Public jobs creation and Unemployment dynamics*

Céline Choulet†
EUREQua-CNRS-Université Paris 1
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Abstract

This paper evaluates the relative impact of labor taxes as propagation channel of the effects of public employment on unemployment dynamics. We use a dynamic matching model and study how public jobs creation affects endogenous workers’ decisions to move on the labor market and private-sector firms’ job creation and destruction decisions. We obtain that creation of one public job destroys 1.5 private jobs when it is financed by distortive labor taxes. A low-skilled public jobs creation leads to more than complete crowding-out of private employment whereas a high-skilled public jobs creation leads to less than complete crowding-out. As an empirical illustration a vectorial autoregressive model is proposed that focuses on the consequences of an upward shift in public employment on disincentives for private jobs creation that labor taxes generate.

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†EUREQua-CNRS-Université Paris 1, Maison des Sciences Economiques, 106/112 Boulevard de l’hôpital, 75013 Paris, Email: choulet@univ-paris1.fr.
1 Introduction

Macroeconomic effects of government spending

The idea that government intervention is an important stimulus for economic activity is popular in traditional Keynesian analysis. In this regard, some papers have focused on the impact of fiscal policies on output. Blanchard and Perotti (2002) study the dynamic effects of shocks in government spending and taxes on economic activity in the United States using a structural VAR approach. Their results show positive government spending shocks as having a positive effect on output, and positive tax shocks as having a negative effect. However, thus far, little attention has been given to the consequences of public spending on job creation for private-sector employment and unemployment. The two surveys on public sector labor markets by Ehrenberg and Schwarz (1986) and Gregory and Borland (1999) highlight that the literature has mainly studied differences between the private and the public sectors in terms of decision making and wage setting. More recent papers have focused on the macroeconomic effects of temporary public employment programs, such as Holmlund and Linden (1993) and Calmfors and Lang (1995). They both conclude that programs have a direct job placement effect, which tends to reduce unemployment. These programs also have an effect on the wage pressure, which could reduce private-sector employment. Thus, the net effect on unemployment may be positive. Finn (1998) deals with the question of the effects of government spending on cyclical economic activity using a quantitative RBC model of the US economy. She distinguishes between the goods purchases and employee compensation components of government spending. A rise in government employment is a contractionary shock to private employment and private output, whereas positive shocks to government good purchases have the opposite effects.

A growing set of studies confirms the ambiguous impact of public employment on labor market performance. One the hand, public jobs creation is expected to offset the scarcity of private jobs. On the other hand, public employment is expected to crowd-out private employment, as it increases labor taxes, produces substitutable goods for private ones and exerts wage pressure (Algan, Cahuc and Zylberberg, 2002). Some papers have tried to identify the nature of the transmission mechanisms of the crowding-out effect. However, thus far, little attention has been given to the relative weight of these propagation channels. We propose, in a dynamic perspective, to measure the relative impact of one of these propagation channels. We have decided to focus here on the relationship between the crowding-out effect of public employment and the disincentives for private jobs creation that distortive labor taxes generate.

Dynamic effects of public jobs creation

Some empirical papers have studied the dynamic effects of public jobs creation on labor market
performance, linking its negative impact to an increase in private wage pressure. Edin and Holmlund (1997) use pooled crossed section and annual time series data for 22 OECD countries over the period 1968-1990. They argue that public sector employment, in the short run, with wages and prices fixed, decreases unemployment, whereas there is no significant long run effect (i.e. in the long run, the crowding-out is complete). Some empirical evidence on dynamic interrelations between aggregate time series on unemployment, real wages and public employment is provided by Malley and Moutos (1998) for Japan, Germany and United-States, Demekas and Kontolemis (2000) for Greece. Demekas et al. (2000) use a static job search model with public and private jobs. Unemployed workers can search for both types of jobs at the same time and all workers are perfectly insured. An increase in public wage or public employment leads through workers’ moves on the labor market to an higher increase in private sector wage. The unemployment rate increases as it positively depends on the wage differential between private and public sectors. Their empirical analysis shows that increases in government employment do not have a significant impact on unemployment, and might even raise it. The mean lag (half-life) of the unemployment rate is close to 3 quarters. They obtain that unemployment is positively correlated with the wage differential but they do not test formally for the positive impact of an increase in the wage differential on unemployment. Moreover, it seems that public employment is negatively correlated with the private wage. Therefore, we can not conclude on the relevance of private wages as propagation channel of public jobs creation. Malley et al. (1998) estimate an unrestricted reduced-form VAR system. They find that the crowding-out effect of public employment on private employment is complete, i.e. unemployment is not affected by changes in government employment, and the mean lag is close to 10 years. However, they argue that this effect doesn’t go through a rise in private real wages. They instead attribute the fall in private employment to the increased disincentives for private-sector firm and job creation. However, they don’t evaluate the relevance of such a propagation channel. We believe that labor taxes are relevant propagation channels. The main contribution of this paper is to evaluate the share of the crowding-out effect which can be attributed to the incentives for private jobs destruction that distortive labor taxes generate.

Outline

The paper is organized as follows. In Section 2, we present the theoretical framework. We consider an economy with public and private jobs. Unemployed workers can search either for a private- or public-
sector job. They can move across the sectors at zero cost, so that the expected utility of an unemployed worker has to be the same in both sectors. We focus on the labor taxes effect of public jobs creation and choose a framework so as to avoid the effects that arise from a competition on goods market. We use a dynamic matching model with endogenous private-sector job creation and destruction decisions. Sections 3 and 4 study the steady-state and out-of steady-state effects of an exogenous public jobs creation on the unemployment rate, with public wage rate fixed. The analysis is organized in two steps. First, we consider that newly created public jobs are financed by proportional labor taxes paid by all workers. As an increase in the public employment rate increases the relative expected utility of an unemployed worker searching for a public job, it induces a move of unemployed workers from the private sector to the public sector. Second, we consider that newly created public jobs are financed by distortive labor taxes. As public jobs creation introduces some distortions in the relative rewards from creating a job, it affects the private jobs creation and destruction decisions in the private sector. Moreover, it induces additional worker flows on the labor market by decreasing the expected utility of an unemployed worker searching for a private job. Thus, we have constructed the model in such a way that public employment exerts its effects on labor market both through an "attracting effect" and a "tax effect".

Section 5 evaluates the relative weight of the "tax effect" as a propagation channel. With a non-distortive tax rate, creation of one public job destroys one private job. The mean lag of the unemployment rate is close to 2 years. Distortive labor taxes increase the size of the crowding-out effect and the mean lag of the unemployment rate, as they affect the probabilities to move on labor market. Empirical data on private-sector and public-sector wages reveal that the low-skilled workers are better off in the public sector whereas the high-skilled workers obtain higher wages in the private sector. Section 6 considers the low- and high-skilled labor markets. Whereas a low-skilled public jobs creation leads to more than complete crowding-out of private employment, we expect, as Holmlund (1997), that a high-skilled jobs creation leads to less than complete crowding-out. The size of the crowding-out effect increases with the public wage premium, as the "attracting effect" of the public sector increases.

In Section 7, as an empirical illustration a vectorial autoregressive model is proposed that focuses on the consequences of an upward shift in the public employment rate. We use French data which were obtained from the OECD Employment Outlooks and INSEE DMMO database. Data cover the period 1986:1-1997:4. The aim is to quantify the share of private jobs destruction and private wage fluctuations which can be attributed to public employment fluctuations in a dynamic perspective. We also propose to measure the size of the crowding-out effect in the long run and the mean lag of the unemployment rate.
to a public employment shock. Section 8 concludes.

2 The Model

Consider an economy with public and private jobs. Our model features three types of agents: private- and public-sector workers, firms and the government. The labor force is constant and normalized to one. Individuals live infinite lives and are risk-neutral. Unemployed workers can search either for a private- or public-sector job, but not for both types of jobs at the same time. However, they can move across the sectors at zero cost, so that the expected utility of an unemployed worker has to be the same in both sectors. There is no job-to-job mobility.

We assume endogenous private-sector job destructions and creations in order to study the effects of distortive labor taxes on job creation and destruction flows. We endogenize the private job destruction decision by considering that the firm chooses a reservation productivity and destroys jobs whose productivity falls below it due to the arrival of an idiosyncratic or aggregate shock.

2.1 Technologies

In the private sector, each firm has one job. When the job is filled, it produces one unit of output (the numeraire) using labor. Profit maximization requires that all new jobs are created at maximum productivity\(^2\). We note \(p x\) the productivity of a private job where \(p\) is a general productivity parameter and \(x\) an idiosyncratic one. \(p\) is a positive constant parameter. When an idiosyncratic shock arrives, the productivity of the job is drawn from a general distribution \(F(x)\), with \(0 \leq x \leq eu\). This productivity is independent of initial productivity and irreversible. Idiosyncratic shocks arrive to jobs at Poisson rate \(q_p\). Note \(J^p(x)\) the value of a filled job with idiosyncratic productivity \(x\). At some of the idiosyncratic productivities that firms now face, production is profitable, but at some others it is not. When a shock arrives, it can be shown that the optimal decision for the firm is to continue production at the new productivity if \(J^p(x) \geq 0\) but to destroy the job if \(J^p(x) < 0\). As \(J^p(x)\) is a monotone continuous increasing function of \(x\), the job destruction rule \(J^p(x) < 0\) satisfies the reservation property with respect to the reservation productivity \(R\), defined by \(J^p(R) = 0\). By the reservation property, firms destroy all jobs with idiosyncratic productivity \(x < R\) and continue producing in all jobs with productivity \(x \geq R\).

