\sigma_{\pi})\) with the “wrong”

\(^{25}\text{The percent change} \%(\Delta L) \text{under the “wrong” instrument rule becomes infinity because it delivers some positive value in the loss function while the targeting rule achieves a zero loss. In percent change that implies an infinite change.}\)
instrument rule is more than twice the value obtained under the “wrong” targeting rule. On the contrary, if the ECB targets the nominal interest rate volatility ($\lambda_R > 0.0$), Taylor-type optimized rules are more robust to preference changes. In these cases, the implementation of the “wrong” targeting rules generate significant increments of $\sigma_R$ and reductions of $\sigma_R$. Nevertheless, differences in standard deviation are never very high. Thus, we could affirm that both types of rules are moderately robust to preferences without habit formation.

A general conclusion can be assessed after discussing the three exercises of robustness. Both targeting rules and optimized instrument rules are moderately robust to changes in price/wage rigidities and households’ preferences. Furthermore they generally show a similar level of robustness because they deviate in analogous magnitudes from the stabilizing properties of the correct optimal rule. 26 Thus, targeting and optimized Taylor-type instrument rules not only reach a similar level of performance, they also show analogous levels of robustness to model changes.

6 Extensions: The welfare-theoretic loss function

Following Woodford (2003a), chapter 8, and Giannoni and Woodford (2003), the welfare-theoretic loss function according to our structural model can be approximated by the second-order function

$$\Sigma_t = \lambda_\pi \pi_t^2 + \lambda_x \pi_t (\pi_t^W) + \lambda_{\delta y_{t-1}}(\bar{y}_t - \delta \bar{y}_{t-1})^2$$

(18')

in which $\pi_t^W$ is nominal wage inflation, $\pi_t = \bar{w}_t - \bar{w}_{t-1} + \pi_t$, and $0 \leq \delta \leq h$. The weights of the policy targets are fully defined from the structural parameters as follows:

$$\lambda_\pi = \frac{\theta_{\pi \kappa_{\pi}}}{\theta_{\pi \kappa_{\pi}} + \theta_{W}(1-\alpha-1(1+\beta)\kappa_{\pi})}, \lambda_x = 1 - \lambda_\pi, \text{ and } \lambda_{\delta y_{t-1}} = \lambda_\pi \frac{\kappa_y}{\theta_{\pi \kappa_{\pi}} + \theta_{W}(1-\alpha-1(1+\beta)\kappa_{\pi})}.$$ 27

The value of $\lambda_{\delta y_{t-1}}$ depends on two new parameters: $\varphi$ is the elasticity of intertemporal substitution with a habit formation structure ($\varphi = \frac{1-\beta h}{\sigma}$), and $\delta$ is the smaller root of the quadratic equation

$$h\varphi(1 + \beta \delta^2) - \left[\frac{\alpha}{1-\alpha} + \frac{\gamma_{ss}}{1-\alpha}\right] + \varphi(1 + \beta h^2) \delta = 0.$$  \hspace{0.8cm} (26)

The figure assigned to the the elasticity of intertemporal substitution $\varphi$ depends on the calibrated parameters $\beta$, $h$, and $\sigma$. Inserting the numbers assigned in the calibration of Section 3 ($\beta = 0.995$, $h = 0.85$, and $\sigma = 1.5$) leads to $\varphi = \frac{(1-\beta h)(1-h)}{\sigma} = 0.015$. With respect to $\delta$, the roots of equation (26) also depends on the values of parameters calibrated in Section 3. Using their numbers, the smaller root was found to be $\delta = 0.60$.

Notice that the Dixit-Stiglitz elasticities $\theta_P$ and $\theta_W$ appear in the determination of the policy weights $\lambda_\pi$, $\lambda_x$, and $\lambda_{\delta y_{t-1}}$. Their values will be calibrated using the elasticity parameters $\kappa_{\pi}$ and $\kappa_w$, defined in Section 2 and estimated for the euro area in Section 3. Their definition and estimation resulted in $\kappa_{\pi} = \frac{(1-\beta \eta_{P})(1-\gamma_{P})}{\eta_{P}(1-\alpha-\beta \eta_{P})} = 0.028$, and $\kappa_w = \frac{(1-\beta \eta_{W})(1-\gamma_{W})}{\eta_{W}(1+\alpha-\beta \eta_{W})} = 0.003$. The Calvo probabilities $\eta_P$ and $\eta_W$ must be calibrated in order to respectively determine the values of $\theta_P$ and $\theta_W$. Here we set $\eta_P = \eta_0 = 2/3$ which

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26 The opposite result is found by Levin, Wieland, and Williams (1999), and Levin and Williams (2003) in robustness exercises with respectively four and three different models calibrated with US data.

