On Convergence in Credit-Rationed Open Economies when Taxes are Progressive

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December 5, 2003

* The author would like to thank, without implicating, Cecilia Garcia-Peñalosa, Richard Rogerson, Jean-François Wen, and two referees for useful comments and suggestions, as well as Amartya Lahiri for stimulating discussions. First draft: may 23, 2003.

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Abstract

This paper studies the impact of progressive taxation on the convergence speed of credit-rationed small open economies. We find that open economies converge faster when taxes are progressive, which extends previous results by Barro, Mankiw, and Sala-i-Martin (1995). However, a regressive tax rate may sharply reduce the convergence speed towards zero and may even destabilize the economy. Therefore, different open economies may converge at different speed to their balanced growth path, depending on how their taxes respond to the tax base, so that tax progressivity may be a potential source of cross-country differences in convergence rates. In a sample of US data, over the period 1920-1990, regional convergence and tax progressivity are found to be positively correlated, which supports our theoretical results. Moreover, we argue that the model is successful in predicting the full range of available convergence estimates only when the tax rate is elastic enough to the tax base.

Keywords: convergence, open economy, progressive taxation.


1 Introduction

It is well known that the neoclassical growth model developed by Ramsey [31], Solow [34] and Swan [36] predicts slow convergence only if the capital share is high enough - about 0.8 - as shown by Barro and Sala-i-Martin [5], Mankiw, Romer, and Weil [25]. Barro, Mankiw, and Sala-i-Martin [4] have argued that the neoclassical model explains more satisfactorily the evidence on convergence when one considers open-economy versions of the model and assumes that capital is only partially mobile. However, their results rely on the assumption of a constant tax rate that is at variance with what one observes for most OECD and developing countries. Most importantly, the latter assumption implies that the model is not fully successful in predicting the range of estimates that has been enlarged by recent studies (see Evans and Karras [14], Quah [29], Temple [38], among others).
In this paper, I consider a distortionary tax rate on labor income that is **progressive** (regressive) - such that the marginal rate is larger (lower) than the average rate. In contrast with the assumption of Barro et al. [4], this captures some features of tax schedules prevailing in OECD countries, e.g. in the U.S. or European countries (see, for instance, the *Statistical Abstracts of the United States*, or *Tax and the Economy: A Comparative Assessment of OECD Countries, OECD Tax Policy Studies n. 6, 2002*), and in developing countries. The major result of this paper is that when the tax rate is progressive (or not too regressive), the convergence speed *increases* with the degree of progressivity: in essence, increasing progressivity amounts to decreasing the after-tax capital share in total income, in which case diminishing returns set in more quickly and convergence is more rapid. Of some importance is the fact that this conclusion holds true independent of the tax rate, which can be set close to zero.

As a consequence, we extend the results of Barro et al. [4] by showing, in their framework, that:

- the model predicts a convergence range of 0.015; 0.030, in agreement with some early studies (see, e.g. Barro [2], Mankiw, Romer, and Weil [25], Levine and Renelt [24], Barro and Sala-i-Martin [5]; see also Galor [15], Quah [28], Klenow and Rodriguez-Clare [22], de la Fuente [13], Bernanke and Gürkaynak [9] for alternative evaluations of the neoclassical implications), provided that the tax rate is not *too elastic* to the tax base (in short, the disposable income-taxable income elasticity must belong to 0.89; 1.11; see Figure 1 in Section 2.2 below).

- however, the model is more successful in predicting the *full* range of available estimates when the tax rate is *elastic enough* to the tax base. For example, a highly progressive tax rate leads to a convergence region of 0.04; 0.08, in agreement with estimates by, for instance, Evans and Karras [14]: the disposable income-taxable income elasticity must then belong to 0.47; 0.79 (see Figure 1). On the other side of the spectrum, a highly regressive tax rate leads to a convergence rate close to zero (in line with some conclusions of Quah [28, 29], Temple [38], among others) and may even imply *divergence* from the balanced growth path (see Figure 1).

In summary, the main theoretical insight of this paper may appear to be useful in light of the *broad* range
of estimates, as it shows that taxation progressivity is an important mechanism that may help to reconcile theory and facts related to convergence. In particular, our results point at elastic tax rates as a potential source of cross-country differences in convergence rates. More precisely, the model predicts, as we just said, an empirically plausible convergence range provided that the tax rate is elastic enough to the tax base.

In section 3, we go through a first step toward providing some evidence on the positive relationship between convergence and tax progressivity. This exercise is based on US data, for the period 1920-1990, provided by Barro and Sala-i-Martin [6] and Stephenson [35]. Our computations establish that data are supportive of a positive correlation between income tax progressivity and convergence speed of per-capita personal income, in agreement with our theoretical results (see Figures 2 and 3 below). We also show that this positive correlation is robust to changes in the set of estimates of both tax progressivity and convergence speed. Our computations also deliver that tax progressivity has ranged in 4%; 11% over the last sixty years (in accord with some recent evaluations by Bénabou [7, pp. 501-2], Cassou and Lansing [10, p. 11], for example). Therefore, the level of income tax progressivity observed in the US over this period implies a higher convergence speed than the reference value of 2%. In other words, tax progressivity may partly explain why some recent estimates argue in favor of more rapid convergence. For example, a range of 5%; 11% for tax progressivity - that is, a disposable income-taxable income elasticity which belongs to 0.89; 0.95 - translates, given our benchmark parameter values, into a convergence range of 2.6%; 3.1% (compare with 2% obtained by Barro et al. [4]), as shown in Figure 1 below.