\(^2\)Newly-created jobs are assumed more productive than existing ones, as they benefit from the best technology in the market.
In the public sector, each job produces one unit of public goods, which are consumed by all individuals. All individuals have the same preferences and the public goods provide \( v(l_g) \) utility. This utility is increasing at a decreasing rate in the amount of the public good, i.e. \( v'(l_g) > 0 \) and \( v''(l_g) < 0 \). The public-sector employment level and wage rate are exogenous.

2.2 Matching function

Our model borrows from Mortensen and Pissarides (1994). In the private sector, we assume a matching function that gives the number of jobs formed at any moment in time as a function of the number of workers looking for jobs and the number of firms looking for workers. A job can be filled or vacant, but only vacant jobs search for workers. Similarly, a worker can be employed or unemployed, but only unemployed workers search for jobs. Hiring a worker and searching for a job are costly activities. Vacant jobs and unemployed workers are brought together in pairs through an imperfect matching process. The steady state is one where the flow of workers into unemployment is equal to the flow of workers out of unemployment.

Consider there are \( l_p \) private jobs. Let \( u_p \) denote the private-sector unemployment rate (the fraction of unmatched workers of the private labor force who searches for a job). \( v_p \) denotes the private vacancy rate (the number of vacant jobs as a fraction of the labor force in the private sector). The matching function exhibits the standard properties. It is increasing in both its arguments, concave, and homogenous of degree 1.

The rate at which private vacant jobs become filled is:

\[
\frac{m(u_p, v_p)}{v_p} \equiv m(\theta)
\]

with \( \theta = \frac{v_p}{u_p} \) and \( m'(\theta) \leq 0 \). In our framework, \( \theta \) denotes the private labor market tightness and \( m(\theta) \) the probability that a match occurs between an unemployed worker and a vacant job in the private sector.

The rate at which private-sector unemployed workers move into employment is:

\[
\frac{m(u_p, v_p)}{u_p} \equiv \theta m(\theta)
\]

with \( (\theta m(\theta))' \geq 0 \). The mean duration of private-sector unemployment is \( 1/\theta m(\theta) \).
### 2.3 Flows

The flow into private-sector unemployment is equal to the product of the fraction of jobs that get hit by a shock, $q_p l_p$, and the probability that the new idiosyncratic productivity is below the reservation productivity, $F(R)$. Therefore the mean number of workers who enters private-sector unemployment during a small time interval is $q_p F(R) l_p$ and the mean number who leaves private-sector unemployment is $\theta m(\theta) u_p$. The evolution of mean private-sector unemployment is given by the difference between the two flows:

$$\dot{u}_p = q_p F(R) l_p - \theta m(\theta) u_p$$  \hspace{1cm} (1)

Let $u_g$ and $l_g$ denote the public-sector unemployment and employment rates respectively. During a small time interval, the flow into public-sector unemployment is given by $q_g l_g$, and the flow out of public-sector unemployment is equal to $s_g u_g$. The evolution of public-sector unemployment is therefore given by:

$$\dot{u}_g = q_g l_g - s_g u_g$$  \hspace{1cm} (2)

---

3 We could endogenize the public jobs creation by assuming that the social planner chooses the level of public employment that maximizes the social optimum. Considering risk-averse workers could make the public-sector wage endogenous. The government would choose the public wage level, which would minimize the size of the public budget and wage inequalities between the two sectors, or would bargain with trade unions.

4 The search for a job is considered as directed. This assumption is convenient with the fact that, in many countries, the public sector has a specific hiring process and workers need some specific information in order to compete for a job in this sector. Thus, we consider that there exists two types of unemployed workers: the public- and the private-sector ones.
So, the evolution of total unemployment reads:

\[ \dot{u} = q_p F(R)l_p + q_g l_g - \theta m(\theta) u_p - s_g u_g \]  

(3)

### 2.4 Expected asset values of private jobs

Using the discount rate \( r \), the present-discounted value of expected profit from an occupied job, with productivity in the range \( R \leq x \leq eu \), satisfies:

\[ r J^p(x) = px - w_p(x)(1 + \tau_F) + q_p \int_R^{eu} J^p(s) dF(s) + q_p F(R) V^p - q_p J^p(x) + \dot{J}^p(x) \]  

(4)

where \( w_p(x) \) is the wage rate, which is determined by a bargain between the firm and the worker for all \( R \leq x \leq eu \). Wages are taxed at the distortive rate \( \tau_F \). Whenever an idiosyncratic shock arrives, the firm continues producing for a new value \( J^p(s) \) if the new idiosyncratic productivity is in the range \( R \leq s \leq eu \), or destroys the job for an expected return \( V^p \) otherwise. \( \dot{J}^p \) is the expected capital gain from changes in job value during adjustment.

\( V^p \) is the present-discounted value of expected profit from a vacant job and it is given by:

\[ r V^p = -pc + m(\theta) [J^p(eu) - V^p] + \dot{V}^p \]

The hiring cost \( pc \) is made proportional to productivity as it is assumed that it is more costly to hire more productive workers.

We assume that all profit opportunities from new jobs are exploited in the steady state and out of it, driving rents from vacant jobs to zero \( V^p = \dot{V}^p = 0 \), which implies:

\[ J^p(eu) = \frac{pc}{m(\theta)} \]

This condition states that in equilibrium, private-sector labor market tightness is such that the expected profit from a new job is equal to the expected cost of hiring a worker.

### 2.5 Expected utilities of workers

We have neglected labor intensity and search costs. A private- or public-sector worker instantaneously enjoys the utility from his net wage rate, \( w_p(x)(1 - \tau_T) \) or \( w_g(1 - \tau_T) \), and from the public goods, \( v(l_g) \).

The unemployed workers enjoy unemployment benefits, \( z(1 - \tau_T) \) and public goods, \( v(l_g) \). Let \( W^p(x) \), \( W^u_z \), \( W^p_u \) and \( W^u_g \) denote, respectively, the expected present values of the lifetime utility for privately employed, publicly employed or unemployed workers.
In the private sector, the returns from working at a job with idiosyncratic productivity $x$ satisfy:

$$rW^p_e(x) = w_p(x)(1 - \tau_T) + v(l_g) + q_p \int_{R}^{eu} W^p_e(s)dF(s) + q_p F(R)W^p_u - q_p W^p_e(x) + W^p_e(x)$$  \hspace{1cm} (5)$$

$$rW^p_u = z(1 - \tau_T) + v(l_g) + \theta m(\theta)[W^p_e(eu) - W^p_u] + \dot{W}^p_u$$  \hspace{1cm} (6)$$

$$rW^g = w_g(1 - \tau_T) + v(l_g) + q_g[W^g_u - W^g_e] + \dot{W}^g_e$$

$$rW^g_u = z(1 - \tau_T) + v(l_g) + s_g[W^g_e - W^g_u] + \dot{W}^g_u$$

Whenever a shock arrives, the private-sector worker remains employed for new returns $W^p_e(s)$ if the new idiosyncratic productivity is in the range $R \leq s \leq eu$, or becomes unemployed for an expected return $W^p_u$ otherwise.

### 2.6 Steady-state equilibrium

In the steady-state, the mean rate of unemployment is constant, $\dot{u} = 0$. Its steady-state value is:

$$u = \frac{q_p F(R)}{\theta m(\theta)}l_p + \frac{q_g l_g}{s_g}$$  \hspace{1cm} (7)$$

Therefore, in equilibrium, the mean number of workers who go on unemployment $q_p F(R)l_p + q_g l_g$ is equal to the mean number of workers who get off unemployment $\theta m(\theta)u_p + s_g u_g$.

#### 2.6.1 Sharing rule

In the steady-state, the expected capital gains from changes in jobs value and in utilities are null. The private wage rate derived from the Nash bargaining solution is the one that maximizes the weighted product of the worker’s and the firm’s net return from the job match $S(x) = W^p_e(x) - W^p_u + J^p(x) - V^p$. It satisfies:

$$w_p(x) = \arg \max (W^p_e(x) - W^p_u)^{\beta} (J^p(x) - V^p)^{1-\beta}$$

where the relative measure of labor’s bargaining strength is in the range $0 \leq \beta \leq 1$.

The first-order maximization condition reads:

$$W^p_e(x) - W^p_u = \frac{\beta(1 - \tau_T)}{\beta(1 - \tau_T) + (1 - \beta)(1 + \tau_T)}(W^p_e(x) - W^p_u + J^p(x))$$  \hspace{1cm} (8)$$
where \( \frac{\beta(1-\tau_T)}{\beta(1-\tau_T) + (1-\beta)(1+\tau_F)} \) is the worker’s share in total surplus, which decreases with a rise in the tax rates.

By substituting \( W_p^e(x), W_u^e \) and \( J_p(x) \) from (5), (6) and (4) into (8), we obtain

\[
w_p(x) = (1-\beta)z + \frac{\beta}{1+\tau_F}p(x + c\theta)
\]

with \( R \leq x \leq cu \).