27 See Woodford (2003a), chapters 6-8, for a characterization of the parameter-based policy weights in alternative welfare-theoretic loss functions.
imply that on average both prices and wages are decided optimally once every three quarters (i.e., nine months).\textsuperscript{28} So, recalling the calibrated parameters $\beta = 0.995$ and $\alpha = 0.36$, and setting the Calvo sticky-price parameter at $\eta_P = 2/3$, the estimated value $\kappa_\pi = 0.028$ in the Phillips curve is obtained with an elasticity of substitution across differentiated goods equal to $\theta_P = 9.3$. Thus, we set $\theta_P = 9.3$. It implies that in steady-state the mark-up of prices over marginal costs is nearly 11%.

Analogously, assuming a Calvo sticky-wage probability $\eta_W = 2/3$, using the calibrated parameters $\beta = 0.995$ and $\gamma n^{ss}/l^{ss} = 8$, the estimate of the slope of the real wage curve $\kappa_w = 0.003$ is obtained for an implied elasticity of substitution across differentiated labor services equal to $\theta_W = 3.4$.

Summarizing, the calibrated-estimated figures of our euro area model parameterization $(\theta_P = 9.3, \kappa_\pi = 0.028, \theta_W = 3.4, \beta = 0.995, \alpha = 0.36, \kappa_w = 0.003, h = 0.85, \varphi^{-1} = 0.015, \delta = 0.60)$ result in the following welfare-theoretic policy weights: $\lambda_\pi = 0.47$, $\lambda_{nW} = 0.53$, and $\lambda_{\bar{y} - \bar{w}} = 0.13$. These numbers are quite different from the estimates of Giannoni and Woodford (2003) for the US economy. Their inflation targeting weight $\lambda_\pi$ is very close to 1.0 and therefore the wage inflation weight $\lambda_{nW}$ is very low. Likewise, the weight on the volatility of the adjusted first difference of the output gap $\lambda_{\bar{y} - \bar{w}}$ is very close to 0.0. Two reasons can be pointed out to explain the differences: first, their implicit estimated value of $\theta_P$ for the US is very high as it takes a value almost 90 times larger than the one assigned to $\theta_W$.\textsuperscript{29} It leads to place much more weight on price inflation stabilization relative to wage inflation targeting. The second difference has to do with the elasticity of intertemporal substitution $\varphi^{-1}$. Giannoni and Woodford (2003) find an implicit estimate of this elasticity for the US equal to $\varphi^{-1} = 1.33$ which is much higher than our calibrated value for the euro area ($\varphi^{-1} = 0.015$). Since the elasticity of intertemporal substitution $\varphi^{-1}$ affects negatively to $\lambda_{\bar{y} - \bar{w}}$ (as defined above), a very high $\varphi^{-1}$ brings about a much lower figure for $\lambda_{\bar{y} - \bar{w}}$. Thus, it can explain why they set such a low weight on $\lambda_{\bar{y} - \bar{w}}$.

Similarly to stabilizing targeting rules, the welfare-based targeting rule is the one obtained by minimizing $E_t \sum_{j=1}^{\infty} \beta^j \Sigma_{t+j}$ subject to the behavioral equations (19)-(21) plus one extra equation defining nominal wage inflation as $\pi_t^W = \hat{w}_t - \hat{w}_{t-1} + \pi_t$. Now we will discuss the implications of taking this welfare-theoretic loss function for the ECB monetary policy. In terms of its stabilizing properties, the welfare-theoretic targeting rule gives rise to the following (annualized) percent standard deviations of the policy targets: $\sigma_\pi = 0.42$, $\sigma_{nW} = 0.20$, and $\sigma_{\bar{y} - \bar{w}} = 0.016$. Figures are reported in part i) of Table 10. Thus, variability of price inflation is low (0.42) and variability of wage inflation even lower (0.20). The variability of the output gap adjusted first difference is close to zero (0.016). Interestingly, variability of the output gap is also near zero ($\sigma_{\bar{y}} = 0.038$). Thus, the welfare-based policy is aimed at reducing very much deviations of output from its (friction-free) potential level. Put differently, the targeting rule that maximizes households’ welfare eliminates most of the price-wage dispersion due to nominal rigidities in order to restore the flexible price-wage

\textsuperscript{28}Sbordone (2002) finds that $\eta_P = 2/3$ leads to a degree of price inertia which is appropriate to fit the US data with a Calvo-type sticky-price model. She also mentions the empirical findings by Blinder et al. (1998) that give support to this calibration of $\eta_P$. As for the sticky-wage parameter $\eta_W$, we set $\eta_W = 2/3$ in order to have a level of wage stickiness identical to price stickiness.