However, there are, in the real world, a number of other taxes with possibly different effects on convergence. First, some progressivity may presumably come into play through transfers, although this is not measured in most studies. On the other hand, some other features of actual tax systems introduce regressivity in otherwise progressive income tax schedules, most notably through proportional consumption taxes, upper bounds that are imposed on social security contributions, absent or low capital income taxes (especially in developing countries; see, e.g., Tanzi and Zee [37]). Therefore, one cannot rule out the case of effective taxes being indeed regressive, even though estimates that would capture all these effects do not seem to be yet available (see, however, Section 3 for some evidence on regressive taxes in the US history). Our model
predicts that regressive taxes lead to slow convergence, in agreement with still another piece of evidence in the literature.

A sensitivity analysis with respect to uncertain parameters - most importantly the degree of credit rationing - ends Section 2.2 and emphasizes the key role played by progressivity. In particular, we show that an economy that is only slightly constrained in international credit markets is predicted to converge at a plausibly low speed only if taxes are regressive. On the contrary, highly constrained economies converge, as do closed economies, at a relatively low speed if taxes are progressive. Moreover, our results also broadly accord with some evidence suggesting that interdependence fosters convergence (see, e.g., Sachs and Warner [33], Ben-David [8], Vamvakidis [39]).

Finally, so as to check the robustness of the main result, we also consider the case of elastic labor supply, in contrast with Barro et al. [4]. Our major insight here is that the convergence rate rises with the labor supply elasticity to the real wage. For instance, the case of indivisible labor, studied by Rogerson [32] and Hansen [19], implies that the predicted convergence speed belongs to 0.015; 0.030 only if the tax rate is regressive. However, the model predicts a convergence rate in agreement with more recent estimates only if the tax rate is progressive. Therefore, our main conclusion remains unchanged: the model’s predictions accord with the evidence on convergence provided that the tax rate is elastic to taxable income.

This paper is also related to recent papers by Altig et al. [1], Bénabou [7], Cassou and Lansing [10], Caucutt, Imrohoroglu and Kumar [11], which study the growth effects of non-uniform tax rates. In particular, Cassou and Lansing [10], Caucutt, Imrohoroglu and Kumar [11] consider progressive taxes on personal income when calibrating their models, therefore ignoring regressive aspects of the actual tax systems. Another difference with the present paper is that the growth rate is not affected, here, by the level of progressivity. However, extending our results would probably lead one to conclude that tax progressivity may, by fostering convergence, reduce volatility (see Pintus [27] for some results along these lines) and, therefore, enhance growth, in agreement with some empirical studies (e.g. Ramey and Ramey [30]). Closest in spirit is the
paper by Gokan [16] which studies how convergence is affected by the way of financing a given stream of
government expenditures, including lump-sum taxes, in a monetary economy. However, all authors cited
above examine closed-economy versions of neoclassical models.

The rest of the paper is organized as follows. Section 2 presents the open-economy model, following Barro
et al. [4], and derives its laws of motion and predicted convergence rate. Section 3 presents some empirical
evidence indicating that convergence and tax progressivity are positively correlated. Section 4 extends the
model to the case of an elastic labor supply and discusses how the convergence rate is affected. Finally, some
concluding remarks and directions for future research are gathered in Section 5.

2 Convergence in a Credit-Rationed Open Economy

2.1 Model and Dynamics

The economy we consider produces tradeable output $Y$ by using three inputs, physical capital $K$, human
capital $H$ and raw labor $L$, according to the following technology:

$$Y = K^\alpha H^\eta (Le^{(n+g)t})^{1-\alpha-\eta},$$

(1)

where, $\alpha \geq 0$, $\eta \geq 0$, $\alpha + \eta < 1$, while $n \geq 0$ is the rate of population growth and $g \geq 0$ is the rate
of labor-augmenting technological progress. As is usual, one can view the production function in intensive
form, that is:

$$y = k^\alpha h^\eta,$$

(2)

where, for instance, $y = Y/(Le^{(n+g)t})$ denotes output measured in efficient-labor units.

The Ramsey households have preferences represented by:

$$\int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-(\rho-n)t} dt,$$

(3)
where $C$ is per capita consumption, $\theta \geq 0$ is the inverse of the elasticity of intertemporal substitution in consumption, and $\rho \geq 0$ is the discount factor. The representative consumer owns the three inputs and rent them to firms through competitive markets. Therefore, we can write down, for sake of brevity, the consolidated budget constraint as:

$$\dot{k} + \dot{h} - \dot{d} = (1 - \tau)(k^\alpha h^\eta - R_kk) + (1 - \tau_k)R_kk - (\delta + n + g)(k + h) - (r - n - g)d - c,$$

(4)

where $d$ is the amount of net debt in efficient-labor units, $c$ is consumption in efficient-labor units, $\tau$ is the tax rate on labor income, to be specified below, $0 \leq \tau_k \leq 1$ is the tax rate on physical capital income $R_kk$, $0 \leq \delta \leq 1$ is the depreciation rate for both types of capital, and $r \geq 0$ is the world interest rate. The initial stocks $k(0) > 0$, $h(0) > 0$ and $d(0)$, and the labor endowment $L^* = 1$ are given to the households.