So, the mean expected wage rate of a private employed worker reads:

\[
E(\frac{w_p(x)}{x} \geq R) = (1-\beta)z + \frac{\beta}{1+\tau_F}p(E(\frac{w_p(x)}{x} \geq R) + c\theta)
\]

### 2.6.2 Arbitrage condition

The expected utilities of unemployed workers must be equal in both sectors. The expected utilities of public- and private-sectors unemployed workers can be rewritten respectively

\[
rW_u^g = z(1-\tau_T) + v(l_g) + s_g \frac{w_g - z)(1-\tau_T)}{r + q_s + s_g}
\]

and

\[
rW_u^p = z(1-\tau_T) + v(l_g) + \frac{\beta pcd\theta}{1-\beta} \frac{1-\tau_T}{1+\tau_F}
\]

Therefore, the unemployed workers are indifferent if and only if the following arbitrage condition is fulfilled

\[
s_g \frac{w_g - z)}{r + q_s + s_g} = \frac{\beta}{(1-\beta)(1+\tau_F)}pcd\theta
\]

The participation constraint imposes \( W_p^e(x) > W_u^e \) and \( W_p^g > W_u^g \).

### 2.6.3 Budget constraint

Taxes finance public employment and unemployment benefits. Therefore, the budget equilibrium condition reads:

\[
\tau_F w_p(x) l_p + \tau_T (w_p(x) l_p + w_g l_g + zu) = w_g l_g + zu
\]

with \( R \leq x \leq cu \).

### 2.6.4 Private-sector job creation and destruction conditions

Substitution of the wage equation (9) into (4) gives

\[
(r + q_p)J_p(x) = (1-\beta)(px - z(1+\tau_F)) - \beta pcd\theta + q_p \int_R^{eu} J_p(s)dF(s)
\]
Evaluating (12) at \( x = R \) and subtracting the resulting equation from (12) after noting \( J^p(R) = 0 \), we get
\[
(r + q_p)J^p(x) = (1 - \beta)p(x - R) \tag{13}
\]
Substituting \( J^p(x) \) from (13) into the integral expression of (12) gives
\[
(r + q_p)J^p(x) = (1 - \beta)(px - z(1 + \tau F)) - \beta pc\theta + \frac{q_p}{r + q_p}(1 - \beta)p \int_R^{eu} (s - R)dF(s) \tag{14}
\]
To derive the condition for private-sector job creation, we evaluate (13) at \( x = eu \) and use the zero-profit condition,
\[
(1 - \beta)\frac{eu - R}{r + q_p} = \frac{e}{m(\theta)} \tag{15}
\]
The expected gain from a new job to the firm must be equal to the expected hiring cost that the firm has to pay. The job creation curve is a downward-sloping curve in the space \((\theta, R)\). Indeed, at higher \( R \), the expected life of a job is shorter, so firms create fewer jobs and \( \theta \) is lower.

To derive the condition for private-sector job destruction, we evaluate (14) at \( x = R \) and use the zero-profit condition,
\[
R - (1 + \tau F)\frac{z}{p} - \frac{\beta}{1 - \beta}pc\theta + \frac{q_p}{r + q_p} \int_R^{eu} (s - R)dF(s) = 0 \tag{16}
\]
The job destruction curve is an upward-sloping curve in the space \((\theta, R)\). Indeed, at higher \( \theta \), the workers’ outside opportunities are better (and wages are higher) and so the reservation productivity \( R \) is higher.

The properties of the steady-state are obtained from the simultaneous solution of the equations of the model. The private-sector job creation (15) and destruction (16) conditions determine the reservation productivity \( R \) and the labor market tightness \( \theta \). The budget constraint (11) determines the tax rate \( \tau F \). The probability to move into public employment \( s_g \) is obtained from the arbitrage condition (10). The private-sector wage rate \( w_p(x) \) is obtained from the sharing rule (9) by substituting \( \theta \) and \( \tau F \). The private and public-sector unemployment rates \( u_p \) and \( u_g \) are given by the respective steady-state conditions (1) and (2) in terms of the private- and public-sector job flows.

2.7 Out-of-steady-state dynamics

The out-of-steady-state dynamics of unemployment are given by equation (3). The matching technology does not allow jumps in job formation (firms and workers can’t create jobs without delay). The matching process is a backward-looking process that is governed by the difference between the job creation and the job destruction flows, making unemployment a predetermined variable at any moment in time.
We assume that firms can open and close vacancies without delay. This assumption implies that the zero-profit condition for new vacancies holds in and out of steady state \( V^p = \dot{V}^p = 0 \). So, labor market tightness \( \theta \) is a jump variable. We also assume that firms can destroy unprofitable jobs without delay. This assumption implies that the zero-profit condition satisfied by \( R \) holds in and out of steady state \( J^p(R) = \dot{J}^p(R) = 0 \). So, \( R \) is a jump variable and must be on its steady state value at all times. We assume that the sharing rule holds in and out of steady state, consistent with the assumption that the firm and worker can renegotiate any time new information. This assumption also requires that wages \( w_p(x) \) are a jump, forward-looking variable. However, according to the budget constraint (11), tax rates \( \tau_F \) and \( \tau_T \) are not jump variables as they depend on a predetermined variable, the unemployment rate.

We relax the budget constraint (11) by assuming that there is a perfect capital market. The government can rent into debt at zero cost during adjustment. Out-of steady-state, the following budget constraint has to be fulfilled:

\[
\int_0^\infty [\tau_F w_p(x) l_p(t) + \tau_T (w_p(x) l_p(t) + w_g l_g + zu(t))] e^{-rt} dt = \int_0^\infty [w_g l_g + zu(t)] e^{-rt} dt \tag{17}
\]

Whenever a shock arrives, the tax rates \( \tau_F \) and \( \tau_T \) jump on their new steady-state values which equilibrate the budget constraint (17)\(^6\). When the new steady-state equilibrium is reached, the constraint (11) is fulfilled.

With a perfect capital market, \( \theta, R \) and \( w_p(x) \) are jump variables. As the arbitrage condition only depends on \( \theta \) and \( \tau_F \), it holds in and out of steady-state, making \( s_g \) a jump variable. Whenever a shock arrives, unemployment starts moving according to (3) only if the new job creation and job destruction rates implied by the change in \( R \) and \( \theta \) are not equal.

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\(^5\)We consider in Appendix (9.2) that the budget constraint has to be fulfilled at each period. In this case, endogenous variables of our model are no more jump variables.

\(^6\)Assuming that \( R, \theta, w_p(x), \tau_F, \tau_T \) and \( s_g \) are jump variables, we integrate the differential equations (3) and (2) between 0 and \( t \):

\[
u(t) = \begin{cases} \frac{b}{a} & + \left[ u_0 + (\theta m(\theta) - s_g) \frac{h}{d} \right] e^{-at} \\ \end{cases}
\]

and

\[
u_g(t) = \begin{cases} \frac{h}{d} \right] e^{-dt} \\
\end{cases}
\]

with \( a = q_F F(R) + \theta m(\theta), b = q_p F(R)(1 - l_g) + q_p l_g, d = s_g, h = q_p l_g. \)

We integrate the budget constraint and substitute \( u_g(t) \) into \( u(t) \):

\[
\int_0^\infty [\tau_F w_p(x)(1 - l_g - u(t)) + \tau_T (w_p(x)(1 - l_g - u(t)) + w_g l_g + zu(t))] e^{-rt} dt = \int_0^\infty [w_g l_g + zu(t)] e^{-rt} dt
\]

with

\[
u(t) = \begin{cases} \frac{h}{d} \right] e^{-dt} \\
\end{cases}
\]

and

\[
u_g(t) = \begin{cases} \frac{h}{d} \right] e^{-dt} \\
\end{cases}
\]

with

\[
u(t) = \begin{cases} \frac{h}{d} \right] e^{-dt} \\
\end{cases}
\]
2.8 Calibration

The model is calibrated in order to match the French economy. Parameter values are chosen so as to reproduce an unemployment rate of 9.8%, a private employment rate of 68.2%, a public employment rate of 22% (source: OECD database, 2000). The private and public employment rates are the ratios of total business sector employment and of general government employment to the labor force, as we consider in our model that labor market participation is given. According to the OECD definition, public employment consists of jobs in central and local administrations, in non-profit organizations controlled or financed by public administrations and in military and diplomatic entities. The definition does not include public firms owned or controlled by the government. The baseline values of the parameters that we use are reported in Table 1. The basis period is taken to be 1 year. We adopt the following constant returns, Cobb-Douglas matching function \( m(u_p, v_p) = u^\eta v^{1-\eta} \). The elasticity of the matching function \( \eta \) and the bargaining power \( \beta \) of the employees amount to 0.5. The distribution of productivity shocks is assumed to be uniform on the support \([\varepsilon_l, \varepsilon_u]\), that is \( F(x) = \frac{x-\varepsilon_l}{\varepsilon_u-\varepsilon_l} \) (Mortensen and Pissarides, 1999).