\textsuperscript{29}Concretely, their implicit estimates of $\theta_P$ and $\theta_W$ for the US are $\theta_P = 251$ and $\theta_W = 2.85$. The estimated figure for $\theta_P$ implies $\frac{\lambda_{\bar{y} - \bar{w}}}{\lambda_{\bar{y} - \bar{w}}} = 1.004$ which results in an extremely low steady-state markup of prices over marginal costs. Consequently, producers have very little monopolistic power and the goods market is very close to perfect competition.
economy. As explained in Erceg et al. (2000), targeting price inflation volatility reduces the gap between the real wage and marginal costs. Similarly, when targeting variability of wage inflation, the gap between the consumption-leisure marginal rate of substitution and the real wage also gets smaller. Both gaps together cause the output gap. Therefore, hitting price and wage inflation indirectly drives the output gap volatility down.30

Extending the analysis to (alternative) instrument rules, let us first compare the performance of the estimated Taylor-type rule \((\mu_1 = 0.47, \mu_2 = 0.13, \text{ and } \mu_3 = 0.53)\) relative to the one of the welfare-theoretic targeting rule. The estimated instrument rule leads to a level of losses in the welfare-based loss function \((18')\) much higher than the one obtained with the targeting rule (see results collected in part ii of Table 10). Losses are nearly 80 times higher \((%\Delta\mathcal{L} = 7931)\). Likewise, the standard deviations of the three policy targets are several times higher than the ones provided by the targeting rule. Figure 4 contains plots of the responses to either demand or supply shocks. It is clearly observed that all responses under the targeting rule (solid lines) are much less volatile than responses under the estimated instrument rule (dashed lines). Consequently, the estimated Taylor-type rule is also far from being optimal when the welfare criterion is considered.

Can the welfare-maximizing properties of our Taylor-type instrument rule \((15)\) be substantially improved by searching the best-performing coefficients? Analogously to the procedure followed in the previous section for the stabilizing loss function, we will find the instrument-rule coefficients for \((15)\) that minimize the welfare-theoretic loss function \((18')\) subject to the model equations \((19)-(21)\) plus the additional equation defining \(\pi_t^W\). The policy exercise will be implemented with the figures of \(\lambda_m, \lambda_{e,w}\), and \(\lambda_{\bar{y} - \bar{y}_{-1}}\) implied from the euro area parameterization of the model.

Noteworthy, the optimized Taylor-type rule has no reaction to inflation deviations \((\mu_1^* = 0.0)\) combined with a very aggressive response to the output gap \((\mu_2^* = 147)\). The level of interest rate smoothing is also quite high \((\mu_3^* = 2.55)\). This policy design is very different from the ones obtained in the previous section which never had a large output gap coefficient. An intuitive explanation to this result would come from the very little output gap volatility that the targeting welfare-based rule permits (as discussed above). The optimized Taylor-type rule now tries to imitate the welfare-based targeting rule and finds the best way to approximate by means of responding very aggressively to the output gap. Numbers provided in part iii of Table 10 show that this aggressive output gap response provides a level of policy achievement close to efficiency. The optimized instrument rule performs very closely to the welfare-theoretic targeting rule. For example, the loss function value increases only by 7\% compared to the targeting rule. As for the standard deviations, there is slightly more volatility of price inflation, \(\sigma_{\pi}\), and less of wage inflation, \(\sigma_{e,w}\), with the optimized instrument rule. The standard deviation of the term involving the output gap, \(\sigma_{\bar{y} - \bar{y}_{-1}}\), is also close to 0 with the optimized Taylor-type rule. In any case, differences are quantitative small (compare part i and part iii of Table 10). These similarities are also found in the impulse response functions of the three targeted variables which are plotted in Figure 4. As observed there for both demand and supply shocks, differences between responses under the

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30 One could believe that the low output gap volatility is also related to the moderately high figure \((0.13)\) assigned to \(\lambda_{\bar{y} - \bar{y}_{-1}}\). However, this is not true because if the rule were designed with \(\lambda_{\bar{y} - \bar{y}_{-1}} = 0.0\), keeping \(\lambda_{\pi}\) and \(\lambda_{e,w}\) constant, the standard deviations would barely change \((\sigma_{\pi} = 0.41, \sigma_{e,w} = 0.21, \text{ and } \sigma_{\bar{y} - \bar{y}_{-1}} = 0.018)\). The policy weight \(\lambda_{\bar{y} - \bar{y}_{-1}}\) seems to have little quantitative significance in the welfare-theoretic targeting rule.
welfare-theoretic targeting rule (solid lines) and the optimized Taylor-type rule (dotted lines) are hardly distinguishable.