Government is assumed to finance public expenditures that do not affect private decisions by taxing output. To simplify the analysis, we assume that the tax rate on physical capital income $\tau_k$ is constant, while the tax schedule on labor income $\tau$ is as follows:

$$\tau = 1 - \nu y^{-\pi},$$

(5)

where $0 < \nu < 1$ is a scaling parameter that plays only a minor role in the following analysis, as shown below, while $\pi < 1$ is our critical paramter. For instance, one may interpret $\nu$ as depending on a base level of income - e.g. the steady state level - that is taken as given. The important feature here is that the marginal tax rate is higher (resp. lower) than the average tax rate when $\pi$ is positive (resp. negative) - in the present formulation, it is also the case that the tax rate $\tau$ increases (resp. decreases) with the tax base when $\pi$ is positive (resp. negative). Therefore, this tax system exhibits progressivity when $\pi$ is positive, and regressivity when $\pi$ is negative. Barro, Mankiw, and Sala-i-Martin [4] considered a benchmark case with $\pi = 0$: the tax rate $\tau = 1 - \nu$ is then constant. As in Guo and Lansing [17], we assume that households take into account how the tax rate will affect their earnings. So as to ensure the existence of an interior balanced growth path, we also assume that taxes are not too regressive, that is, $\pi > \overline{\pi}$, where $\overline{\pi}$ is a (negative) lower bound to be determined in the next section (see Proposition 2.1).

Our simplifying assumption that physical capital income is taxed at a constant rate is in line with the
observation that there exists a progressivity differential in the US tax code as well as in most OECD tax systems: labor income taxes are more progressive (see, e.g. Hall and Rabushka [18]). Moreover, this hypothesis may be justified on the grounds that redistribution policies often rely on income sources that are among the less internationally mobile (as raw labor and, to some extent, human capital). However, Appendix B shows how peripheral this assumption is, as it concludes that our qualitative results remain valid when taxes on physical capital income are as progressive as labor income taxes.

We also assume, following Barro et al. [4], that international borrowing is limited to the amount of physical capital, that is, $d \leq k$. This hypothesis is, for instance, fulfilled when only physical capital is accepted as collateral for international credit, whereas human capital is not. We focus on the case in which $r = \rho + g\theta$, the interest rate that would prevail in the corresponding closed economy (that is, when $d = 0$). Moreover, we assume that one has $k(0) + h(0) - d(0) < h^*$, at the outset of the period, where $h^*$ is the steady state value of human capital stock. In other words, the initial net asset position is small enough so that the credit constraint binds, that is, $d = k$ (when the latter inequality does not hold, the economy jumps immediately to the steady state; see Barro et al. [4, p. 109]).

The after-tax return on physical capital therefore equals $r + \delta$, which yields $k = (1 - \tau_k)\alpha y/(r + \delta)$, and, therefore, by using equation (2) and the equilibrium condition that $L = 1$,

$$y = Bh^\varepsilon,$$

where $B = [(1 - \tau_k)\alpha/(r + \delta)]^{\alpha/(1-\alpha)}$ and $\varepsilon = \eta/(1 - \alpha)$. Our assumption that $0 \leq \alpha + \eta < 1$ implies $0 \leq \varepsilon < \alpha + \eta$.

Therefore, the budget constraint (4) may be written as the following, by observing that $y - R_k k = (1 - \alpha)y$ and that interest payments $(r + \delta)d = \alpha(1 - \tau_k)y$ when $d = k$:

$$\dot{h} = (1 - \alpha)(1 - \tau)y - (\delta + n + g)h - c.$$  

Households decisions are then obtained by maximizing (3) subject to the budget constraint (7), using
equations (6) and (5), given the initial stock \( h(0) \) assumed to be less than \( h^* \). The first-order Euler condition is then:

\[
\theta \dot{c} = \nu \varepsilon (1 - \alpha)(1 - \pi) \frac{y^{1-\pi}}{h} - \rho - \delta - \theta g. 
\]  

We may rewrite the budget constraint, from (7) and (5), as:

\[
\dot{h} = \nu (1 - \alpha) \frac{y^{1-\pi}}{h} - \delta - n - g - \frac{c}{h}. 
\]  

Equations (8)-(9) characterize the dynamics of intertemporal equilibria. On the other hand, it is not difficult to check that the associated transversality constraint is met, in the following analysis, provided that \( \rho - n + g(\theta - 1) > 0 \), as we consider orbits that converge towards the interior steady state in a monotonous way. Moreover, direct inspection of equations (5) and (6) reveal that after-tax income is concave in \( h \) whenever \( \varepsilon (1 - \pi) < 1 \), that is \( \pi > (\alpha + \eta - 1)/\eta \) (when the tax rate is not too regressive).

2.2 Convergence when Taxes are Progressive

In this section, we discuss how the level of progressivity of the income tax schedule affects the convergence rate of our credit-constrained open economy. From equations (6) and (8)-(9), one can get the basic intuition behind our results: the human capital share now multiplies a factor \( (1 - \pi) \) - that is smaller than one when \( \pi \) is positive, but greater than one when \( \pi \) is negative. Therefore, the higher the level of tax progressivity \( \pi \), the smaller the after-tax capital share \( \varepsilon (1 - \pi) \), that is, the more quickly diminishing returns set in and the more rapid convergence. Of some importance is the fact that this conclusion holds true independent of the steady-state tax rate, which can be set close to zero.

The first appendix derives the convergence rate by linearizing equations (8)-(9) around the unique interior steady state. For a steady state to be feasible, we need the additional condition that \( \rho + \delta + g\theta > \varepsilon (1 - \pi)(\delta + n + g) \) to be met (see the appendix for a proof), which imposes a negative lower bound on \( \pi \), that is, \( \pi > \overline{\pi} \equiv 1 - (\rho + \delta + \theta g)/[\varepsilon (\delta + n + g)] \). Under the latter assumption, one gets, after some computations, the following \( \beta \)-convergence rate (which is simply the opposite of the negative eigenvalue of the Jacobian matrix
of equations (8)-(9), evaluated at the steady state; see for instance Barro and Sala-i-Martin [6]).