General productivity \( p \) is normalized at unity.

\[
\begin{array}{cccc}
\tau & \varepsilon_l & \eta & \beta & p \\
0.05 & 0 & .5 & .5 & 1 \\
\end{array}
\]

Table 1: Baseline values of the parameters

The probability to move into public unemployment \( q_g \) is calibrated in order to reproduce the duration of a public job. The share of long-term contracts in public sector amounts to roughly 87% (the duration of long-term public job is close to 35 years) and the share of short-term contracts amounts to roughly 13% (the average duration is 36 months) (Enquête Emploi, 2000, INSEE). Thus \( q_g \) is fixed such as \( 1/q_g = 30.8 \) years.

In the same way, we calibrate the probability for a private job to be hit by a shock \( q_p \) in order to reproduce the mean duration of a private job. The share of long-term contracts in private sector amounts to roughly 87.6% (with a mean duration of 11 years) and the share of short-term contracts amounts to roughly 12.4% (the average duration is 18 months) (Enquête Emploi, 2000, INSEE). Thus \( q_p \) is fixed such as \( 1/q_p F(R) = 9.8 \) years. We calibrate the search cost \( c \) of a firm to represent 30% of the mean productivity of an employee to be consistent with survey results reported by Hamermesh (1993). The tax rate \( \tau_F = 35% \) is fixed using French legislation on taxes in the business sector. A lump sum tax \( \tau_T = 5% \) is introduced such as the public budget constraint is balanced. The value of unemployment income \( z \) is
set to reproduce a replacement rate of \( \frac{z}{w_p(x)} = 38\% \) (Cahuc and Zylberberg, 2004). The public wage \( w_g \) is calibrated to reproduce the mean wage gap between the public and private sectors (1.06)\(^7\). Maximum productivity \( \varepsilon u \) is set to reproduce the unemployment rate \( u \).

3 Steady-state effects of public jobs creation

We study in this section the steady-state effects of a public jobs creation on the unemployment rate. We alternatively assume that newly created public jobs are financed by a non-distortive tax (increase in \( \tau_T \) with \( \tau_F \) constant) and then by a distortive one (increase in \( \tau_F \) with \( \tau_T \) constant). The impact of public employment goes through an "attracting effect", i.e. voluntary workers' flows between sectors, in the non-distortive case, whereas it also goes through a "tax effect", i.e. private-sector job creation and destruction flows, in the distortive case.

3.1 Non-distortive taxation

First, consider that newly created public jobs are financed by lump sum taxes or proportional wage taxes, paid by all workers. The tax rate \( \tau_F \) remains unchanged. Therefore the tax system does not introduce any distortion in the workers' relative rewards from taking a job.

An increase in the public employment rate \( l_g \) induces an "attracting effect" on the private-sector unemployed workers. Indeed, for given public unemployment rate, the public labor market tightness increases. Thus the expected utility of unemployed workers searching for a public job tends to increase as the probability to move into public employment \( s_g \) increases. The public jobs creation induces a move of unemployed workers from the private sector to the public sector such as the probability to move out of public unemployment \( s_g \) remains unchanged. Voluntary unemployed workers' flows between sectors permit to maintain the indifference of unemployed workers to the two sectors. When the new steady-state equilibrium is reached, the number of unemployed workers searching for a public job \( u_g \) has increased whereas the number of unemployed workers searching for a private job \( u_p \) has decreased (Figure 2).

Moreover, as the move of private-sector unemployed workers to the public sector decreases the private unemployment, the private labor market tightness \( \theta \) tends to increase. So, the probability that a match occurs between an unemployed worker and a vacant job in the private sector decreases, making the expected cost of hiring a worker higher. Moreover, it increases the private wage pressure as the exit rate

\(^7\)Empirical data on net wages in France show that the low-skilled workers obtain higher wages in the public sector \((w_g/w_p = 1.18)\) whereas the high-skilled workers obtain higher wages in the private sector \((w_g/w_p = 0.95)\) (Enquête Emploi, 2000, INSEE). We consider these cases in Section 6.
from private unemployment increases. Firms open less vacancies such as the labor market tightness $\theta$ and the private wage $w_p(x)$ remain unchanged. Therefore, when the new steady-state equilibrium is reached, the number of private employed workers $l_p$ has decreased (Figure 2).

So, the steady-state effects of public jobs creation are to decrease the private-sector labor force ($l_p$ and $u_p$) and to increase the public-sector labor force ($l_g$ and $u_g$). Assuming a mean wage gap, there is relatively less unemployed workers $u^8$ (Figure 2).

We have tested the robustness of our results in relation to changes in parameters’ values.

3.2 Distortive taxation

Second, consider that newly created public jobs are financed by an increase in the distortive tax rate $\tau_F$.

The increase in the public employment rate $l_g$ induces, as in Subsection 3.1, an "attracting effect" on the private-sector unemployed workers. Workers’ moves on the labor market tend to increase the number of unemployed workers searching for a public job $u_g$ and to decrease the number of private employed workers $l_p$ and unemployed workers searching for a private job $u_p$.

But, now, public jobs creation affects the private-sector job creation and destruction conditions through the rise in the distortive tax rate. The tax increase increases the private-sector after-tax cost of labor, decreasing job creations and increasing job destructions. The reservation productivity $R$ increases and the labor market tightness $\theta$ decreases (Figure 3). The reason is that with higher net cost of labor, the expected return from a job $J^P(x)$ declines. The level of private unemployment $u_p$ tends to increase and the level of private employment $l_p$ to decrease due to a "tax effect". Moreover, the increase in the tax

*It will be shown that this result depends on the relative values of private- and public-sector wages.
rate $\tau_F$ and the decrease in the labor market tightness $\theta$ reduce the worker’s bargaining strength. So, the private wage rate $w_p(x)$ and the expected utility of a private unemployed worker $W_p^u$ decrease. Therefore, the "attracting effect" is reinforced: more unemployed workers move from the private sector to the public sector. These additional workers’ flows induce a higher public unemployment rate $u_g$, decreasing the public labor market tightness. The exit rate from public unemployment $s_g$ falls such as unemployed workers remain indifferent to the two sectors (Figure 3).

![Figure 3: The steady-state effects on $R$, $\theta$, $sg$ and $\tau_F$ with distortive taxation](image)

So, the "tax effect" reinforces the impact of the "attracting effect" on the number of public unemployed workers and of private employed workers. When the new steady-state equilibrium is reached, there are more unemployed workers in the public sector $u_g$ and less employed workers in the private sector $l_p$ (Figure 4). Whereas the "attracting effect" decreases the private unemployment rate $u_p$ through workers’ flows between sectors, the "tax effect" increases it due to the fall in the expected return from a private job $J^p(x)$. The net effect will depend on the parameter values which affect the probabilities to move on the labor market and the private-sector job creation and destruction conditions. With our calibration, the "attracting effect" dominates the "tax effect", as the private unemployment rate reaches a lower new steady-state value (Figure 4). Assuming a mean wage gap, there are more unemployed workers $u$ in this economy (Figure 4). The impact of public employment on labor market performance works through

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9 Let us consider an higher replacement rate. As previously, public jobs creation induces moves of unemployed workers from the private sector to the public sector, decreasing the private unemployment rate $u_p$. Increase in the distortive tax rate rises the reservation productivity, increasing private job destruction. But, as the expected return of a job negatively depends on unemployment benefits, the rise in $R$ is as higher as the replacement rate is high. The reason is that a higher replacement rate increases the worker’s bargaining strength. Therefore, the increase in the private unemployment rate $u_p$ is reinforced compared to the previous case. Moreover, the expected utility of an unemployed worker positively depends on unemployment benefits. So, this utility reaches a higher value with a higher replacement rate. Then, less private unemployed workers move into the public sector. The decrease in the private employment rate $u_p$ can be offset such as the impact of the "tax effect" on $u_p$ dominates the impact of the "attracting effect".
voluntary workers’ flows between sectors and distortions introduced by the tax system\textsuperscript{10}.

Figure 4: The steady-state effects on $u$, $ug$, $lp$ and $up$ with distortive taxation

4 Out-of Steady-State Dynamics

We analyze, in this section, the out of steady-state dynamics of the unemployment rate. We alternatively assume that newly created public jobs are financed by a non-distortive tax and a distortive one. The study of unemployment dynamics will permit to evaluate the sensitivity of the mean lag of the unemployment rate to distortive labor taxes.

4.1 Non-distortive taxation

Consider a 10\% increase in the public employment rate. The increase in public employment rate is financed through a rise in the tax rate $\tau_T$ such as the intertemporal budget constraint (17) remains balanced. Instantaneously, newly created public jobs are filled by the unemployed workers who search for a job in the public sector. Thus, the public-sector unemployment rate $ug$ instantaneously falls (Figure 5).

As the two-equation system (15) and (16) that determines equilibrium values of $R$ and $\theta$ is independent of the public employment rate, the public jobs creation has no effect on the reservation productivity $R$ and on the private labor market tightness $\theta$. Therefore, the private-sector job creation and destruction conditions remain unchanged. At the time of the shock, the private-sector labor market isn’t affected

\textsuperscript{10}We could consider that unemployment benefits are index-linked on private wages. The "tax effect" would be reduced as the private-sector job creation and destruction conditions would not more depend on the distortive tax rate. In this case, the increase in the tax rate would be entirely paid by workers.
and the private wage rate \( w_p(x) \) is unchanged. In the same way, as the probability to move into public employment \( s_g \) only depends on the private labor market tightness, the arbitrage condition (10) requires a unique value of \( s_g \) whatever the level of public employment.