Recalling the discussion on operational instrument rules from the end of subsection 5.1, the unobservable output gap term \( \hat{y}_t \) from the Taylor-type instrument rule (15) should be replaced by some observable variable to make the rule fully operational. One strong candidate to enter the instrument rule is wage inflation \( \pi_t^w \) both as observable and as a policy target appearing in the welfare-theoretic loss function (18).\(^{31}\) Results for an operational instrument rule with \( \pi_t^w \) instead of \( \hat{y}_t \) are shown in part iv of Table 10. The optimized coefficients found are very high figures for both price inflation (\( \mu_1^* = 52 \)) and wage inflation (\( \mu_2^* = 108 \)), and a moderate interest rate smoothing (\( \mu_3^* = 1.15 \)). The optimized instrument rule with wage inflation performs nearly as efficient as the targeting rule. Actually, it performs even better than the optimized Taylor-type rule (15). As documented in part iv of Table 10, the percent change in the loss function is very low (2.2%) and the standard deviations of the three policy targets very much resemble the ones obtained with the targeting rule. They are identical for wage inflation \( \sigma_{\pi w} \), almost identical for price inflation \( \sigma_{\pi} \), and slightly higher for \( \sigma_{\bar{y} - \delta \bar{y}_{-1}} \). In conclusion, the welfare-theoretic targeting rule can be well represented by a (simple) instrument rule on the nominal interest rate. The rule would consist of reacting very aggressively to deviations of price inflation and, even more strongly, to deviations of wage inflation. In addition, the rule should have a long time inertia.

Since large optimized coefficients were found, the instrument rules were re-optimized constrained to be designed for not-so-high figures. Otherwise, very high coefficients found could generate much instability in case of model mis-specification or other mistakes in policy design. Thus, let us bind the optimized coefficients to be lower than 5.0, as a moderate reference value. Results appear for the optimized Taylor-type rule (15) in part v of Table 10. Now the coefficient on inflation is \( \mu_1^* = 2.3 \), the output gap coefficient is at its upper bound (\( \mu_2^* = 5.0 \)), and there is a unit lagged nominal interest rate coefficient (\( \mu_3^* = 1.0 \)). The welfare-maximizing properties of the constrained optimized rule are not as good as the unconstrained rule (compare part v with part iii of Table 10). Nevertheless, the unconstrained rule still performs quite well. The increase in the welfare-theoretic loss function is 16.9% (it was 7% with the unconstrained rule). The figures of \( \sigma_{\pi} \) and \( \sigma_{\pi w} \) are similar to the ones obtained with the unconstrained rule whereas \( \sigma_{\bar{y} - \delta \bar{y}_{-1}} \) is a bit higher.

Regarding the operational instrument rule with \( \pi_t^w \) (instead of \( \hat{y}_t \)) with optimized coefficients equal or lower than 5.0, the optimal coefficients are \( \mu_1^* = 5.0 \) (upper bound), \( \mu_2^* = 5.0 \), (upper bound), and \( \mu_3^* = 0.96 \). Again, the rule without large coefficients does not reach the very good stabilizing properties of the unconstrained rule although differences are not very large (compare part iv and vi of Table 10). The loss function is 31.8% higher than the value obtained under the targeting rule (it was 2.2% higher with the unconstrained rule). As for the standard deviations, they are higher for \( \sigma_{\pi w} \) and \( \sigma_{\bar{y} - \delta \bar{y}_{-1}} \), and, by contrast, slightly lower for \( \sigma_{\pi} \). In conclusion, the optimized instrument rule constrained to be designed with coefficients lower than 5.0 does not perform noticeably worse than the unconstrained rule and still represents a reasonably good approximation to the welfare-theoretic targeting rule.