**Proposition 2.1 (Convergence Rate and Tax Progressivity)**

In the economy with tax progressivity, the convergence rate is given by:

\[
\beta \equiv 0.5 \left\{ (\phi^2 + 4(\rho + \delta + \theta g) \frac{1 - \varepsilon(1 - \pi)}{\theta}) \left[ \frac{\rho + \delta + \theta g}{\varepsilon(1 - \pi)} - \delta - n - g \right] \right\}^{0.5} - \phi, \tag{10}
\]

where \( \phi = \rho - n + g(\theta - 1) > 0 \).

Under the assumption that the tax rate is not too regressive, that is, \( \pi > \overline{\pi} \equiv 1 - (\rho + \delta + \theta g)/[\varepsilon(\delta + n + g)] \), the convergence rate \( \beta \) given by equation (10) increases, when positive, with the progressivity parameter \( \pi \).

In particular, convergence is more rapid (resp. less rapid) than in the case of a constant tax rate when the tax rate is progressive (resp. regressive). The convergence speed tends to zero when \( \pi \) approaches the negative value \( (\alpha + \eta - 1)/\eta \).

We now study the quantitative implications of our progressive tax schedule and, to facilitate direct comparison, use the benchmark parameter values supported by Barro et al. [4, p. 107], that is:

\[
n = 0.01, \quad g = 0.02, \quad \delta = 0.05, \quad \theta = 2, \quad \rho = 0.02, \quad \alpha = 0.3, \quad \eta = 0.5. \tag{11}
\]

In the following analysis, the second root of the characteristic polynomial is positive, hence unstable. Therefore, the steady state is a saddle when \( \beta > 0 \) and is unstable (that is, a source) when \( \beta < 0 \). Note that the scaling parameter \( \nu \) - see equation (5) - does not appear in equation (10) and is, therefore, not relevant in the following analysis.

Figure 1 reports the convergence rate \( \beta \), computed by using (10), as a function of the progressivity parameter \( \pi > \overline{\pi} = -0.925 \). It shows that \( \beta \) is quite sensitive to \( \pi \): \( \beta \) belongs to the range 0.015; 0.030 only if \( \pi \) is confined to a narrow range, that is, \(-0.11; 0.11 \). When \( \pi \) is outside this range, then the convergence rate is either higher or smaller, depending on the sign of \( \pi \), as shown in Proposition 2.1: for example, \( \beta = 0.1 \) when \( \pi = 0.62 \) and \( \beta = 0 \) when \( \pi = (\alpha + \eta - 1)/\eta = -0.4 \) (see Proposition 2.1).
Three important conclusions can be drawn from this experiment. First, the model’s predictions regarding the convergence speed are consistent with the range of estimates obtained by Barro and Sala-i-Martin [5], Mankiw et al. [25] only if the tax rate is almost flat.

Second, the model also delivers a broader range for the convergence speed, provided that the tax rate is quite responsive to the taxable base. For instance, the predicted range is 0.04; 0.08, in agreement with estimates by Evans and Karras [14], if \( \pi \) belongs to 0.21; 0.53. On the other hand, Temple [38] shows that assessing the robustness of growth regressions casts some serious doubt on the estimated narrow range of the convergence rate, while Quah [29] argues against the evidence of convergence in data. Our analysis shows that when progressivity of taxes is considered, the predicted convergence range is much broader and consistent with this view. It may help to reinterpret the quantitative implications of our results by discussing in terms of the disposable income-taxable income elasticity. In view of equation (5), disposable income is 

\[(1 - \tau)y = \nu y^{1-\pi}.\]

Therefore, the disposable income-taxable income elasticity is given by \(1 - \pi\). When progressivity is high, the latter elasticity is close to zero, whereas it is larger than one when taxes are
regressive. More precisely, the disposable income-taxable income elasticity must belong to 0.89; 1.11 in order for the model to predict the convergence region of 0.015; 0.030. Similarly, the convergence speed is close to zero when the latter elasticity is close to 1.4, while the predicted convergence range is 0.04; 0.08 when the disposable income-taxable income elasticity belongs to 0.47; 0.79.

Finally, Figure 1 shows that when the tax rate is strongly regressive ($\pi < -0.4$), the economy may diverge from the balanced growth path. In fact, self-fulfilling fluctuations are likely to occur in this case (see Pintus [27] for a related discussion on the aggregate instability of open economies, in the presence of increasing returns, which is outside the scope of the present paper).

Our next step is to examine the sensitivity of our quantitative experiment to the baseline values in (11). As $\beta$ is quite unsensitive to $\rho$, $\delta$, $n$ and $g$, we discuss how the above conclusions change when different values of $\theta$, $\alpha$ and $\eta$ are adopted. We are led to interesting conclusions if we vary the severity of credit rationing, that is, if we vary $\alpha$, the share of capital serving as collateral, given $\alpha + \eta$, the share of total capital. The basic mechanism behind the following analysis is that fixing $\alpha + \eta$ and increasing $\alpha$ implies, by decreasing $\varepsilon$, that convergence is more rapid (see equation (10)).

The values in equation (11) imply that $\alpha/(\alpha + \eta) = 0.375$. For sake of brevity, we use the two alternative values of 0 (highly constrained open economy case) and 0.6 (slightly constrained open economy case) for $\alpha$, given $\alpha + \eta = 0.8$. When $\alpha = 0$, then $\varepsilon = \eta = 0.8$, and $\beta$ belongs to 0.015; 0.030 when $\pi$ belongs to 0.01; 0.20. For instance, $\beta = 0.014$ in the case of a constant tax rate ($\pi = 0$). This is also the convergence speed experienced by the corresponding closed economy with a constant tax rate. On the other hand, when $\alpha = 0.6$, then $\varepsilon = 0.5$ and $\alpha/(\alpha + \eta) = 0.75$, and $\beta$ belongs to 0.015; 0.030 when $\pi$ belongs to $-0.71; -0.28$.

