Sticky prices, fair wage, and the co-movements of unemployment and labor productivity growth

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Abstract

In this paper, we study the co-movements of unemployment and labor productivity growth for the United States economy. Measures of co-movements in the frequency domain indicate that co-movements between these variables differ strongly according to the frequency. First, they are concentrated in the long-run and business cycle frequencies rather than in the short-run frequencies. Second, they are negative in the short- and the long-run and positive along the business cycle. A New Keynesian model that combines a nominal rigidity on the goods market (namely sticky prices) and a real rigidity on the labor market (namely fair wage) is shown to be quantitatively consistent with observed co-movements in the long-run and the business cycle. However, the model fails to explain the short-run co-movements.

Keywords: sticky prices, fair wage, co-movement, spectral analysis

JEL Classification: C32, E31, E32, J41

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1 Introduction

The relationship between employment and labor productivity has been debated for a long time and has recently became even more puzzling\(^1\). It is now widely accepted that labor market fluctuations are driven by technological shocks, at least partially\(^2\), but there is no consensus regarding their effects. On the one hand, the studies on permanent technological shocks of Basu et al. (2004) and Gali (1999) have highlighted their negative effects on the use of labor. This result has been recently criticized by Christiano et al. (2003, 2004). On the other hand, the studies on the Phillips Curve dynamic in the United States of Ball and Moffitt (2001) and Staiger et al. (2001) explain the exceptionally low values of the unemployment rate during the 1990s by the equally exceptional productivity gains during this period. These results suggest two contradictory views on the relationship between unemployment and the growth of labor productivity. In this paper, we argue that these two views are not contradictory, but are rather complementary to understanding the overall relationship between these variables. To reconcile these views, we distinguish the short-run co-movements of the long-run co-movements.

To make the distinction between the short- and the long-run, we study the co-movements in the frequency domain by means of spectral analysis. Spectral analysis has become very popular in macroeconomics for describing the dynamic properties of time series as well as the co-movements between two series (see Watson, 1993, Diebold et al., 1998, Wen, 1998, and Croux et al., 2001). Spectral analysis has also provided basis of filters of time series (such as the high-pass filter of Hodrick and Prescott, 1997, or the band-pass filters of Baxter and King, 1999, and Christiano and Fitzgerald, 2003). For our purposes, the usefulness of spectral analysis is twofold. First, it gives us an overall view of the co-movements of unemployment and labor productivity growth, which can be appraised according to different

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\(^1\)See the important debates on the question of embodied vs. disembodied technical progress in matching models of unemployment initiated by Pissarides (2000), Aghion and Howitt (1994), and Mortensen and Pissarides (1998).

\(^2\)Even if the initial proposition of real-business-cycle theory (namely that technological shocks can explain almost all the macroeconomic fluctuations) was moderated, their contribution is widely recognized.
frequencies: from the short to the long run. Second, one can evaluate models on their ability to reproduce empirical co-movements at different frequencies.

We begin by describing the empirical co-movements of unemployment and labor productivity growth for the U.S. economy. We take the co-spectrum as the main measure of co-movements. This measure indicates that these two variables are closely related and that the sign of their co-movements strongly differ according to the frequency: they co-move negatively in the short- and long-run and positively along the business cycle. In addition, co-movements are concentrated rather in the long-run and business cycle frequencies than in the short-run frequencies. The application of alternative measures of co-movements confirm these results: co-movements are strong and there is a shift of their sign with the periodicity. We propose a New Keynesian explanation of these facts, which is based on the interaction of nominal and real rigidities.

We assume that nominal goods prices move sluggishly because firms face a quadratic cost of price adjustment, as originally suggested by Rotemberg (1982)\(^3\). As emphasized by Basu et al. (2004) and Gali (1999), the nominal rigidities constitute the most relevant mechanism to explain the short-run negative effects of technological improvements on the use of labor. The real rigidity on the labor market comes from the fair wage hypothesis as originally suggested by Arkerlof (1982) and recently extended by Collard and de la Croix (2000) and de la Croix et al. (2000). The extended fair wage model leads to persistence in the workers’ wage aspiration that positively links a current increase in productivity with future employment increases. These two rigidities were recently combined by Ball and Moffitt (2001) (who provide an estimation of Phillips Curve with fair wage) and Danthine and Kurmann (2004) (who give results for the business cycle). We leave aside physical capital accumulation and model labor productivity as purely exogenous phenomena.

The model is estimated to reproduce the spectra of labor productivity growth and unemployment. We evaluate next its empirical relevance on the basis of its ability to replicate the empirical co-spectrum between these variables. When a positive productivity shock hits

\(^3\)This assumption has been incorporated into dynamic, stochastic, and general equilibrium models, notably by Hairault and Portier (1993), Rotemberg (1996) and Ireland (2000).
the economy, the model predicts an increase in unemployment (due to the nominal rigidity) followed by a decrease in unemployment (due to the real rigidity). This theoretical timing of events appears consistent with the empirical relationship between labor productivity and unemployment for the long-run and the business cycle, but not for the short-run. This conclusion is confirmed by the study of two sub-models. A model with fair wage and flexible prices can only account for the long-run co-movements whereas a model with sticky prices and indivisible labor supply can only account for the business cycle co-movements. However, any versions of this model is able to explain the short-run co-movement.

The remainder of the paper is organized as follows. The empirical facts are described in Section 2. The model is exposed in Section 3. The results are presented in Section 4. Section 5 concludes.