Finally, we will investigate the stabilizing properties of the welfare-theoretic targeting rule. This exercise can be readily done by finding the weights of the stabilizing loss function (18) which would best approximate the volatilities implied by the welfare-theoretic loss

\(^{31}\) It is also one of the policy recommendations suggested by Erceg et al. (2000).
function (18’). As previously mentioned in this section, the welfare-theoretic targeting rule seems to situate the economy near its natural-rate level of output defined as the amount that would prevail in an economy free of nominal rigidities. Translating this welfare criterion in terms of stabilizing criteria, the weight of the output gap target ought to be high enough so that variability of the output gap became low. Confirming this guess, it was found that the stabilizing targeting rule that best resemble the welfare-theoretic rule is that with a very high output gap weight ($\lambda_G = 1.88$) and no interest rate weight ($\lambda_R = 0.0$). Under these stabilizing parameters, the monetary policy is conducted in a really similar way to the welfare-theoretic targeting rule. Table 11 shows this good matching. The standard deviations of the three targeted variables ($\sigma_\pi$, $\sigma_p w^*$, and $\sigma_\delta y_{t-1}$) are all strikingly alike. The welfare-theoretic loss function value obtained with a stabilizing targeting rule with $\lambda_G = 1.88$ and $\lambda_R = 0.0$ is just 2.6% higher than the one obtained with the welfare-theoretic targeting rule. Therefore, social welfare could be nearly maximized by implementing a stabilizing targeting rule with a very strong output gap targeting and no interest rate targeting. The welfare-maximizing policy would then result in an economy with low inflation volatility, near zero output gap volatility, and a very high volatility in the nominal interest rate (see the standard deviations reported in Table 11).\textsuperscript{32}

7 Conclusions

This paper has examined the behavior of instrument (Taylor-type) rules relative to targeting rules in a euro area New Keynesian model. The results can be summarized into two major findings:

i) Rules quasi-equivalence for stabilizing purposes. Instrument rules à la Taylor can be designed with optimized coefficients in order to perform closely to the efficient targeting rules. The quasi-equivalence in performance holds for a variety of ECB preferences on stabilizing inflation, the output gap, and the nominal interest rate. The evaluation of instrument rules has also served to characterize the optimal coefficients of Taylor-type rules for the euro area. Thus, the coefficient of the responses to inflation deviations goes significantly up as ECB targets inflation variability more strongly, the coefficient of the responses to the output gap is generally low, and the coefficient on the lagged nominal interest rate is slightly above one (except under strict inflation targeting). This last feature implies a great extent of monetary policy inertia. The rules quasi-equivalence still maintains when considering instrument rules that only involve fully observable variables (e.g., excluding the output gap). In addition to their similar performance, both types of rules are also closely equivalent in their robustness to model changes. There were a few exceptions to the latter reported in the text.

ii) Rules quasi-equivalence for maximizing welfare. Optimized instrument rules can also be designed to perform very closely to a targeting rule based on maximizing households’ welfare. The resulting welfare-theoretic rule situates the economy near its free-rigidity (natural rate) equilibrium as it yields a low variability of the output gap. By contrast, the welfare-theoretic rule leads to a very high nominal interest rate variability. This policy behavior\textsuperscript{32}Obviously, this result would no longer hold if variability of the nominal interest rate would damage households’ utility. That could be the case, for example, if there were transactions-facilitating money in the model or a zero lower bound on the nominal interest rate was considered. Woodford (2003b) gives some justifications for the introduction of the interest-rate variability in the central bank loss function and analyses the behavior of the corresponding targeting rules.
can also be attained by applying a stabilizing targeting rule with a unit weight on inflation volatility, a large weight on the output gap volatility, and no weight on the nominal interest rate volatility.

These results lead to a double message for monetary policy making in the euro area. First, instrument rules à la Taylor can represent quite closely the optimal policy behavior of either stabilizing or welfare-theoretic targeting rules. Therefore, instrument rules can be designed as simple rules that imply policy reactions very close to those provided by the optimal targeting rules. Second, the design of a monetary policy rule for the euro area depends very much on the ECB policy preferences. Thus, the definition of the ECB policy targets and their relative weights play a crucial role in the optimal design of monetary policy rules for the euro area.

The whole monetary policy analysis conducted here is unavoidably linked to the closed-economy New Keynesian model described in the text. This particular model was selected for the analysis due to both its theoretical appeal as based on optimizing behavior, and to its capacity to replicate many regularities observed in the euro area business cycle. However, it would be interesting to repeat the exercises of policy evaluation using other models that were also able to explain reasonably well the euro area business cycle. Actually, that would be necessary to validate the robustness of our results. Unfortunately, it lies beyond the scope of this paper, remaining open for future research.
REFERENCES


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——, and E. Nelson 2003, Targeting vs. instrument rules for monetary policy, manuscript.