These results come from the fact that fixing $\beta$ implies that the higher $\alpha$, the smaller $\varepsilon$ and, therefore, the smaller the required value for $\pi$. Here again, progressivity plays a key role. In particular, the model predicts that an economy that is only slightly constrained in international credit markets ($\alpha/(\alpha + \eta)$ large) converges at a plausibly low speed only if taxes are regressive. On the contrary, highly constrained economies converge, as do closed economies, at a relatively low speed if taxes are progressive.
When, other things equal, $\theta = 0.5$ (a value supported, for instance, by Hansen and Singleton [20]), then $\beta$ belongs to the range $0.015; 0.030$ when $\pi$ is in $-0.23; -0.07$. On the other hand, $\beta$ belongs to the narrow range when $\theta = 5$ (see Campbell [12]), if $\pi$ is in $0; 0.24$. Therefore, with different values of $\theta$, the model’s implications of predicting the narrow range $0.015; 0.030$ are very different with respect to progressivity: with low intertemporal substitution in consumption (high $\theta$), the conventional range is obtained only if the tax rate is slightly *progressive*. In contrast, *regressive* taxation is needed when $\theta$ is small. This comes from the fact that $\beta$ decreases with $\theta$ but increases with $\pi$. Therefore, fixing $\beta$ implies that the higher $\theta$, the higher the required value for $\pi$.

Therefore, the main lesson from this experiment is that the model predicts the full convergence range when taxes are *elastic enough* to the tax base.

### 3 Some Evidence on Convergence Speed and Tax Progressivity

To the best of our knowledge, no attempt has been made to uncover what the data reveal about the relationship between tax progressivity and convergence speed. This section presents a first attempt in this direction and provides some evidence on the *positive* correlation between both variables. Our primary data are provided by Barro and Sala-i-Martin [6] and Stephenson [35] and are described in a more detailed way in Appendix C. Although we focus on a particular data set, in this section, we discuss in Appendix C the robustness of our results when alternative estimates are used.

In Barro and Sala-i-Martin [6, Table 11.1, p. 388], we find some estimates of the convergence rate of per-capita income for the US states, over decades from 1920 to 1990, which we reproduce in the second column of Table 1 below. On the other hand, Stephenson [35, Table 1, p. 391] provides us with some estimates, for the US, of both the average tax rate and the marginal tax rate on income. We then compute, from these data, estimates of tax progressivity, as we now described, and then average them over a decade. From equation (5), it is not difficult to compute the marginal tax rate $\tau_m = \partial(\tau y)/\partial y$, which can be written as $\tau_m = \tau + \pi \nu y^{-\pi}$. 
<table>
<thead>
<tr>
<th>Decade</th>
<th>Convergence Speed (%)</th>
<th>Tax Progressivity</th>
</tr>
</thead>
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<tr>
<td>1920-30</td>
<td>-1.49</td>
<td>-0.022</td>
</tr>
<tr>
<td>1930-40</td>
<td>1.41</td>
<td>-0.025</td>
</tr>
<tr>
<td>1940-50</td>
<td>4.31</td>
<td>0.012</td>
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<tr>
<td>1950-60</td>
<td>1.90</td>
<td>0.018</td>
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<tr>
<td>1960-70</td>
<td>2.46</td>
<td>0.010</td>
</tr>
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<td>1970-80</td>
<td>1.98</td>
<td>0.007</td>
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<tr>
<td>1980-90</td>
<td>0.11</td>
<td>-0.007</td>
</tr>
</tbody>
</table>

Table i: convergence speed and tax progressivity in the US: 1920-1990. Source: author’s computations, based on Sala-i-Martin [6, Table 11.1, p. 388] and Stephenson [35, Table 1, p. 391]

So as to avoid any scaling effect in our tax schedule, we now define $\nu = \chi \overline{y}^\pi$, where $\overline{y}$ is steady-state income and $\chi$ is a positive scaling parameter. In that case, one gets that $\tau_m = \tau + \pi \chi (\overline{y}/y)^\pi$. Therefore, one can get a steady-state estimate of tax progressivity, $\pi$, from average tax rate, $\tau$, and marginal tax rate, $\tau_m$: it is simply given by $\pi = (\tau_m - \tau)/(1 - \tau)$. The corresponding estimates appear in the third column of Table i.

Finally, Figures 2 and 3 report, in a scatter plot of convergence speed against tax progressivity, the estimates of Table i. Figure 2 also reproduces the linear trend obtained from ordinary least-squares, linear regression. The slope is significantly positive (at a critical level of 5%), in agreement with the positive correlation predicted by our analysis of Section 2.2: the critical $t$–value is approximately 2.18 so that the $p$–value is about 4%. For illustrative purpose, Figure 3 reports the non-linear trend obtained from second-order polynomial regression (compare with Figure 1).

Figure 2: insert here.

Figure 3: insert here.
4 The Model with Elastic Labor Supply

4.1 Model and Dynamics

The purpose of this section is to study an extended version of the model in which labor is elastic to the real wage. For simplicity, we assume, following a large strand of the literature (e.g. King and Rebelo [21]), that preferences are now given by:

\[\int_0^\infty \left\{ \frac{C(t)^{1-\theta}}{1-\theta} - \frac{L(t)^{1+\gamma}}{1+\gamma} \right\} e^{-(\rho-n)t} dt,\] (12)

where \(\gamma \geq 0\) is the inverse of the (Frisch) elasticity of labor supply with respect to the real wage.