2 Empirical facts

Our bivariate data set comes from the Bureau of Labor Statistics and covers the period 1948:1-2000:4 at a quarterly periodicity for the United States. The series\(^4\) are the first difference of the logged labor productivity (we remove its mean value) and the logdeviation of the unemployment rate from its empirical mean\(^5\). They are denoted \(g\) and \(u\), respectively.

2.1 Spectra and co-spectrum

We consider\(^6\) the real, stochastic, zero-mean, and stationary process \([g, u]\). Let \(S(\Omega) = \int_{\Omega} S(\omega) d\omega\) be the spectral density matrix of the process at frequency band \(\Omega = [\underline{\omega}, \overline{\omega}]\), where \(\underline{\omega} \in [-\pi, \pi]\) is the lower-bound of the frequency range of \(\Omega\) and \(\overline{\omega} \in [-\pi, \pi]\) the upper-bound, with \(\underline{\omega} < \overline{\omega}\). The spectra of \(g\) and \(u\) are denoted \(S_g(\Omega)\) and \(S_u(\Omega)\), respectively, and the cross spectrum from \(u\) to \(g\) is denoted \(S_{gu}(\Omega)\). Frequencies can be interpreted in

\(^4\)BLS series ID are LFS21000000 and PRS85006092.
\(^5\)We take the logarithm of unemployment (and not its level as is usual) to facilitate comparison with the model’s predictions, where the logdeviation (and not deviation) from steady state of the variables are simulated. This has no consequences on our results.
\(^6\)This section is mainly based on Hamilton (1994).
terms of cycle duration in the time domain. The frequency $\omega$ corresponds to the cycles of period $2\pi/\omega$ (the length of time required for the process to repeat a full cycle).

The spectra give us information about the portion of the variance of $[g, u]$ attributable to cycles of frequencies $\Omega$. To obtain information about their covariance, we have to study the cross spectrum’s properties. Generally, the cross spectrum is not real and is thus difficult to interpret. It is therefore more convenient to study the real part of the cross spectrum which is known as the co-spectrum of $g$ and $u$

$$C_{gu} (\Omega) = \text{real} (S_{gu} (\Omega))$$

The co-spectrum can be interpreted as the portion of the covariance between $u$ and $g$ that is attributable to cycles of frequencies $\Omega$.

To compute the spectra of $g$ and $u$ and the cross spectrum from $u$ to $g$, we have to estimate the spectral density matrix of $[g, u]$. To this end, we estimate a bivariate VAR$^7$ and deduce the associated spectral density matrix (see Hamilton, 1994). To interpret the results, we retain a common decomposition of frequencies. The frequency band $\Omega = [0, \pi/16]$ is defined as the long-run movements (which correspond to cycles with a longer period than 8 years), the frequency band $\Omega = [\pi/16, \pi/3]$ is defined as the business cycle movements (which corresponds to cycles with a period contained between 1.5 and 8 years), and the last frequency band $\Omega = [\pi/3, \pi]$ is defined as the short-run movements (which corresponds to cycles with a period less than 1.5 years). To interpret the results, we compute 90% confidence intervals$^8$ for the spectra and the co-spectrum.

Figure 1 depicts the spectra of $g$ and $u$. The spectrum of labor productivity growth has three peaks: two at the extremities of the frequency band (for $\omega = 0$ and $\omega = \pi$) and one within the business cycle frequencies. The spectrum of unemployment$^9$ is concentrated

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$^7$The lag in the VAR is chosen by minimization of the Schwartz criterion and is equal to 6.

$^8$These were computed using the bootstrap Monte Carlo procedure described in Edelberg et al. (1999) with 5000 replications.

$^9$This figure can not be easily compared with other studies (e.g. Watson, 1993, for the spectrum of total hours), because we calculate the spectrum of the level of unemployment and not of its first difference. This choice is motivated by our interest for the co-spectrum between the rates of unemployment (in level and not in first difference) and of labor productivity growth.
within the long-run frequencies and takes its greatest value for $\omega = 0$.

Figure 2 depicts the co-spectrum of $g$ and $u$. The co-spectrum takes its smallest value with the zero frequency and reaches a peak in the business cycle frequencies. Between these two frequencies, the co-spectrum is increasing and crosses the zero line around the frequency $\pi/24$ (which corresponds to cycles of 12 years). Its values are always positive for the business cycle co-movements. The co-spectrum is smaller (in absolute value) for the short-run frequencies, but becomes negative and significantly different from zero for the highest frequencies. Two facts emerge from this figure. First, the rates of unemployment and labor productivity growth seem more connected in the long-run and the business cycle than in the short-run. Second, the relation between these variable strongly differs according to the frequency considered: co-movements are positive along the business cycle, whereas they are negative in the short- and long-run frequencies. In addition, these co-movements are significantly different from zero for the main part of the business cycle frequencies and the lowest and the highest frequencies (see the 90% confidence intervals). We now investigate the robustness of this finding to the choice of the measure of co-movements.

2.2 Alternative measures of co-movement

In this section, we consider alternative measures of co-movement and begin with correlation measures in the frequency domain. To measure bivariate co-movements, Croux et al. (2001) define the dynamic correlation which combines the spectra and the co-spectrum according to $D_{gu}(\Omega) = C_{gu}(\Omega) / \sqrt{S_g(\Omega) \cdot S_u(\Omega)}$. As shown by the authors this measure of co-movements can be interpreted as the correlation coefficient between $u$ and $g$ that is attributable to cycles of frequency $\Omega$. Table 1 reports the dynamic correlation coefficient over the two frequency bands of interest. It is strongly negative for the long-run co-movements, strongly positive for the business cycle co-movements, and negative (and not very strong)
for the short-run co-movements\textsuperscript{11}. To obtain these values we use the same spectral density matrices as above (see Figures 1 and 2). To assess the robustness of our results we also compute the dynamic correlation coefficient for a larger VAR inspired from Gali (1999) (two variables are added: the real interest rate and the inflation rate). Both VAR lead to very close values of dynamic correlation for each of the two frequency bands (see Table 1).