The total unemployment rate \( u \) instantaneously falls (Figure 5), due to the jump in the public-sector unemployment rate.

Following the induced change in the public-sector unemployment rate, the public job creation rate, defined as the ratio of total public job creation to public employment \( \frac{s_g u_g}{1 - u - l_p} \), decreases whereas the public job destruction rate, defined as the ratio of total public job destruction to public employment \( \frac{q_g(1 - u - l_p)}{(1 - u - l_p)} \), does not change. The public-sector unemployment rate \( u_g \) has to increase (Figure 5) until the public job creation rate rises up to the level of the public job destruction rate (equation 2). The mean lag of the public unemployment rate is close to 2 years\(^{11} \). The increase in \( u_g \) is obtained through a move of unemployed workers from the private sector to the public sector, due to an "attracting effect" (Subsection 3.1).

These workers' flows between sectors induce a decrease in the private-sector levels of unemployment \( u_p \) and employment \( l_p \) (Subsection 3.1), such as the steady-state conditions in terms of the private-sector jobs flows (1) are fulfilled. The private unemployment and employment rates fall until they reach their new steady-state values (Figure 5).

Total unemployment \( u \) starts moving according to (3). Assuming a mean wage gap, the new steady-state level of unemployment is lower than in the initial steady-state equilibrium (Figure 5). The mean lag of the unemployment rate is close to 2 years.

### 4.2 Distortive taxation

Consider a 10\% increase in the public employment rate. Instantaneously, as in Subsection 4.1, newly created public jobs are filled by the public-sector unemployed workers. Thus, the public-sector unemployment rate \( u_g \) falls (Figure 7).

The increase in public employment rate is financed through a rise in the distortive tax rate \( \tau_F \) such as

\[^{11}\text{To derive the mean lag of the public unemployment rate } u_g, \text{ we solve the differential equation (2). We obtain:} \]

\[ u_g(t) = u_g^* + (u_{g0} - u_g^*)e^{-s_g t} \]

with \( u_g^* \) the steady-state level of public unemployment, and \( u_{g0} \) the initial level of public unemployment.

The mean lag (half-life) \( T_g \) is defined by:

\[ u_g^* - u_g(T_g) = (1 - 50\%)(u_g^* - u_{g0}) \]

that is

\[ T_g = \frac{-\ln 0.5}{s_g} \]

The mean lag of the public unemployment rate negatively depends on the exit rate from public unemployment \( s_g \).
the intertemporal budget constraint (17) remains balanced. This tax increase rises the private-sector net cost of labor $w_p(x)(1 + \tau_F)$. Therefore, it affects the private-sector job creation and destruction decisions.

The differentiation of (16) and of (15) with respect to $\tau_F$ gives:

$$\left[1 - \frac{q_p}{r + q_p}[1 - F(R)]\right] \frac{\partial R}{\partial \tau_F} = \frac{z}{p} + \frac{\beta c}{1 - \beta} \frac{\partial \theta}{\partial \tau_F}$$

(18)

$$\frac{c\eta(\theta)}{\theta m(\theta)} \frac{\partial \theta}{\partial \tau_F} = - \frac{1}{r + q_p} \frac{\partial R}{\partial \tau_F}$$

(19)

with $\eta(\theta) = - \frac{\partial m(\theta)}{\partial \theta} \frac{\theta}{m(\theta)}$.

The substitution of $\frac{\partial \theta}{\partial \tau_F}$ from (19) into (18) gives:

$$\left[1 - \frac{q_p}{r + q_p}[1 - F(R)]\right] \frac{(r + q_p)\eta(\theta)}{(1 - \beta)\theta m(\theta)} + \frac{\beta c}{1 - \beta} \frac{\partial R}{\partial \tau_F} = \frac{z}{p} > 0$$

Higher public employment $l_g$ shifts up the job destruction curve in the plane $(\theta, R)$, implying more job destruction at given $\theta$ (Figure 6). The reservation productivity $R$ increases and the labor market tightness $\theta$ decreases.

For given private unemployment, the private job destruction rate increases and the private job creation rate decreases. The job destruction rate goes up to a higher $q_pF(R)$, but instantaneously all jobs whose idiosyncratic productivity $x$ is below the new value of $R$ are destroyed. Thus, at the time of the impact, the private job destruction rate jumps to a higher value and then returns to its new steady state value. Given the mass of jobs destroyed, there is instantaneously an over-adjustment of the job destruction rate relatively to its real new steady state value. The private job creation rate falls, but because private
unemployment rate rises with the instantaneous destruction of new unprofitable jobs, it does not fall by the full amount that would have fallen at given \( u_p \)\(^{12}\). Thus, instantaneously, the private-sector unemployment rate \( u_p \) increases and the private employment rate \( l_p \) decreases (Figure 7).

The \( \tau_F \) increase and \( \theta \) decrease unambiguously induce a decrease in the private-sector wage rate \( w_p(x) \) (equation (9)). Moreover, the fall in the tax base reinforces the required increase in \( \tau_F \). The differentiation of the arbitrage condition (10) with respect to \( \tau_F \) gives:

\[
\frac{(w_g - z)(r + q_g)}{(r + q_g + s_g)^2} \frac{\partial s_g}{\partial \tau_F} = \frac{\beta pc}{(1 - \beta)(1 + \tau_F)} \left( \frac{\partial \theta}{\partial \tau_F} - \frac{\theta}{(1 + \tau_F)} \right) < 0 \tag{20}
\]

Therefore, according to (20), the exit rate from public unemployment \( s_g \) decreases in order to offset the decrease in \( \theta \) (Subsection 3.2).

The total unemployment rate \( u \) instantaneously falls (Figure 7). This fall is lower than the one in the Subsection 4.1 due to the instantaneous private job destructions.

As in Subsection 4.1, following the induced change in the public-sector unemployment rate, the public-sector unemployment rate \( u_g \) has to increase (Figure 7). The mean lag of the public unemployment rate is close to 2 years\(^{13}\). The increase in \( u_g \) is obtained through a move of unemployed workers from the private sector to the public sector, due to an "attracting effect" (Subsection 3.1), and because of the decrease in \( s_g \), due to a "tax effect" (Subsection 3.2).

Dynamic adjustment in the private unemployment rate follows the jumps in \( R \) and \( \theta \), until the private job creation rate \( \frac{\theta m(\theta)u_p}{(1 - u - l_g)} \) rises to the level of the higher private job destruction rate. The private unemployment rate \( u_p \) increases and the private employment rate \( l_p \) decreases. However, due to voluntary private workers' moves, the private-sector unemployment rate \( u_p \) decreases. The net impact will depend

\(^{12}\)See Pissarides (2000) for more details.

\(^{13}\)As the exit rate from public unemployment \( s_g \) is lower in this case compared to the non-distortive case, the mean lag of the public unemployment rate is higher.
on some parameter values. Here, the "attracting effect" dominates the "tax effect", as \( u_p \) reaches a lower new steady-state value (Figure 7). The private employment rate \( l_p \) unambiguously decreases (Figure 7).

Total unemployment \( u \) starts moving according to (3). Assuming a mean wage rate, the new steady-state level of unemployment \( u \) is higher than in the initial steady-state equilibrium (Figure 7). The mean lag for unemployment’s adjustment is close to 2 years.

![Figure 7: Out-of steady-state dynamics of the unemployment and employment rates with distortive taxation](image)

5 Relative weight of labor taxes as propagation channel

In this section, we evaluate the relative impact of distortive labor taxes on the size of the crowding-out effect and on the speed of adjustment of the unemployment rate.

The size of the crowding-out effect

Non-distortive taxation. When the new steady-state equilibrium is reached, the economy faces a loss in the private employment rate \( l_p \) of 2.2 percentage points and a decrease in the unemployment rate \( u \) of 0.1 percentage points. We can measure the size of the crowding-out effect of the public sector on the private sector by dividing the loss in the private employment rate by the exogenous change in the public employment rate.

An increase in the public employment rate unambiguously decreases the private employment rate, due to moves of unemployed workers from the private sector to the public sector (Subsection 3.1). The differentiation of (7) with respect to \( l_g \) gives:

\[
\left( \frac{dl_p}{dl_g} \right)_{\text{nondis}} = - \left( 1 + \frac{q_g}{s_g} \right) \frac{\theta m(\theta)}{\theta m(\theta) + q_p F(R)} < 0 \quad (21)
\]
as $R$, $\theta$ and $s_g$ do not depend on the public employment rate.

The crowding-out effect is complete (Figure 8): creation of one public job destroys 0.962 private jobs and decreases the number of unemployed workers by 0.038. There are relatively more unemployed workers searching for a public job (the number rises by 0.076) and relatively less private-sector unemployed workers (the number falls by 0.114) compared to the initial steady-state equilibrium.