Table 1. Calibration/Estimation of parameters

<table>
<thead>
<tr>
<th>Calibrated parameters</th>
<th>Estimated parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.36$</td>
<td>$\mu_1 = 0.47$</td>
</tr>
<tr>
<td>$\beta = 0.995$</td>
<td>$\mu_2 = 0.13$</td>
</tr>
<tr>
<td>$\sigma = 1.5$</td>
<td>$\mu_3 = 0.53$</td>
</tr>
<tr>
<td>$h = 0.85$</td>
<td>$\kappa_\pi = 0.028$</td>
</tr>
<tr>
<td>$\gamma = 16$</td>
<td>$\kappa_w = 0.003$</td>
</tr>
<tr>
<td>$\rho_z = 0.95$</td>
<td>$\sigma_{\varphi\pi} = 0.95$</td>
</tr>
<tr>
<td>$\rho_\zeta = 0.6$</td>
<td>$\sigma_{\varphi\zeta} = 5.62$</td>
</tr>
</tbody>
</table>

Table 2. Business cycle statistics. Model-to-data comparison.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Euro area data</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{y}$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>Annualized Std. deviations (%)</td>
<td>2.00</td>
<td>2.73</td>
</tr>
<tr>
<td>Coef. of autocorrelation</td>
<td>0.96</td>
<td>0.90</td>
</tr>
<tr>
<td>Coef. of correlation to $\hat{y}$</td>
<td>1.0</td>
<td>0.16</td>
</tr>
<tr>
<td>Coef. of correlation to $\pi$</td>
<td>0.16</td>
<td>1.0</td>
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Table 3. Efficiency I. Comparing stabilizing targeting rules with the estimated Taylor-type instrument rule.

<table>
<thead>
<tr>
<th>Targeting rule</th>
<th>Estimated Taylor-type rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\lambda_{\hat{y}}, \lambda_R)$</td>
<td>$(\mu_1, \mu_2, \mu_3)$</td>
</tr>
<tr>
<td>$(\sigma_{\pi}, \sigma_{\hat{y}}, \sigma_R)$</td>
<td>$(\sigma_{\pi}, \sigma_{\hat{y}}, \sigma_R)$</td>
</tr>
<tr>
<td>$(0.0, 0.0)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
<tr>
<td>$(0.0, 0.0)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
<tr>
<td>$(0.1, 0.0)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
<tr>
<td>$(0.0, 0.1)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
<tr>
<td>$(0.0, 0.1)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
<tr>
<td>$(0.1, 0.1)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
<tr>
<td>$(0.0, 0.5)$</td>
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<tr>
<td>$(0.0, 0.5)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
<tr>
<td>$(0.1, 0.5)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
<tr>
<td>$(2.37, 1.82)$</td>
<td>$(0.47, 0.13, 0.53)$</td>
</tr>
</tbody>
</table>

26
Table 4. Efficiency II. Comparing stabilizing targeting rules with optimized Taylor-type instrument rules.

<table>
<thead>
<tr>
<th>Targeting rule</th>
<th>Optimized Taylor-type rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\lambda, \lambda_R)$</td>
<td>$(\sigma_\pi, \sigma_\tilde{y}, \sigma_R)$</td>
</tr>
<tr>
<td>(0.0,0.0)</td>
<td>0.0</td>
</tr>
<tr>
<td>(0.01,0.0)</td>
<td>0.15</td>
</tr>
<tr>
<td>(0.1,0.0)</td>
<td>0.29</td>
</tr>
<tr>
<td>(0.0,0.1)</td>
<td>0.59</td>
</tr>
<tr>
<td>(0.01,0.1)</td>
<td>0.57</td>
</tr>
<tr>
<td>(0.1,0.1)</td>
<td>0.51</td>
</tr>
<tr>
<td>(0.0,0.5)</td>
<td>0.85</td>
</tr>
<tr>
<td>(0.01,0.5)</td>
<td>0.83</td>
</tr>
<tr>
<td>(0.1,0.5)</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Table 5. Efficiency III. Comparing stabilizing targeting rules with optimized Taylor-type instrument rules with $\tilde{y}_t$ (instead of $\tilde{y}_t$).