To complete our set of measures of co-movements, we abandon the estimation of VAR and directly compute the correlation coefficient of filtered series. To preserve our temporal decomposition of co-movements, we use band pass filters. Table 1 reports the results obtained with the filters proposed by Baxter and King (1999) and Christiano and Fitzgerald (2003). In both cases, we use the parametrization of the filters suggested by the authors so as to extract the short-run movements (less than 1.5 years), the business cycle movements (between 1.5 and 8 years) and the long-run movements (above 8 years). Both filters lead to similar values, which again confirms our two facts. First, the correlation between unemployment and labor productivity growth is stronger in the long-run and business cycle frequencies than in the short-run frequencies. Second, this correlation is positive for the business cycle and negative otherwise.

To end this section, we propose to use the den Haan’s (2000) measure of co-movement based on the VAR forecast errors of variables. The method consists in estimating the empirical VAR\textsuperscript{12} and then looking at the co-movement between forecast errors of variables at different horizons. The co-movement can be depicted with two measures. The first is the correlation coefficient of the forecast errors which, for stationary variables, converges to the unconditional correlation coefficient of the two series as the forecast horizon goes to infinity. The second is the covariance of the updates of the forecast errors which has a very attractive property: it can be directly compared with the product of the impulse response functions to the shocks. We use the previously described bivariate VAR to compute the correlation of forecast errors and covariance of the updates of forecast errors. Results are

\textsuperscript{11}In Tripier (2002), we show that this pattern is robust to the choice of the estimation procedure of the spectral density matrix.

\textsuperscript{12}Den Haan (2000) underlines the great flexibility of its method at this step. In particular, it does not require assumptions about the order of integration of series used. See also Summers and den Haan (2004).
reported in Figure 3 for forecast horizons range from 1 quarter to eight years with the 90% confidence intervals. They confirm the results previously obtained in the frequency domain for the short-run and the business cycle. The two first forecast errors are negatively related, whereas the next are positively related. Co-movements are significantly from the zero for some forecast horizons contrary to the unconditional correlation coefficient. As shown in Figure 4, covariance of the updates of forecast errors are concentrated in the first forecast horizons (more precisely, in the first six) and the correlation of forecast errors converges quickly toward its final value. Hence, this measure does not describe the shift in the sign of co-movement between the business cycle and the long run observed with measure in the frequency domain. Even if we consider forecast horizons higher than eight years, there is no changes in the co-movement as the Figure 3 suggests (indeed, for forecast horizons about six years the correlation is already flat and covariance close to zero).

2.3 Discussion

Our results for the business cycle are close to the puzzling empirical fact known as the Dunlop-Tarshis observation. Contrary to the prediction of standard real business cycle models, the empirical correlation between the cyclical components of labor productivity and employment is not strongly positive, but rather close to zero or even negative according to the data set used (see Christiano and Eichenbaum, 1992, and Hansen and Wright, 1992). This observation was recently strengthened by Basu et al. (2004) and Gali (1999) who obtain an even more negative correlation when only technological shocks are taken into account. However, these studies do not consider the unemployment rate to describe the labor market dynamics, but rather total hours worked. Hence, it is an important issue to assess the robustness of our empirical results to the choice of the measure of labor market activity. We then propose to perform the same analysis of co-movements with the total
hours worked\textsuperscript{13} variable instead of unemployment. The results\textsuperscript{14} are reported in Figures 4 and 5 and in Table 2. The empirical pattern with total hours worked is really very close to the pattern previously described with unemployment. First, co-movements are still stronger in the long-run and business cycle frequencies than in the short-run frequencies (both for the co-spectrum and the correlation coefficient). Second, co-movements are positive in the short- and long-run frequencies and negative along the business cycle.

Concerning the long term, it is more difficult to link our results to the literature due to the lack of studies on this dimension. A notable exception is Staiger et al. (2001) who confront the long-run trends of growth and of unemployment and conclude that there is negative correlation, qualified as "striking and intriguing" whilst remaining cautious on the implications of this result (See also Caballero, 1993, for similar analyses). A second exception is Pissarides and Vallanti (2003) who also conclude that faster productivity growth increases employment\textsuperscript{15} using an annual panel data for United-States, Japan, and Europe. Interestingly, the positive impact of faster growth comes after a negative impact. These results are similar to ours.

To sum up, the use of spectral analysis provides us with an overall view of the co-movements of unemployment and labor productivity growth. Our empirical analysis highlights and brings together several crucial facts which were already observed, but in either short or long term analyses and not in an integrated framework as presented here. We now turn toward the theory to attempt to explain these empirical facts.

\textsuperscript{13}The data is the hours of all persons in business sector (from BLS) divided by the civilian noninstitutional population for the same period (1948:1-2000:4).
\textsuperscript{14}We do not report the spectra of these variables which are similar to those reported in Figure 1. In particular, there is a concentration of the variance of total hours worked in the low frequencies.
\textsuperscript{15}They interpret these results as evidences in favor of disembodied technical progress in new job rather than embodied.
3 The model

The economy is inhabited by infinitely lived households which supply an effort at work according to the fair wage principle, which consume, accumulate money, and receive firms profits. There is an intermediate good sector in which monopolistically competitive firms operate using labor as sole input and face a quadratic cost of price adjustment. Production technology is stochastic and the productivity of labor follows a random walk process with drift.