![Figure 8: The size of the crowding-out effect with non-distortive taxation](image)

**Distortive taxation.** We expect that the crowding-out effect of the public sector on the private sector is higher when the tax system introduces distortions. When the new steady-state equilibrium is reached, the economy faces a loss in the private employment rate $l_p$ of 3.4 percentage points and an increase in the unemployment rate $u$ of 1.1 percentage points.

An increase in the public employment rate unambiguously decreases the private employment rate, due to moves of unemployed workers from the private sector to the public sector and to private job destruction increase and job creation decrease (Subsection 3.2). The differentiation of (7) with respect to $l_g$ gives:

$$\left(\frac{dl_p}{dl_g}\right)_{\text{dis}} = \frac{\theta_m(\theta)}{\theta_m(\theta) + q_p F(R)} \left[ - \left( 1 + \frac{q_g}{s_g} \right) - l_p \frac{q_p \theta_m(\theta) F \left( \frac{dR}{dl_g} \right) - \left[ \frac{dF}{dl_g} m(\theta) + \theta \frac{dF}{dl_g} \right] q_p F(R)}{\left[ \theta_m(\theta) \right]^2} \right]$$

$$+ \frac{\theta m(\theta)}{\theta m(\theta) + q_p F(R)} \left[ l_g q_g \left( \frac{ds_g}{dl_g} \right) \right] < 0$$

as $R$, $\theta$ and $s_g$ depend on the public employment rate.

With our calibration, creation of one public job destroys 1.5 private jobs and increases the number of unemployed workers by 0.5 (Figure 9). There are relatively more public-sector unemployed workers (the number rises by 0.59) and less private-sector unemployed workers (the number falls by 0.09)\footnote{Here, we obtain a decrease in the number of private unemployed workers as the "attracting effect" dominates the "tax effect".} compared to the initial steady-state equilibrium.

**Relative weight.** We can show with (21) and (22) that the size of the crowding-out effect $\frac{dl_p}{dl_g}$ is
Figure 9: The size of the crowding-out effect with distortive taxation

higher in the distortive case\textsuperscript{15}, as the negative impact on \( l_p \) of the "attracting effect" is reinforced by the "tax effect". In the same way, as explained above, the "tax effect" reinforces the impact on \( u_g \) of the "attracting effect". Creation of one public job increases the number of unemployed workers searching for a public job by 0.51 compared to the non-distortive case and destroys 0.54 more private jobs. As the "tax effect" has an ambiguous impact on the private unemployment rate, the net effect is not clear. Here, the effect is null.

Moreover, we can show that the size of the crowding-out effect depends on the size of the shock. The differentiation of (21) and (22) with respect to \( l_g \) gives:

\[
\left( \frac{d \frac{d l_p}{d l_g}}{d l_g} \right)_{\text{nondis}} = 0 \quad \text{and} \quad \left| \frac{d \frac{d l_p}{d l_g}}{d l_g} \right|_{\text{dis}} > 0
\]

The size of the shock has no effect on the size of the crowding-out effect when public jobs creation is financed by a non-distortive tax. The reason is that the fall in the private employment rate exactly offsets the "attracting effect", such as the probabilities to move on the labor market remain unchanged. In the distortive case, the offset is more than complete: an higher rise in \( l_g \) increases the crowding-out effect.

For example, consider a 10%, 15% and 20% increases in the public employment rate. Creation of one

\textsuperscript{15}In the distortive case, the differentiation of (7) with respect to \( l_g \) gives:

\[
\left( \frac{d l_p}{d l_g} \right)_{\text{dis}} = \frac{\theta m(\theta)}{\theta m(\theta) + q_p F(R)} \left[ -\left( 1 + \frac{q_g}{s_g} \right) l_p \left( \frac{d \theta m(\theta) F(R)}{d l_g} \right)_{\text{dis}} - \left( \frac{d \theta m(\theta)}{d l_g} + \frac{6 \frac{d m(\theta)}{d l_g}}{d l_g} \right)_{\text{dis}} q_p F(R) \right]
\]

\[
+ \frac{\theta m(\theta)}{\theta m(\theta) + q_p F(R)} \left[ \frac{l_g q_g}{s_g} \left( \frac{ds_g}{d l_g} \right)_{\text{dis}} \right]
\]

Therefore, \( \left| \frac{d l_p}{d l_g} \right|_{\text{dis}} > \left| \frac{d l_p}{d l_g} \right|_{\text{nondis}} \).
The speed of adjustment

To derive the mean lag of the unemployment rate $u$, we solve the two-differential equation system (3) and (2):

\[
\begin{align*}
\dot{u}(t) &= q_p F(R)[1 - l - u(t)] + q_g l_g - \theta m(\theta)[u(t) - u_g(t)] - s_g u(t) \\
\dot{u}_g(t) &= q_g l_g - s_g u_g(t)
\end{align*}
\]

We obtain:

\[
u(t) = u^* + (u_0 - u^*)e^{-[q_p F(R) + \theta m(\theta)]t} + \frac{1}{s_g - q_p F(R) - \theta m(\theta)} [\theta m(\theta) - s_g] \left( u_{g0} - u^*_g \right) \left( e^{-[q_p F(R) + \theta m(\theta)]t} - e^{-(s_g)t} \right)
\]

with $u^*$ and $u^*_g$ the steady-state levels of total and public unemployment, and $u_0$ and $u_{g0}$ the initial levels of total and public unemployment, respectively.

The mean lag $T$ is given by:

\[
u^* - u(T) = (1 - 50\%) (u^* - u_0) \text{ or } u(T) - u_0 = (1 - 50\%) (u^* - u_0)
\]

The speed of adjustment of the unemployment rate depends on the endogenous probabilities to move on the labor market, so it changes with a change in parameters which affect these probabilities. In the non-distortive case, the mean lag of the unemployment rate is the same as the mean lag of the public unemployment rate. It only depends on the exit rate from public unemployment. The reason is that the new steady-state equilibrium is only reached through workers' moves into the public sector. In the distortive case, as the probabilities to move on the private labor market are affected, the mean lag of the unemployment rate is not the same as the mean lag of the public unemployment rate. The new steady-state is reached through workers’ moves into the public sector and workers’ flows on the private labor market. We obtain that an increase in the exit rate from employment (unemployment) increases (decreases) the mean lag, i.e. it reduces (increases) the speed of adjustment of the unemployment rate. Distortive labor taxes increase the mean lag as the exit rate from private employment, $q_p F(R)$, increases and the exit rates from unemployment, $\theta m(\theta)$ and $s_g$, decrease.

Moreover, we can show that the size of the shock $\Delta l_g$ affects the mean lag. In the non-distortive case, the probabilities to move on the labor market are unchanged. Therefore, the mean lag is the same whatever the increase in the public employment rate. In the distortive case, public jobs creation increases
Table 2: High-skilled versus low-skilled public jobs creation

<table>
<thead>
<tr>
<th></th>
<th>non distortive taxation</th>
<th>distortive taxation</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean wage gap (w_g/w_p = 1.06)</td>
<td>(-0.038) (-0.962)</td>
<td>(+0.5) (-1.5)</td>
</tr>
<tr>
<td>high-skilled public job creation (w_g/w_p = 0.95)</td>
<td>(-0.118) (-0.882)</td>
<td>(+0.28) (-1.28)</td>
</tr>
<tr>
<td>low-skilled public job creation (w_g/w_p = 1.18)</td>
<td>(+0.048) (-1.048)</td>
<td>(+0.75) (-1.75)</td>
</tr>
</tbody>
</table>

the exit rate from private employment and decreases the exit rates from unemployment. Therefore, the mean lag of the unemployment rate is as higher as the shock is high. Consider a 10%, 15% and 20% increases in the public employment rate. The respective mean lags are close to 2 years, 3 years and 4 years.

6 High-skilled versus low-skilled public jobs creation

Empirical data on private-sector and public-sector net wages in France reveal that the low-skilled workers are better off in the public sector (the mean wage gap is 1.18) whereas the high-skilled workers obtain higher wages in the private sector (0.95) (Enquêtes Emploi, 2000). In our paper, we consider all workers with a mean wage gap near 1. We expect that the initial public-sector unemployment rate is as higher as public wages are higher and exit rate from public employment is lower. Indeed, higher public wages and less "risky" jobs will attract more workers into wait unemployment in the public sector, ceteris paribus.

We obtain, in section 5, that public jobs creation leads to a complete crowding-out of private jobs. We expect that it leads to more than complete crowding-out when there is a public-sector wage premium, as Holmlund (1997)\(^{16}\). In the opposite, the creation of high-skilled public jobs should lead to less than complete crowding-out. We report in Table 2 the effect of creation of one public job. We obtain that the crowding-out effect is higher (lower) than 1 when the wage ratio \(w_g/w_p\) is higher (lower) than 1, whenever \(q_g < q_p F(R)\).