With a slight change of notation, we now define technology as:

\[y = k^\alpha h^\eta L^{1-\alpha-\eta},\] (13)

where, for instance, \(y \equiv Ye^{-(n+g)t}\) denotes output deflated to account for population and technological growth. In our credit-constrained economy, one still has that \(k = (1 - \tau_k)\alpha y/(r + \delta)\). One can solve the latter equation in \(k\) and replace in equation (13), to get:

\[y = Bh^\epsilon L^{1-\epsilon},\] (14)

where \(B = [(1 - \tau_k)\alpha/(r + \delta)]^{\alpha/(1-\alpha)}\) and \(\epsilon = \eta/(1 - \alpha)\), with \(0 \leq \epsilon < \alpha + \eta\).

Households decisions are then obtained by maximizing (12) subject to the budget constraint (7), using equations (14) and (5), given the initial stock \(h(0) < h^*\). The usual first-order conditions are then:

\[\theta \frac{\dot{c}}{c} = \nu \varepsilon(1 - \alpha)(1 - \pi)\frac{y^{1-\pi}}{h} - \rho - \delta - \theta g,\] (15)

\[L^\gamma C^\theta = \nu (1 - \alpha)(1 - \pi)(1 - \varepsilon)\frac{y^{1-\varepsilon}}{L} e^{\theta t}.\]

The first appendix derives the convergence rate by linearizing equations (9)-(15) around the unique interior steady state. As usual, requiring \(L\) to be constant at the steady state imposes \(\theta = 1\) (see, e.g. Barro and Sala-i-Martin [6] or King and Rebelo [21]). For a steady state to be feasible, we need here again that \(\pi > \pi \equiv 1 - (\rho + \delta + g)/[\varepsilon(\delta + n + g)]\). To compute the \(\beta\)-convergence rate, one has first to solve for labor
by using the second equation of (15) and equation (14). Some calculations are then needed to get the following proposition.

**Proposition 4.1 (Convergence Rate when Labor is Elastic)**

In the economy with elastic labor supply, the convergence rate \( \beta \) is given, when \( \theta = 1 \), by:

\[
\beta \equiv 0.5\{(\phi^2 + 4(\rho + \delta + g)(\frac{\gamma + 1}{\gamma + 1 - (1 - \varepsilon)(1 - \pi)})(\frac{\rho + \delta + g}{\varepsilon(1 - \pi)} - \delta - n - g))^{0.5} - \phi\},
\]

where \( \phi = \rho - n > 0 \).

Under the assumption that the tax rate is not too regressive, that is, \( \pi > \pi \equiv 1 - (\rho + \delta + g)/[\varepsilon(\delta + n + g)] \), the convergence rate \( \beta \) given by equation (16) increases, when positive, with the labor supply elasticity to the real wage \(1/\gamma\).

Moreover, the convergence rate \( \beta \) increases, when positive, with the progressivity parameter \( \pi \) when the human capital share is large enough, that is, when \( \eta > 0.5(1 - \alpha) \). In particular, convergence is then more rapid (resp. less rapid) than in the case of a constant tax rate when the tax rate is progressive (resp. regressive). The convergence speed tends to zero when \( \pi \) approaches the negative value \((\alpha + \eta - 1)/\eta\).

In Proposition 4.1, the case of inelastic labor may be recovered by setting \( \gamma \) to infinity: one then gets the simpler expression in (10). We now study how the convergence rate \( \beta \) depends, when positive, on the progressivity parameter \( \pi \) when labor supply is elastic, that is, when \( \gamma < \infty \).

**4.2 Convergence when Labor is Elastic**

We now examine the quantitative implications of our progressive tax schedule and use the benchmark parameter values in (11), with the exception that \( \theta = 1 \). We focus on two scenarios when increasing labor supply elasticity \((1/\gamma)\): \( \gamma = 1 \) or 0. Proposition 4.1 shows that the more elastic labor supply, the more rapid convergence and, therefore, the less progressive should be the tax rate.
Figure 4: the convergence rate $\beta$ as a function of tax progressivity $\pi$: the case of elastic labor.

For the first value $\gamma = 1$, implying that labor supply has unitary elasticity, Figure 4 shows that convergence is more rapid than when labor supply is inelastic (that is, when $\gamma = \infty$). In particular, the predicted convergence rate belongs to the narrow range $0.015; 0.030$ only if the tax rate is regressive, i.e. when $\pi$ is in $-0.20; -0.02$. However, it also falls within the more recent range of estimates supported, for instance, by Evans and Karras [14]: Figure 4 shows that $\beta \approx 0.08$ when the tax rate is quite progressive, that is, when $\pi = 0.4$.

Smaller values of $\gamma$ increase the speed of convergence and, therefore, require taxes to be even more regressive: when $\gamma = 0$ (the case of indivisible labor studied by Rogerson [32] and Hansen [19]), $\beta$ is predicted to belong to $0.15; 0.030$ only if $\pi$ is in $-0.23; -0.06$. On the other hand, $\beta \approx 0.08$ when $\pi = 0.37$. Here again, progressivity plays a key role in the model’s predictions of the convergence speed.

Therefore, the main conclusion from this exercise is that the model’s predictions accord well with the existing evidence on convergence, when labor supply is elastic. However, this requires the tax rate to be quite responsive to the tax base.
5 Conclusion

This paper has studied a neoclassical, open-economy growth model with progressive taxation and has shown that the predicted convergence rate increases with, and is highly sensitive to the progressivity parameter. In particular, the model’s predictions include a narrow range of estimates (supported, for instance, by Barro and Sala-i-Martin [5], Mankiw, Romer, and Weil [25]) provided that the tax rate is not too sensitive to the tax base. However, the model predicts the full convergence range, which includes more recent estimates (see, e.g., Evans and Karras [14], Quah [29], Temple [38]), when the tax rate is elastic enough to income. A preliminary attempt to assess the empirical evidence on the convergence speed-tax progressivity correlation revealed a positive relationship, in agreement with our theoretical analysis.