3.1 Households

The representative household seeks to maximize the expected discounted utility function with respect to consumption, $c_t$, real balances, $m_t$, and work effort, $s_t$:

$$\max_{c_t, m_t, s_t} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^{t+k} [u(c_{t+k}, m_{t+k}) - v(s_{t+k})]$$

where $\mathbb{E}$ is the expectation operator and $\beta$ is the subjective discount factor, with $0 < \beta < 1$. The per-period stream of utility is the sum of two functions. In the first function $u(c_t, m_t)$, $c_t$ denotes the household’s consumption and $m_t = M_t/P_t$ denotes the household’s real balances, where $M_t$ are the nominal balances and $P_t$ the final good price. We take a standard specification of this function

$$u(c_t, m_t) = \log(c_t) + \gamma \log(m_t)$$

The second function determines the well known ”effort function” of efficiency wage models. We specify it as in Collard and de la Croix (2000):

$$v(s_t) = q_t \left[ s_t - \delta_c - \delta_a \log \left( \frac{w_t}{w_t^a} \right) - \delta_s \log \left( \frac{w_t}{w_t^s} \right) \right]^2$$

where $q_t$ is a dummy variable which is equal to 1 when the worker is employed and 0 otherwise. When the worker is employed, both the effort and the job satisfaction enter in the utility function. In return of the supplied costly effort, $s_t$, the worker receives a satisfaction related to the job which depends on three elements. The first element is constant and
measured by $\delta_c$. The two other elements are connected to the real wage paid by the firm to the worker, namely $w_t = W_t/P_t$ with $W_t$ the nominal wage. The worker compares his real wage to his current alternative opportunities on the labor market, $w_t^a$, and to a reference index of past wages, $w_t^s$. The higher the real wage (compared with the intertemporal and intratemporal wage norms), the more the worker is satisfied with his job. The two norms are weighted in the effort function by the parameters $\delta_a$ and $\delta_s$. The current alternative opportunities and the reference index of past wages are defined by

$$w_t^a = n_t w_t$$  \hspace{1cm} (5)$$

$$w_t^s = \rho_s \sum_{j=1}^{\infty} (1 - \rho_s)^{j-1} (w_{t-j})$$  \hspace{1cm} (6)$$

where $n_t$ is the employment rate, $w_t$ is the real average wage, and $\rho_s$ measures the persistence of past wages in the reference index, with $0 < \rho_s < 1$.

The first order condition of (2) with respect to $s_t$ gives the equilibrium effort function\(^\text{16}\)

$$s_t = \delta_c + \delta_a \log \left( \frac{w_t}{w_t^a} \right) + \delta_s \log \left( \frac{w_t}{w_t^s} \right)$$  \hspace{1cm} (7)$$

Let us make a few comments about these specifications. As in Collard and de la Croix (2000) and Ball and Moffitt (2001), logarithm is used in the effort function to simplify the model’s resolution (see de la Croix et al., 2000, for an alternative assumption). Danthine and Kurmann (2004) consider a more general effort function, which decompose each parameter ($\delta_a$ and $\delta_s$) into two parameters: one concerning wage and the other employment. Contrary to Danthine and Donaldson (1990), unemployment benefits are not taken into account in the current alternative opportunities, see eq. (5). In the same way, we do not follow de la Croix et al. (2000) who consider that past alternative opportunities also enter in the intertemporal wage norm. The reference index of past wages, eq. (6), corresponds to the particular case of habit formation studied by Collard and de la Croix (2000). Notice that Ball and Moffitt (2001) also consider a habit formation process, but on the growth of wages not on the level of wages as we do. Finally, contrary to Collard and de la Croix (2000), we

\(^{16}\)As in Collard and de la Croix (2000), the equilibrium value of $v(s_t)$ is zero.
will consider only the social norm case and not the personal norm case where the presence of past wages is explicitly taken into account within the labor contract. Here, the past wages act as a pure externality.

To avoid households’ heterogeneity induced by the individual history on the labor market, we assume that a perfect insurance market exists (see the appendix of Collard and de la Croix, 2000, for an explicit treatment). The real labor market revenues of households are \( w_t n_t \). The household carries \( M_{t-1} \) units of money and \( B_{t-1} \) bonds into period \( t \) and receives a lump-sum transfer \( T^r_t \) from the monetary authority and nominal profits \( D_t \) from the intermediate goods producers. Its revenues are used to consume, to purchase bonds, and to store money. The gross nominal interest rate of bonds between period \( t \) and \( t+1 \) is denoted \( r_t \). The budget constraint is

\[
m_t + \frac{b_t}{r_t} + c_t \leq m_{t-1} + b_{t-1} + \tau^*_t + w_t n_t + d_t
\]

where \( b_t = B_{t-1}/P_t \), \( \tau^*_t = T^r_t/P_t \), and \( d_t = D_t/P_t \) are the real values of bonds, transfer, and profits.