We can show that the size of the crowding-out effect increases with the ratio \(w_g/w_p\). As an increase in the wage premium attracts more workers into the public sector, the impact of an increase in the public employment rate on the private labor market tightness, as well as on the expected cost of hiring a worker, is higher. More private employees \(l_p\) become unemployed workers. Making use of the arbitrage condition (10) and the wage rate (9), the differentiation of (21) with respect to the wage differential \(w_g - w_p(x)\)

\(^{16}\) Under some assumptions on labor mobility, he shows that intensified recruitments in the public sector lead to more than complete crowding-out when there is a public-sector wage premium, the reason being that such a wage differential reinforces the increase in private wage pressure.
under constraint \((1 + \tau_F)((1 - \beta)w_g - w_p(x)) + \beta px > 0\), i.e. \(s_g > 0\). An increase in the wage ratio increases the crowding-out effect. Therefore, in the case of a wage gap higher than 1, \((w_g/w_p(x)) > 1\), the higher is the public wage premium, the higher is the crowding-out effect, so the higher is the increase in the unemployment rate. In the case of a wage ratio lower than 1, \(0 < (w_g/w_p(x)) < 1\), the lower is the wage differential, the lower is the crowding-out effect, so the higher is the decrease in the unemployment rate.

7 Empirical analysis

The aim of this section is to test formally for the existence of a long-term relationship between public employment and private job creation and destruction flows. We also attempt to measure the size of the crowding-out effect in the short and long terms and the adjustment time of the unemployment rate. We use a structural vector auto-regression (SVAR) technique on French data to obtain the impulse response functions of unemployment \(U\), private job creation \(C\) and destruction \(D\), and private wage rates \(W_p\) to an innovation in public employment rate \(L_g\).

The data were obtained from the OECD Employment Outlooks database. The public employment rate is the ratio of general government employment to the labor force. According to the OECD definition, public employment consists of jobs in central and local administrations, in non-profit organizations controlled or financed by public administrations and in military and diplomatic entities. The definition does not include public firms owned or controlled by the government. The unemployment rate is the ratio of unemployment to the total labor force (Figure 10). The private wages are the wage rates in the business sector. Job creation and destruction rates are obtained from the "Déclaration des mouvements de main d’oeuvre" database (DMMO)\(^{18}\). They measure workers’ inflows and outflows for the French establishments of 50 employees and more in the manufacturing sector\(^{19}\). Sample runs from 1986:1 to 1997:

\[
\left| \frac{dL_p}{dL_g} \right| \cdot \frac{d(w_g - w_p)}{dL_g} > 0
\]

(23)

\[
\left( \frac{dL_p}{dL_g} / dL_g \right) = \left( \frac{dL_p}{ds_g} \cdot \frac{ds_g}{d(w_g - w_p)} \right)
= \left( \frac{aq}{s_g} \frac{\theta_m(\theta)}{\theta_m(\theta) + \theta_p F(R)} \right) \left( \frac{(1 + \tau_F)\beta pc(\theta + q_g)}{(1 + \tau_F)(1 - \beta)w_g - w_p(x) + \beta px} \right) < 0
\]

\(^{17}\)The DMMO database is jointly managed by INSEE and the French Ministry of Labor.

\(^{18}\)The DMMO database is jointly managed by INSEE and the French Ministry of Labor.

\(^{19}\)As the manufacturing sector not produces substitutable goods for public ones, we expect that the effect of public employment on private job flows only goes through wages and taxes.
Figure 10: Public employment and unemployment rates in France (1986-1997). Source: OECD

Figure 11: Private job destruction and creation rates in France (1986-1997). Source: DMMO

Data are available at a quarterly frequency and are seasonally adjusted. ADF (and Phillips-Perron) unit root and KPSS stationarity tests are performed. In the case of the ADF test, the null hypothesis describes a non-stationary process (if t-statistic < critical value, the null can be rejected). It corresponds to the assumption of stationarity under the KPSS test (if t-statistic > critical value, the null can be rejected). They indicate that public employment series is stationary in first difference (white-noise process with 1 lag) and unemployment series is stationary in first difference (1 lag) (Tables 3 and 4).
The preliminary ADF and KPSS tests conclude that wages series is integrated of degree 2. However, Perron (1989) shows that unit root tests are biased toward accepting the null hypothesis of a unit root in the presence of a structural break. We use Perron’s methodology to test for a unit root against the alternative hypothesis of a stationary series with a one-time change in the mean. The Dickey-Fuller tests on the wage series in first difference indicate that the null of a unit root can’t be rejected. We use the Zivot-Andrews test to detect the date of the structural break (Table 5).

Then we use the Perron methodology to filter the wage series in first difference and perform a Dickey-Fuller test of resids of the filtered wage series. The value of the t-statistic is higher than the critical value (-3.96): the filtered series does not contain a unit root. Wages series is stationary in first difference with a constant, a trend and a structural break (2 lags).

First, we propose to estimate the following VAR system:

\[ Y_t = (L_g, U, W_p)_t \]

where \( L_g \) is the public employment rate, \( U \) the unemployment rate and \( W_p \) the wage rate in the private sector. If we obtain that an innovation in public employment has no significant impact on private wages, but a significant one on unemployment, it could suggest the relevance of labor taxes as propagation channel. This system contains a trend, a constant and a wage control dummy. The optimal VAR lag-
Series Lags | ADF α5% = −1.95  | ADF μ5% = −2.93  | ADF τ5% = −3.50
---|---|---|---
$C_t$ | 3 | I(1) | I(0) | I(1)
$D_t$ | 0 | I(1) | I(1) | I(1)

Table 6: Augmented Dikey-Fuller unit root tests
Notes: The first column denotes unit root test statistics based on a regression without constant, the second with a constant, the third with a constant and a trend.

Series Lags | KPSS μ5% = 0.46 | KPSS τ5% = 0.14
---|---|---
$C_t$ | 3 | I(0) | I(0)
$D_t$ | 0 | 1.50 | 0.29 | I(1)

Table 7: Kwiatowski, Phillips, Schmidt and Shin stationarity tests
Notes: The first statistic tests stationarity around a level, the second tests trend stationarity.

length is derived from the AIC and BIC criteria leading to a choice of three lags. However, the Johansen cointegration test confirms that there exists one cointegrating relation (between public employment and unemployment). We have to impose restrictions on the cointegrating vector and/or on the adjustment coefficients. This second type of restrictions would permit to test whether one of the endogenous variables is weakly exogenous with respect to the cointegrating vector. Then we will be able to estimate this VEC model.

Then we propose to estimate the following VAR system:

$$ y_t = (Lg, C, D, W_p)_t $$

where $C$ is the job creation rate and $D$ the job destruction rate in the private sector.

Job flows can be considered as stationary, although displaying an important persistence (Tables 6 and 7). ADF unit root and KPSS stationarity tests indicate that private job creation rate series is stationary in level with a constant (3 lags) and private job destruction rate series is stationary in first difference (0 lag).

Our aim is to quantify the share of private jobs destruction and private wage fluctuations which can be attributed to public employment fluctuations in a dynamic perspective. If we obtain that an innovation in public employment has no significant impact on the private wage, a significant impact on private job creations and destructions would suggest the relevance of labor taxes as propagation channel. In this regard, it would be very informative to report the impulse response function of private job creation and destruction rates to a shock in private wage rates.
To be completed: first, we have to report the impulse response functions (IRF) of public employment, unemployment and wages to an innovation in each shock equivalent to a 1% point rise. Second, we will have to analyze the contribution of each structural shock to the variance of the k-quarter ahead forecast error for each endogenous variable. Third, we propose to decompose the historical evolution of unemployment into its three structural components. Eventually, it could be informative to repeat the exercise on US data (Davis and Haltiwanger, 1992).

8 Conclusion

This paper evaluates the relative weight of distortive labor taxes as propagation channel of the effects of public employment on unemployment dynamics. We have used a dynamic matching model which permits to show how public jobs creation affects the labor market structure through endogenous workers’ decisions to move on the labor market and private-sector job creation and destruction decisions. We study the effects of an exogenous public jobs creation on the unemployment rate, with public wage rate fixed. In a first part, we consider that newly created public jobs are financed by proportional wage taxes paid by all workers. An increase in the public employment rate increases the public labor market tightness. So, the expected utility of an unemployed worker searching for a public job increases. It induces a move of unemployed workers from the private sector to the public sector such as the exit rate from public unemployment remains constant. As the private unemployment rate decreases, the private labor market tightness tends to increase. So the probability that a match occurs decreases, making the expected cost of hiring a worker higher. Firms open less vacancies such as the private labor market remains constant. When the new steady-state equilibrium is reached, the number of public-sector workers has increased whereas the number of private-sector workers has decreased. The net effect on the unemployment rate depends on the wage differential and on the relative exit rates from employment. Assuming a mean wage gap, there is less unemployed workers. The impact of the public sector on the labor market works through an "attracting effect". In a second part, we consider that newly created public jobs are financed by distortive labor taxes. The distortive labor taxes increase rises the private-sector net cost of labor, decreasing job creation and increasing job destruction. It decreases the private wage rate, as well as the expected utility of a private unemployed worker. This induces additional workers flows to the public sector. When, the new steady-state equilibrium is reached, the number of public-sector workers has increased whereas the number of private employed workers has decreased. The impact on the number of private unemployed workers is ambiguous. Assuming a mean wage gap, there is more unemployed
workers. The impact of the public sector on the labor market works through an "attracting effect" and a "tax effect".