One important lesson of our paper is that although international capital mobility tends to raise the speed of convergence towards the balanced growth path, in comparison with the closed-economy configuration, different open economies may converge rapidly or slowly, depending on the way taxation responds to income. In the real world, countries seem to differ significantly with respect to their tax system, for various reasons including distribution concerns (see Bénabou [7]). Therefore, an interesting direction for future research would be to estimate a model that incorporates tax progressivity and to assess its predictions with respect to the speed of convergence. Moreover, our results point at tax progressivity as a potential source of cross-country differences in convergence rates and this conclusion should be quantified. I plan to pursue this project in the near future.

On the theoretical side, a relevant extension of our analysis would be to assume, in agreement with existing tax schedules, that capital income taxes are somewhat less progressive than labor income taxes. Moreover, it would also be relevant to take into account the fact, documented by some studies (e.g. Krusell et al. [23]), that skilled labor and physical capital are complements while raw labor and physical capital are substitutes. In view of some results by Barro et al. [3, pp. 23-28], one expects that the stronger complementarity between physical and human capital, the higher the convergence rate. Therefore, adding tax progressivity
to the model would probably lead us to predict that convergence will be slow provided that the tax rate is regressive enough. I hope this indicates some directions for future and fruitful research.

A Deriving the Convergence Rate $\beta$

This appendix derives the expression of the convergence rate $\beta$. The strategy is to derive this rate in the more general model of section 4 and to study the special case of inelastic labor supply, that is, $\gamma = \infty$.

The dynamical system describing intertemporal equilibria consists of equations (9)-(15). The first step is to rewrite, from the static condition in (15), the following equation:

$$\left[\gamma + 1 - (1 - \varepsilon)(1 - \pi)\right]\hat{L} = \text{cst} + \varepsilon(1 - \pi)\hat{h} - \theta \hat{c} + g(1 - \theta)t,$$

(17)

where, for example, $\hat{L}$ denotes $\ln L$.

Moreover, taking logs in equation (14) yields $\hat{y} = \text{cst} + \varepsilon \hat{h} + (1 - \varepsilon)\hat{L}$, which delivers, by using equation (17):

$$(1 - \pi)\hat{y} - \hat{h} = \lambda_0 + \lambda_1 \hat{h} + \lambda_2 \hat{c} + \lambda_3 g(1 - \theta)t,$$

(18)

where

$$\lambda_0 = (1 - \pi) \ln B + \frac{(1 - \varepsilon)(1 - \pi)\ln(\nu(1 - \varepsilon)(1 - \alpha)(1 - \pi)B^{1 - \pi})}{\gamma + 1 - (1 - \varepsilon)(1 - \pi)} , \quad \lambda_1 = \frac{\gamma + 1 - (1 - \varepsilon)(1 - \pi)}{\gamma + 1 - (1 - \varepsilon)(1 - \pi)},$$

(19)

$$\lambda_2 = \frac{-\theta(1 - \varepsilon)(1 - \pi)}{\gamma + 1 - (1 - \varepsilon)(1 - \pi)}, \quad \lambda_3 = \frac{(1 - \varepsilon)(1 - \pi)}{\gamma + 1 - (1 - \varepsilon)(1 - \pi)}.$$

By rewriting equations (9)-(15) in logs, it is easy to get:

$$\dot{\theta} \hat{c} = \nu(1 - \alpha)(1 - \pi)e^{\lambda_0 + \lambda_1 \hat{h} + \lambda_2 \hat{c} + \lambda_3 g(1 - \theta)t} - \rho - \delta - \theta g,$$

(20)

$$\dot{\hat{h}} = \nu(1 - \alpha)e^{\lambda_0 + \lambda_1 \hat{h} + \lambda_2 \hat{c} + \lambda_3 g(1 - \theta)t} - \delta - n - g - e^{\hat{c} - \hat{h}}.$$

We now consider two cases: $\gamma = \infty$ and $\theta \geq 0$ (section 2) or $\gamma < \infty$ and $\theta = 1$ (section 4).
A.1 Inelastic Labor

We may recover the dynamical system arising in the model with inelastic labor supply (section 2) by setting $\gamma = \infty$, which leads, in view of (19) to: $\lambda_2 = \lambda_3 = 0$ and $\lambda_1 = \varepsilon(1 - \pi) - 1$. It is not difficult to show that the associated dynamical system has a unique interior steady state if $\rho + \delta + g\theta > \varepsilon(1 - \pi)(\delta + n + g)$, that is, if $\pi > \pi \equiv 1 - (\rho + \delta + \theta g)/[\varepsilon(\delta + n + g)]$. The Jacobian matrix associated with equations (20), evaluated at the steady state, has trace $T$ and determinant $D$ such that:

$$T = \rho - n + g(\theta - 1), D = -(\rho + \delta + \theta g)(\frac{1 - \varepsilon(1 - \pi)}{\theta})[\frac{\rho + \delta + \theta g}{\varepsilon(1 - \pi)} - \delta - n - g].$$

(21)

When it exists, the negative eigenvalue of the Jacobian matrix is $0.5\{T - (T^2 - 4D)^{0.5}\}$, which gives, in view of equations (21), the expression of the convergence rate $\beta = -0.5\{T - (T^2 - 4D)^{0.5}\}$ in equation (10).