### 3.2 Firms

The final good sector is perfectly competitive and uses \( y_t(i) \) unites of intermediate good \( i \) to produce \( y_t \) units of final good according to a constant return to scale technology

\[
y_t = \left( \int_0^1 y_t(i)^{(\varepsilon-1)/\varepsilon} \, di \right)^{\varepsilon/\varepsilon-1}
\]

where the elasticity of substitution between goods is \( \varepsilon \), with \( \varepsilon > 1 \). The profits’ maximization program of the final good sector representative firm gives us the intermediate good demand function

\[
y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} y_t
\]

where \( P_t(i) \) is the intermediate good \( i \) nominal price and the final good price \( P_t \) satisfied

\[
P_t = \left( \int_0^1 P_t(j)^{1-\varepsilon} \, dj \right)^{1/(1-\varepsilon)}
\]
The intermediate good producer faces a quadratic cost of adjusting its nominal price which is measured in final good

\[ \frac{\psi}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 P_t y_t \]  

(12)

where \( \pi \) is the steady-state gross rate of inflation. The intermediate good quantity is produced according to the technology

\[ y_t(i) = z_t n_t(i) s_t(i) \]  

(13)

where \( z_t \) is the stochastic labor productivity at date \( t \) common to all producers and \( n_t(i) s_t(i) \) is the effective labor input, that is the product of the workers and their individual effort. The labor productivity law of motion is

\[ \log(z_t) = \log(g) + \log(z_{t-1}) + \zeta_t \]  

(14)

where \( g \) is the steady state gross rate of labor productivity and \( \zeta_t \) is the productivity shock, with \( \zeta_t \sim iid \left( 0, \sigma_\zeta^2 \right) \). In the sequel we denote \( g_t = \log(z_t / (gz_{t-1})) = \zeta_t \) the logdeviation of the growth factor of \( z_t \) from its steady value \( g \).

The per period nominal profits flow of producer \( i \) is

\[ D_t(i) = P_t(i) y_t(i) - W_t(i) n_t(i) - \frac{\psi}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 P_t y_t \]  

(15)

where \( \psi > 0 \). Due to the presence of efficiency wage, the wage enters in the intermediate goods producer maximization program

\[ \max E_0 \sum_{t=0}^{\infty} \left( \beta^t \lambda_t \frac{D_t(i)}{P_t} \right) \]  

(16)

with respect to \( y_t(i), n_t(i) \) and \( W_t(i) \), subject to the constraints (7), (10), (13) and the relation \( s_t(i) = s \left[ W_t(i) \right] \) implied by the efficiency wage hypothesis. \( \lambda_t \) is the multiplier’s value of the budget constraint in the maximization program of the representative household.

In a symmetric equilibrium, all intermediate goods producers make identical decisions and the solution of (16) leads to the equilibrium relations

\[ \frac{\pi_t}{\pi} \left( \frac{\pi_t}{\pi} - 1 \right) = \beta E_0 \left\{ \frac{\lambda_t y_{t+1}}{\lambda_t} \left( \frac{\pi_{t+1}}{\pi} \right) \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \right\} - \left( \frac{\frac{\varepsilon}{\varepsilon - 1} - 1}{\psi} \right) \left( 1 - \frac{\varepsilon}{\varepsilon - 1} \frac{w_t}{z_t s_t} \right) \]  

(17)
\[
\frac{\partial s_t}{\partial w_t} s_t = 1 \tag{18}
\]

Eq. (17) describes the sluggishness adjustment of the inflation gross rate \(\pi_t\) and eq. (18) is the well-known Solow’s condition.

### 3.3 The monetary authority

Since we focus our attention on the effects of technological shocks, the monetary authority is assumed to ensure a constant growth rate of money \(M_t = \mu M_{t-1}\) (the newly created money is given to households in the form of transfer \(M_t = M_{t-1} + T^*_t\)).

### 3.4 Equilibrium

To end the model description, we present the loglinearized, stationary, and dynamic equilibrium. Let \(\pi_t = x_t \varphi^t\) the stationarized value of the growing variable \(x_t\) and \(\hat{\pi}_t = \log(\pi_t / \pi)\) the logdeviation of this variable from its steady state value \(\pi\) (the bar is omitted for stationary variables). Endogenous variables \(\{m_t, w_t, w^s_t, n_t, \pi_t\}\) satisfy the following five equations

\[
\left( \hat{m}_t - \hat{m}_t^s + g_t \right) \frac{\delta_t}{\delta a} - \hat{n}_t = 0 \tag{19}
\]

\[
\rho_s \hat{w}_t (1 - \rho_s) \frac{\hat{m}_t}{g} (\hat{w}_t^s - g_t) - \hat{w}_t \hat{w}_t^{s+1} = 0 \tag{20}
\]

\[
\hat{m}_t + g_t + \hat{n}_t - \hat{m}_{t-1} = 0 \tag{21}
\]

\[
\hat{n}_t - \left(1 - \frac{\beta}{g}\right) \hat{m}_t - \frac{\beta}{g} E_t (\hat{n}_{t+1} + g_{t+1}) = 1 \tag{22}
\]

\[
\beta E_t \{\hat{n}_{t+1}\} + \left(\frac{\varepsilon - 1}{\psi} \right) \hat{m}_t - \hat{n}_t = 0 \tag{23}
\]

where \(g_t\) is the innovation to the productivity process as defined by Eq. (14).

Eq. (19) is the labor market equilibrium and expresses the current value of (the logdeviation of) labor as a function of the wage and the wage norm. Eq. (20) describes the law of motion of the wage norm. Eq. (21) and (22) concerns the supply and demand of money. Finally, Eq. (23) is the (new) Phillips curve.
4 Results

We first present the procedures of calibration and of estimation and then describe our results. To solve and simulate the model, we use the method of Uhlig (1999).