<table>
<thead>
<tr>
<th>Targeting rule</th>
<th>Optimized instrument rule with $\tilde{y}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\lambda, \lambda_R)$</td>
<td>$(\sigma_\pi, \sigma_\tilde{y}, \sigma_R)$</td>
</tr>
<tr>
<td>(0.0,0.0)</td>
<td>0.0</td>
</tr>
<tr>
<td>(0.01,0.0)</td>
<td>0.15</td>
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<tr>
<td>(0.1,0.0)</td>
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<td>0.59</td>
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<td>(0.1,0.5)</td>
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Table 6. Efficiency IV. Comparing stabilizing targeting rules with optimized Taylor-type instrument rules without output-related response ($\mu_2 = 0$).

<table>
<thead>
<tr>
<th>Targeting rule</th>
<th>Optimized instrument rule with $\mu_2 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\lambda, \lambda_R)$</td>
<td>$(\sigma_\pi, \sigma_\tilde{y}, \sigma_R)$</td>
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<tr>
<td>(0.0,0.0)</td>
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<td>(0.01,0.0)</td>
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<tr>
<td>(0.1,0.0)</td>
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<tr>
<td>(0.0,0.5)</td>
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<td>(0.01,0.5)</td>
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27
Table 7. Robustness I. Robustness to price flexibility.

<table>
<thead>
<tr>
<th>Targeting rule</th>
<th>Targeting rule</th>
<th>Optimized Taylor-type rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>with flexible prices (right)</td>
<td>with sticky prices (wrong)</td>
<td>with sticky prices (wrong)</td>
</tr>
<tr>
<td>((\lambda^*, \lambda^R))</td>
<td>(\sigma_\pi)</td>
<td>(\sigma_{\bar{Y}})</td>
</tr>
<tr>
<td>(0.0,0.0)</td>
<td>0.0</td>
<td>1.01</td>
</tr>
<tr>
<td>(0.01,0.0)</td>
<td>0.05</td>
<td>0.99</td>
</tr>
<tr>
<td>(0.1,0.0)</td>
<td>0.45</td>
<td>0.85</td>
</tr>
<tr>
<td>(0.0,0.1)</td>
<td>3.55</td>
<td>0.47</td>
</tr>
<tr>
<td>(0.01,0.1)</td>
<td>3.55</td>
<td>0.46</td>
</tr>
<tr>
<td>(0.1,0.1)</td>
<td>3.55</td>
<td>0.41</td>
</tr>
<tr>
<td>(0.0,0.5)</td>
<td>3.96</td>
<td>0.52</td>
</tr>
<tr>
<td>(0.01,0.5)</td>
<td>3.96</td>
<td>0.51</td>
</tr>
<tr>
<td>(0.1,0.5)</td>
<td>3.95</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 8. Robustness II. Robustness to wage flexibility.

<table>
<thead>
<tr>
<th>Targeting rule</th>
<th>Targeting rule</th>
<th>Optimized Taylor-type rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>with flexible wages (right)</td>
<td>with sticky wages (wrong)</td>
<td>with sticky wages (wrong)</td>
</tr>
<tr>
<td>((\lambda^*, \lambda^R))</td>
<td>(\sigma_\pi)</td>
<td>(\sigma_{\bar{Y}})</td>
</tr>
<tr>
<td>(0.0,0.0)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(0.01,0.0)</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.1,0.0)</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>(0.0,0.1)</td>
<td>1.14</td>
<td>0.14</td>
</tr>
<tr>
<td>(0.01,0.1)</td>
<td>1.14</td>
<td>0.14</td>
</tr>
<tr>
<td>(0.1,0.1)</td>
<td>1.12</td>
<td>0.12</td>
</tr>
<tr>
<td>(0.0,0.5)</td>
<td>2.07</td>
<td>0.28</td>
</tr>
<tr>
<td>(0.01,0.5)</td>
<td>2.07</td>
<td>0.28</td>
</tr>
<tr>
<td>(0.1,0.5)</td>
<td>2.06</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 9. Robustness III. Robustness to preferences without habit formation.