A.2 Elastic Labor

The dynamical system arising in the model with elastic labor supply (section 4) is obtained by setting $\theta = 1$, with $\gamma < \infty$, in equations (20). Here again, the associated dynamical system has a unique interior steady state if $\pi > \pi \equiv 1 - (\rho + \delta + g)/[\varepsilon(\delta + n + g)]$. Straightforward computations lead to the following expressions of $T$ and $D$, respectively the trace and determinant of the Jacobian matrix associated with equations (20), evaluated at the steady state:

$$T = \rho - n, D = -(\rho + \delta + g)(\frac{(\gamma + 1)(1 - \varepsilon(1 - \pi))}{\gamma + 1 - (1 - \varepsilon)(1 - \pi)})[\frac{\rho + \delta + g}{\varepsilon(1 - \pi)} - \delta - n - g].$$

(22)

When it exists, the negative eigenvalue of the Jacobian matrix is $0.5\{T - (T^2 - 4D)^{0.5}\}$, which gives, in view of equations (22), the expression of the convergence rate $\beta = -0.5\{T - (T^2 - 4D)^{0.5}\}$ in equation (16).
B The Case of Progressive Physical Capital Taxes

This appendix shows that our qualitative results - that tax progressivity fosters convergence - remain valid when taxes on physical capital income also are elastic. For sake of brevity, we assume that $\tau_k = \tau$, which implies that all sources of income are now taxed at the same rate $\tau$, which varies with the base income. Moreover, we embed this feature in the model with inelastic labor (see section 2).

Equation (4) transforms into:

$$\dot{k} + \dot{h} - \dot{d} = (1 - \tau)k^\alpha h^\eta - (\delta + n + g)(k + h) - (r - n - g)d - c.$$  

(23)

Moreover, the after-tax return on physical capital equals $r + \delta$ so that $k = (1 - \tau)\alpha y/(r + \delta)$. Therefore one has, by using equation (2),

$$y = Bh^\varepsilon,$$  

(24)

where $B = [\alpha \nu/(r + \delta)]^{\alpha/(1-\alpha(1-\pi))}$ and $\varepsilon = \eta/(1 - \alpha(1 - \pi))$. Finally, equations (8) and (9) still describe the dynamics of consumption and human capital, together with equation (24). It is then clear that after-tax income is increasing and concave in $h$ only if $\pi > (\alpha + \eta - 1)/(\alpha + \eta)$ which we assume to hold (this is stronger than the condition $\pi > (\alpha + \eta - 1)/\eta$ imposed in section 2).

Some computations then give the expression of the convergence rate $\beta$ which is the analog of equation (10), except that $\varepsilon(1 - \pi) = \eta(1 - \pi)/(1 - \alpha)$ should now be replaced by $\eta(1 - \pi)/[1 - \alpha(1 - \pi)]$. The most important conclusion of this exercise is that the convergence speed $\beta$ is still an increasing function of progressivity $\pi$. Therefore, the basic mechanism also works when taxes on physical capital income are progressive: increasing progressivity means decreasing the after-tax capital share, in which case diminishing returns set in more quickly, that is, convergence is more rapid.

Figure 5 reports the graph of the convergence rate $\beta$, using the parameter values in equation (11).

In particular, Figure 5 shows that $\beta$ belongs to the range 0.015;0.030 only if $\pi$ is confined to a narrow
Figure 5: the convergence rate $\beta$ as a function of tax progressivity $\pi$: the case of progressive taxes on all sources of income.
range, that is, $-0.074:0.074$ (compare with $-0.11:0.11$ in section 2.2). When $\pi$ is outside this range, then $\beta$ is either higher or smaller, depending on the sign of $\pi$: for example, $\beta = 0.1$ when $\pi = 0.53$ and $\beta = 0$ when $\pi = (\alpha + \eta - 1)/(\alpha + \eta) = -0.25$.

C Data and Robustness of OLS estimates

This section describes the data used in Section 3 and discusses robustness of our OLS estimates. First, the data on convergence are taken from Barro and Sala-i-Martin [6, Table 11.1, rows 4-10, p. 388]. In the main text, as well as in Table i and Figures 2-3, we use the estimates $\hat{\beta}$ obtained from their basic equation (11.6) (see Barro and Sala-i-Martin [6, p. 387]). On the other hand, our estimates of tax progressivity, as explained in Section 3, are calculated using data from the second and last columns of Stephenson [35, Table 1, p. 391] for the average tax rate ($ATR$) and the marginal tax rate on income ($AMEITRPI$), respectively.

I have checked robustness by using different estimates of convergence speed and tax progressivity. In particular, I have used estimates $\hat{\beta}$ from the fourth and sixth columns in Barro and Sala-i-Martin [6, Table 11.1, p. 388] (which add, respectively, regional dummies only and structural variables and regional dummies to the basic equation), as well as the third column ($AMEITR$) of Stephenson [35, Table 1, p. 391].

This leads to six possible linear regressions, out of which two yield insignificant coefficients: this is the case when we use the estimates in the sixth column of Barro and Sala-i-Martin [6, Table 11.1, p. 388], obtained by adding only regional dummies. For the remaining four data set, using the second and fourth columns of Barro and Sala-i-Martin [6, Table 11.1, rows 4-10, p. 388] (with or without regional dummies and structural variables included in the OLS regression), the following results hold. The correlation coefficient is positive and belongs to $0.62:0.70$ while the determination coefficient belongs to $0.39:0.49$. Therefore, the positive correlation between tax progressivity and convergence speed is found to be reasonably robust when alternative definitions are considered. More precisely, the $p-$value is approximately $4\%$ and $5\%$ when progressivity is measured by the $AMEITRPI$ series, while the $p-$value is about $7\%$ when $AMEITR$ is used.
References


[34] R. Solow, A contribution to the theory of economic growth, Quart. J. Econ. 70 (1956), 65-94.


Figure 2: some estimates of convergence speed and tax progressivity in US data (seven decades, 1920-90): OLS regression with correlation=0.7, $R^2=0.49$ (p-value=4%).
Figure 3: some estimates of convergence speed and tax progressivity in US data (seven decades, 1920-90): polynomial regression.