4.1 Calibration and estimation of parameters

Once the model is loglinearized around its steady state, we can directly deduce its spectral density matrix and compute the spectra and co-spectrum of the model. To assign quantitative values to the parameters, we follow Wen (1998) and adopt a strategy that mixes calibration and estimation procedures. We estimate the parameters for which we have little empirical evidence: the real rigidities on the labor market (the weight parameters in the effort function), the nominal rigidities on the good market (the degree of sluggishness of price adjustments), and the variance of the shock. The remaining parameters are calibrated as described below. The values of all parameters are reported in Table 3.

The calibration procedure is based on several constraints. The steady-state values of unemployment, the (gross) quarterly rate of labor productivity growth, and the (gross) quarterly rate of inflation are set to their empirical mean value on our sample (hence, \( n = 0.9435 \), \( g = 1.0054 \), and \( \pi = 1.0088 \)). The parameters \( \varepsilon \) and \( \beta \) are chosen according to conventional estimates. The \( \varepsilon \) value implies a realistic mark-up rate of 10% (see Basu and Fernald, 1997). The \( \beta \) value implies an annual interest rate at 6.9% as suggested by King and Rebelo (1999).

For the effort function, notice that only the ratio \( \delta_s/\delta_a \) matters in the dynamic system and that one steady-state restriction remains free (namely the labor market equilibrium). We adopt the following strategy. Instead of estimating the ratio \( \delta_s/\delta_a \), we estimate only \( \delta_s \) and set the parameter \( \delta_a \) to 0.90 as in Danthine and Donaldson (1990) and Collard and de la Croix (2000). Finally, the remaining steady-state restriction is used for calibrating the scale parameter \( \delta_c \). Only two parameters of the effort function are estimated: \( \delta_s \) and \( \rho_s \).

For the sticky prices process, since our model is based on price adjustment costs, \( \psi \) is a scale parameter which is not easily interpreted. Nevertheless, as pointed out by Ireland
(2000) amongst others, there is a relation between this parameter and the probability that a producer can revise its price in a model à la Calvo (1983)\textsuperscript{17}. In the sequel, the parameter ψ is estimated and we also indicate the associated average duration of price fixity so as to facilitate comparisons with the literature.

Finally, the standard deviation of the shock σζ is also estimated.

The method of estimation is very close to that of Wen (1998). The parameters are estimated to reproduce the spectra of unemployment and labor productivity growth. The empirical spectral density matrix is denoted \( S(\omega) \). The theoretical spectral density matrix, denoted \( T(\omega, \xi) \), depends on the vector of structural parameters \( \xi = [\sigma_\zeta, \delta_s, \rho_s, \psi] \). A simple criterion for the choice of the parameters is to minimize the distance between the spectral density matrices of the data and the model. Over the frequency band \( \Omega \), the objective is to find \( \xi \) that minimizes

\[
a(\xi) = \int_\Omega \Gamma(\omega) \odot |A(\omega, \xi)| d\omega,
\]

where \( \odot \) is an element by element multiplication operator and \( \Gamma(\omega) \) is a frequency weighing function which determines the weight attached to each frequency. Three supplementary steps are necessary. First, it is relevant to consider a weight matrix. We set \( W = [S_g(\Omega), 0; 0, S_u(\Omega)]^{-1} \) to give an equal weight to the two spectra. Second, we take the trace of the distance matrix to evaluate its size. Hence, we define the metric \( \Delta = \text{tr}(W \cdot a(\xi)) \) that we minimize with respect to \( \xi \). Finally, as Wen (1998), we give to each frequency a weight proportional to the percentage contribution of that frequency to the total variance of the data:

\[
\Gamma(\omega) = S(\omega) \odot \int_0^\pi S(\omega') d\omega'
\]

where \( \odot \) is an element by element division operator.

The results for an estimation over the frequency band \( \Omega = [0, \pi] \) are reported in Table 3. The parameter ψ is slightly higher than the conventional value retained (the average fixity

\textsuperscript{17}The two New Phillips Curves are equivalent when \( \psi = \theta (\varepsilon - 1) / [(1 - \beta \theta)(1 - \theta)] \), where \( \theta \) is the probability of not to revise its prices in a Calvo (1983) model and the parameters ψ, ε and β are defined in Section 3.
or price is 4.8 quarters instead of 4 quarters as suggested by King and Wollman, 1996). For the effort function our values differ from the values of Collard and de la Croix (2000). The value of $\rho_s$ is 0.032, which is lower than the value retained by the authors (0.10). For this parameter, our estimation appears to be rather close to Ball and Moffitt (2001) (but still higher) who suggest a value of 0.05 for $\rho_s$. So, our estimation describes a strong persevering process for the reference index of past wages. But, at the same time, it indicates that the reference index of past wages has a modest weight in the effort function: the estimated value of 0.430 for $\delta_s$ is notably lower than the value of 2.45 in Collard and de la Croix (2000).

4.2 Model evaluation

Once the value of the parameters are chosen, we can compute the theoretical spectral density matrix and compare it with the empirical density matrix. We first check that the model performs well for the spectra (it has been estimated to reproduce them) and then we look at the co-spectrum.