<table>
<thead>
<tr>
<th>Targeting rule</th>
<th>Targeting rule</th>
<th>Optimized Taylor-type rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>with (h = 0) (right)</td>
<td>with (h = 0.85) (wrong)</td>
<td>with (h = 0.85) (wrong)</td>
</tr>
<tr>
<td>((\lambda^*, \lambda^R))</td>
<td>(\sigma_\pi)</td>
<td>(\sigma_{\bar{Y}})</td>
</tr>
<tr>
<td>(0.0,0.0)</td>
<td>0.0</td>
<td>1.10</td>
</tr>
<tr>
<td>(0.01,0.0)</td>
<td>0.14</td>
<td>0.35</td>
</tr>
<tr>
<td>(0.1,0.0)</td>
<td>0.26</td>
<td>0.14</td>
</tr>
<tr>
<td>(0.0,0.1)</td>
<td>0.47</td>
<td>3.14</td>
</tr>
<tr>
<td>(0.01,0.1)</td>
<td>0.40</td>
<td>2.34</td>
</tr>
<tr>
<td>(0.1,0.1)</td>
<td>0.49</td>
<td>1.10</td>
</tr>
<tr>
<td>(0.0,0.5)</td>
<td>0.64</td>
<td>3.42</td>
</tr>
<tr>
<td>(0.01,0.5)</td>
<td>0.64</td>
<td>2.97</td>
</tr>
<tr>
<td>(0.1,0.5)</td>
<td>1.06</td>
<td>1.88</td>
</tr>
</tbody>
</table>

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Table 10. Extensions I. Comparing the welfare-theoretic targeting rule with instrument rules.

<table>
<thead>
<tr>
<th>Rule Type</th>
<th>Parameters</th>
<th>( \sigma_\pi )</th>
<th>( \sigma_{wR} )</th>
<th>( \sigma_{yR} )</th>
<th>( \sigma_{R} )</th>
<th>( %\Delta \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Welfare-theoretic targeting rule</td>
<td></td>
<td></td>
<td>( \lambda_\pi, \lambda_{xw}, \lambda_{\bar{y}_t} )</td>
<td>( (0.47, 0.53, 0.13) )</td>
<td></td>
<td>0.42</td>
</tr>
<tr>
<td>ii) Estimated Taylor-type rule</td>
<td></td>
<td></td>
<td>( \mu_1, \mu_2, \mu_3 )</td>
<td>( (0.47, 0.13, 0.53) )</td>
<td></td>
<td>2.73</td>
</tr>
<tr>
<td>iii) Optimized Taylor-type rule</td>
<td></td>
<td></td>
<td>( \mu_1^<em>, \mu_2^</em>, \mu_3^* )</td>
<td>( (0.0, 147, 2.55) )</td>
<td></td>
<td>0.52</td>
</tr>
<tr>
<td>iv) Optimized Taylor-type rule with ( \pi_t^W ) (instead of ( y_t ))</td>
<td></td>
<td></td>
<td>( \mu_1^<em>, \mu_2^</em>, \mu_3^* )</td>
<td>( (52, 108, 1.15) )</td>
<td></td>
<td>0.43</td>
</tr>
<tr>
<td>v) Optimized Taylor-type rule with coefficients ( \leq 5.0 )</td>
<td></td>
<td></td>
<td>( \mu_1^<em>, \mu_2^</em>, \mu_3^* )</td>
<td>( (2.30, 5.0, 1.0) )</td>
<td></td>
<td>0.44</td>
</tr>
<tr>
<td>vi) Optimized Taylor-type rule with ( \pi_t^W ) (instead of ( y_t )) and coefficients ( \leq 5.0 )</td>
<td></td>
<td></td>
<td>( \mu_1^<em>, \mu_2^</em>, \mu_3^* )</td>
<td>( (5.0, 5.0, 0.96) )</td>
<td></td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 11. Extensions II. Approximating the welfare-theoretic targeting rule with a stabilizing targeting rule.

<table>
<thead>
<tr>
<th>Rule Type</th>
<th>Parameters</th>
<th>( \sigma_\pi )</th>
<th>( \sigma_{xwR} )</th>
<th>( \sigma_{yR} )</th>
<th>( \sigma_R )</th>
<th>( %\Delta \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Welfare-theoretic targeting rule (18')</td>
<td></td>
<td></td>
<td>( \lambda_\pi, \lambda_{xw}, \lambda_{\bar{y}_t} )</td>
<td>( (0.47, 0.53, 0.13) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ii) Stabilizing targeting rule (18)</td>
<td></td>
<td></td>
<td>( \lambda_{\bar{y}_R} )</td>
<td>( (1.88, 0.0) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Euro area data, 1980.1-2002.4. Output and the real wage are linearly detrended, whereas inflation and the nominal interest rate are the original series.
Figure 2: Model-to-data comparison. Correlation functions among output ($y$), inflation ($\pi$), the real wage ($w$), and the nominal interest rate ($R$). Eight-quarter ($j$) lag and lead.
Figure 3: Impulse response functions. Comparing a stabilizing targeting rule with Taylor-type instrument rules.
Figure 4: Impulse response functions. Comparing the welfare-theoretic targeting rule with Taylor-type instrument rules.