We obtain that the size of the crowding-out effect is higher in the distortive case, as the "tax effect" reinforces the decrease in the private employment rate. Creation of one public job destroys 1 private job in the non-distortive case and 1.5 private jobs in the distortive case. Therefore, the destruction of 0.5 private job can be attributed to distortive taxes. The crowding-out increases with the size of the shock in the distortive case, as it induces more destructions. Dynamic analysis reveals that a distortive tax decreases the speed of adjustment of the unemployment rate from its initial steady-state value to its new steady-state value, through changes in the endogenous probabilities to move on the labor market. We expect that a low-skilled public jobs creation leads to more than complete crowding-out of the private employment rate whereas a high-skilled jobs creation leads to less than complete crowding-out. The crowding-out effect increases with the public wage premium, as the attracting effect of the public sector increases with the relative level of the public wage.

9 Appendix

9.1 Phase diagram

In Section 4, the behavior of the unemployment rate \( u(t) \) has been studied using analytical and numerical methods. We propose here to use a graphical method in order to illustrate the stability of our solution.

The two differential equations (3) and (2) of our model read:

\[
\begin{align*}
\dot{u}(t) &= q_p F(R)(1 - l_g - u_g(t)) + q_g l_g - \theta_m(\theta)(u(t) - u_g(t)) - s_g u_g(t) \\
\dot{u}_g(t) &= q_g l_g - s_g u_g(t)
\end{align*}
\]

And can be rewritten:

\[
\begin{align*}
\dot{u}(t) &= -\theta_m(\theta) u(t) + (\theta_m(\theta) - q_p F(R) - s_g) u_g(t) + q_p F(R)(1 - l_g) + q_g l_g \\
\dot{u}_g(t) &= -s_g u_g(t) + q_g l_g
\end{align*}
\]

The diagonal matrix of eigenvalues \( D \) is given by

\[
D = \begin{pmatrix}
\frac{-2(\theta_m(\theta) + s_g) + \sqrt{2\theta_m(\theta)s_g}}{2} & 0 \\
0 & \frac{-\sqrt{2\theta_m(\theta)s_g}}{2}
\end{pmatrix}
\]

The negative sign of both eigenvalues confirm that the system is stable (Figure 12).

First case

The locus of points for which \( \dot{u} \) equals 0 is the upward-sloping line \( u_g = \frac{\theta_m(\theta) u - q_p F(R)(1 - l_g) - q_g l_g}{\theta_m(\theta) - q_p F(R) - s_g} \), under the constraint \( \theta_m(\theta) - q_p F(R) - s_g > 0 \). The \( \dot{u} = 0 \) crosses the horizontal axis at point
\[ \dot{u} = \frac{q_p F(R)(1 - l_g) + q_g l_g}{\theta m(\theta)} \]  

If we start at a point on the \( \dot{u} = 0 \) schedule and increase \( u \) a bit, then the right-hand side of the expression for \( \dot{u} \) decreases. Hence, \( \dot{u} \) becomes negative and \( u \) is decreasing in that region. The arrows in this region point south. A symmetric argument implies that the arrows point north for points to the right of the \( \dot{u} = 0 \) schedule (Figure 12).

The \( \dot{u}_g(t) = 0 \) locus is given by \( u^*_g = \frac{q_g l_g}{s_g} \), a vertical line. The expression for \( \dot{u}_g \) implies that if \( u_g \) rises, then \( \dot{u}_g \) decreases. Hence, to the right of the \( \dot{u}_g = 0 \) locus, \( \dot{u}_g \) is negative, and the arrows point west. The arrows point east for points to the left of the \( \dot{u}_g = 0 \) schedule (Figure 12).

The steady state is the point at which the two loci cross, a condition that corresponds in this case to

\[
\begin{align*}
u^* &= u^*_g + \frac{q_p F(R)(1 - l_g - u^*_g)}{\theta m(\theta)} \\
\theta m(\theta) - q_p F(R) - s_g < 0 \quad (20) \\
\end{align*}
\]

The \( \dot{u}_g(t) = 0 \) locus is given by \( u^*_g = \frac{q_g l_g}{s_g} \), a vertical line. In the case \( u^*_g < \bar{u}_g \), the system is stable as for

Second case

The locus of points for which \( \dot{u} \) equals 0 is the downward-sloping line \( u_g = \frac{\theta m(\theta) u - q_p F(R)(1 - l_g) - q_g l_g}{\theta m(\theta) - q_p F(R) - s_g} \), under the constraint \( \theta m(\theta) - q_p F(R) - s_g < 0 \). The \( \dot{u} = 0 \) schedule crosses the horizontal axis at point \( \bar{u} = \frac{q_p F(R)(1 - l_g) + q_g l_g}{\theta m(\theta)} \) 21 and the vertical axis at point \( \bar{u}_g = \frac{q_p F(R)(1 - l_g) + q_g l_g}{\theta m(\theta) - q_p F(R) - s_g} \). As previously, the \( \dot{u}_g(t) = 0 \) locus is given by \( u^*_g = \frac{q_g l_g}{s_g} \), a vertical line. In the case \( u^*_g < \bar{u}_g \), the system is stable as for

---

20 The steady-state value \( u^* \) is higher than \( \bar{u} \) under the constraint \( \theta m(\theta) - q_p F(R) - s_g > 0 \).

21 The steady-state value \( u^* \) is lower than \( \bar{u} \) under the constraint \( \theta m(\theta) - q_p F(R) - s_g < 0 \).

---

Figure 12: Phase diagram

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any initial values of \( u \) and \( u_g \), the dynamics of the system takes it back to the steady state (Figure 13). In the case \( u_g^* > \bar{u}_g \), the two loci do not cross. There is no steady state.

\[
\dot{u}_g = 0
\]

\[ \begin{array}{c|c}
\text{u} & \text{u}^* \\
\mid & \mid \\
\text{u} & \text{u}_g \\
\hline
\end{array} \]

\[ \begin{array}{c|c}
\text{u}\_g & \text{u} \\
\mid & \mid \\
\bar{u}_g & \text{u}^* \\
\hline
\end{array} \]

\[ \dot{u} = 0 \]

Figure 13: The phase diagram in the second stable case

### 9.2 Out-of steady-state dynamics

We assume that the government can’t rent into debt. The budget constraint (11) has to be fulfilled at any moment in time. As in Subsection (2.7), the out-of steady-state dynamic of unemployment is given by equation (3). The unemployment rate is a predetermined variable. Therefore, the tax rates \( \tau_F \) and \( \tau_T \) are not jump variables.

We allow wages to be renegotiated continually such as the sharing rule holds in rates of change \( \dot{S}(x) = \dot{W}_p(x) - \dot{W}_u(x) + \dot{J}^p(x) - \dot{V}^p \). Therefore, the wage equation (9) holds in and out of the steady-state. Out-of steady-state, the expected capital gains from changes in jobs value and in utilities are not null. The exit rate from public-sector unemployment is given by the following arbitrage condition:

\[
s_g\frac{(w_g - z)(1 - \tau_T)}{r + q_g + s_g} + s_g\frac{\dot{W}_g^e - \dot{W}_g^u}{r + q_g + s_g} + \dot{W}_g^u = \frac{\beta(1 - \tau_T)}{(1 - \beta)(1 + \tau_F)}pc\theta + \dot{W}_p^u
\]

We assume that firms can destroy unprofitable jobs without delay. This assumption implies that the zero-profit condition satisfied by \( R \) holds in and out of steady state \( J^p(R) = \dot{J}^p(R) = 0 \). The condition for private-sector job destruction is given by:

\[
R - (1 + \tau_F)\frac{z}{p} - \frac{\beta}{1 - \beta}pc\theta + \frac{q_p}{r + q_p} \int_{s}^{c} \left( s - R + \frac{\dot{J}^p(s)}{(1 - \beta)p} \right) dF(s) = 0
\]
We also assume that firms can open and close vacancies without delay. This assumption implies that the zero-profit condition for new vacancies holds in and out of steady state $V^p = \dot{V}^p = 0$. The job creation condition reads:

$$(1 - \beta) \frac{cu - R}{r + q_p} - \frac{cm'(\theta)}{(r + q_p)m^2(\theta)} \dot{\theta} = \frac{c}{m(\theta)}$$

(26)

The out-of steady-state dynamic is obtained from the simultaneous solution of the equations (9), (11), (24), (25), (26), (2) and (3). The out-of steady-state dynamics of the private-sector unemployment and employment rates are deduced from (2) and (3), given that the public-sector employment rate is assumed constant.

Let us describe the effects of an increase in public employment. As in Subsections (4.1) and (4.2), newly created public jobs are filled by the public-sector unemployed workers. Therefore, the public and total unemployment rates $u_g$ and $u$ instantaneously fall. But now, the budget constraint does not allow jumps in tax rates $\tau_F$ and $\tau_T$. Therefore, at the time of the shock, $R, \theta, w_p(x)$ and $s_g$ do not instantaneously jump on their new steady-state values. The "tax effect" does not induce a jump in the private-sector unemployment and employment rates. All variables are driven by a backward-looking process.

We simulate the effect of a 10% public employment increase on the out-of steady-state dynamics of $u$ in the non-distortive case. We use the same calibration as previously. Fluctuations are low and all variables reach the same new steady-state level as in the case with perfect capital market. The unemployment rate highly increases in a first step and then fluctuates around its new steady-state value (Figure 14).
References


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