Figures 6 and 7 depict the spectra of labor productivity growth and of unemployment, respectively, for the model and the data. The model fits the empirical spectrum of unemployment very well by putting the main part of its variance into the low frequencies, and it belongs to the empirical CI for the major part of frequencies considered. The results are also satisfactory for the labor productivity growth. We have modelled it as a white noise, hence its flat spectrum. The model cannot naturally reproduce the rich dynamics observed in the data, notably the presence of peaks. However, expect for a small band of frequencies (near $\pi/16$), the theoretical spectrum belongs to the empirical CI.

The theoretical co-spectrum has a similar pattern to its empirical counterpart: it is concentrated rather in the long-run and business cycle frequencies than in the short-run frequencies (see Figure 3). The co-spectrum is negative for the zero frequency, it increases to reach a peak and then decreases within the business cycle frequencies to converge toward a small positive value in the short-run frequencies. Nevertheless, there are two failures. First, a shifting toward the low frequencies appears when we compare it with the empirical co-spectrum. The frequencies for which the theoretical co-spectrum crosses the zero line.
and reach its peak are lower than their empirical equivalents. The theoretical co-spectrum is even outside the empirical CI for these frequencies. Second, for the short-run frequencies, the model overestimates the value of the co-spectrum. It predicts positive co-movements whereas they are empirically negative (and significantly different from zero). To conclude, the model reproduces well the pattern of the co-spectrum for the medium and low frequencies (especially the shift in the sign of the co-movement), but does not account for the short-run co-movements. This conclusion is confirmed if we look at the alternative measure of co-movement presented in section 2.

Table 4 reports the values of dynamic correlation for the model with sticky prices and fair wage. If we take all frequencies, the model clearly overestimates the correlation between growth and unemployment (0.307 for the model against 0.037 for the data). Looking at the decomposition by frequency band of this correlation indicates that this overestimation mainly comes from the short-run frequencies. For these frequencies the theoretical correlation is close to one whereas it is empirically negative. For the others frequency bands, the model performances are better. Even if it does not replicate the values, the sign of the co-movement is correctly reproduced.

Results are less satisfying for the measure of co-movement based of forecast errors, see Figure 8. The correlation of forecast errors is overestimated for all forecast horizons. As for the dynamic correlation, we obtain a strong overestimation of the unconditional correlation coefficient which, once again, comes from the model’s failure at the short-run. We then turn toward the covariance of forecast errors for which different forecast horizons can be studied separately. For the first forecast horizon, we find again the short-run failure of the model. For higher forecast horizons, the covariance is mechanically zero since productivity growth has no persistence in the model. Adding persistence of growth slightly improves the fit of the model. Labor productivity law of motion is still described by Eq. (14), but $\zeta_t$ follows:

$$
\zeta_t = \rho \zeta_{t-1} + \tilde{\zeta}_t
$$

where $\tilde{\zeta}_t \sim iid\left(0, \sigma^2\right)$. This setup permits us to consider a persistence process for the

18They are deduced from an estimated VAR using one very long simulation of the model.
growth of the labor productivity together with the random walk property for its level. In addition we impose \( \sigma^2_\zeta = \frac{\sigma^2_e}{1 - \rho_\zeta} \) to leave unchanged the variance of productivity growth. Results are reported in Figure 9 for \( \rho_\zeta = \{0.00, 0.40, 0.80\} \). Adding persistence of growth reduces the overestimation problem (both for the first and the last forecast horizons) and induces non-negative covariance of forecast errors for the first forecast horizons, which are close to their empirical counterpart. However, if one consider a strong persistence process for growth (\( \rho_\zeta = 0.80 \)), co-movement becomes too negative in the business cycle frequencies and lead to an underestimation of the unconditional correlation coefficient.

### 4.3 IRFs and models comparison

To help to understand the behavior of the model, we study the impulse response functions (IRF) and consider two alternative versions of the model. In the first, prices are flexible. In the second, the fair wage hypothesis is abandoned in favor of the model of labor supply with preferences à la Hansen (1985). In the former, \( \psi \) is simply set to zero. In the latter, the utility function defined by the eq. (4) becomes \( v(n_t) = hn_t \) (where \( h \) is the constant hours by worker) and the production function defined by eq (13) becomes \( y_t(i) = z_t hn_t(i) \). The parameter \( h \) is set to the equilibrium value of the effort at work with the fair wage hypothesis. Figure 10 reports the models’ IRF and Figure 11 the models’ co-spectrum. See also Table 4 for the values of dynamic correlation.

Because there is only a shock in our model, inspecting the IRFs in the time domain is particularly useful, so as to understand the results previously described within the frequency domain. The short-term response corresponds to the high-frequency properties, whereas the long-term behavior corresponds to the properties at low frequencies. Figure 5 depicts the IRFs of unemployment for the three models. On the basis of this figure one can easily understand the model’s behavior.

In the model, the rates of unemployment and labor productivity growth co-move positively in the short-run because prices are sticky. Basu et al. (2004) and Gali (1999) describe the underlying mechanism to explain the instantaneous negative effects of productivity shocks on the use of the labor input. When the shock hits the economy, due to
the nominal rigidities, the aggregate demand is partially fixed. To satisfy their demand, producers need less labor due to the improvement in the labor productivity. This effect vanishes along the complete adjustment of prices.

In the model, the rates of unemployment and labor productivity growth co-move negatively in the long-run for two reasons. The first reason is that job satisfaction depends on the wage growth as in Collard and de la Croix (2000), de la Croix et al. (2000), Ball and Moffitt (2001) and Danthine and Kurmann (2004). In our model the equilibrium value of the effort at work is constant. To ensure a given level of effort, there is a compensation between the intertemporal term (which depends on wage growth) and the intratemporal term (which depends on unemployment). Hence, the higher the wage growth, the weaker the required unemployment rate. The second reason is that the comparison of the present and past wages by workers adjusts very slowly during time. Due to our habit-formation assumption, a current increase in the wage will be compared to a large set of past wages and then lead to unemployment decreases that will last for a long time.

We have presumed that the sticky prices hypothesis explains the behavior of the economy in the short-run whereas the fair wage hypothesis explains it in the long-run. The comparison of the theoretical co-movement for three models confirms this assertion. The co-spectrum and the dynamic correlation of the model with the real rigidity are negative for all frequencies studied and that of the model with the nominal rigidity are always positive. Consequently, to explain only the long-run co-movements one can take the model with fair wage and flexible prices. In the same manner, to explain only the co-movements along the business cycle one can consider the model with sticky prices and indivisible labor. Contrariwise, to explain co-movements in the long-run and along the business cycle, both fair wage and sticky prices are necessary. Finally, any version of this model provides a satisfactory explanation of the short-run co-movement.
5 Conclusion

We have studied the co-movements of the rates of unemployment and of labor productivity growth for the United States economy by means of spectral analysis, which gives us an overall view of the co-movements of these variables. We have put together several crucial facts which had never been described together in an integrated framework: the positive co-movement along the business cycle and the negative co-movement in the long- and the short-run. We have then proposed a theoretical explanation of these facts based on New Keynesian mechanisms: the fair wage principle for the labor market and sticky prices for the goods market. The combination of these two rigidities provides a satisfactory explanation of the empirical co-movements for the long-run and the business cycle, but not for the short-run. Due to the real rigidity on the labor market, the rates of unemployment and of labor productivity growth co-move negatively in the long run and due to the nominal rigidity on the goods market, they co-move positively along the business cycle. These results would be usefully completed by further research on another countries (especially the European countries whose rigidities on the labor market have been widely studied). We have also left aside some crucial features which could be fruitfully examined in further research such as monetary policy shocks and physical capital accumulation. These features could also enhance the model’s ability to account for the short-run co-movements which remains unexplained by our model.
References


### Table 1. Correlation between labor productivity growth and unemployment

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>All</th>
<th>Long-Run</th>
<th>Business Cycle</th>
<th>Short-Run</th>
</tr>
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<tr>
<td>Dynamic correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Bivariate VAR with 6 lags</td>
<td>+0.037</td>
<td>−0.458</td>
<td>+0.515</td>
<td>−0.139</td>
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<td>Large VAR with 8 lags</td>
<td>+0.050</td>
<td>−0.444</td>
<td>+0.503</td>
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<tr>
<td>Correlation between filtered series</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Christiano and Fitzgerald (2003)</td>
<td>+0.061</td>
<td>−0.373</td>
<td>+0.570</td>
<td>−0.105</td>
</tr>
<tr>
<td>Baxter and King (1999) K = 12</td>
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<td>−0.498</td>
<td>+0.501</td>
<td>−0.123</td>
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<td>K = 36</td>
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### Table 2. Correlation between labor productivity growth and total hours worked

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>All</th>
<th>Long-Run</th>
<th>Business Cycle</th>
<th>Short-Run</th>
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<tbody>
<tr>
<td>Dynamic correlation</td>
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<tr>
<td>Bivariate VAR with 3 lags</td>
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<td>Large VAR with 7 lags</td>
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<td>−0.536</td>
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<tr>
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<td>K = 36</td>
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<td>+0.432</td>
<td>−0.538</td>
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Table 3. Parameter values

<table>
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<tr>
<th>Effort function</th>
<th>$\delta_a = 0.900, \delta_s = 0.430, \delta_c = 1.450, \rho_s = 0.032$</th>
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<tr>
<td>Goods market</td>
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<td>Technology and preferences</td>
<td>$g = 1.005, \sigma_\zeta = 0.0086, \beta = 0.9887$</td>
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Table 4. Correlation between labor productivity growth and unemployment

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<td>Bivariate VAR with 3 lags</td>
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<td>-0.458</td>
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<td>-0.139</td>
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<tr>
<td>Large VAR with 7 lags</td>
<td>+0.050</td>
<td>-0.444</td>
<td>+0.503</td>
<td>-0.126</td>
</tr>
<tr>
<td>Dynamic correlation of models with</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>fair wage and sticky prices</td>
<td>+0.307</td>
<td>-0.117</td>
<td>+0.859</td>
<td>+0.897</td>
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<td>fair wage and flexible prices</td>
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<td>Indivisible labor and sticky prices</td>
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<td>+0.933</td>
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<td>+0.835</td>
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</tbody>
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Captions for figure

**Figure 1.** Spectra for the data. Panel a: the spectrum of labor productivity growth. Panel b: the spectrum of unemployment. The shaded areas show confidence intervals.

**Figure 2.** Co-spectrum of labor productivity growth and unemployment for the data. The shaded area shows confidence intervals.

**Figure 3.** Correlation of forecast errors (panel a) and covariance of the updates of forecast errors (panel b) for labor productivity growth and unemployment. The shaded area shows confidence intervals.

**Figure 4.** Co-spectrum of labor productivity growth and total hours worked for the data. The shaded area shows confidence intervals.

**Figure 5.** Correlation of forecast errors (panel a) and covariance of the updates of forecast errors (panel b) for labor productivity growth and total hours worked. The shaded area shows confidence intervals.

**Figure 6.** Spectra for the data and the model. Panel a: the spectrum of labor productivity growth. Panel b: the spectrum of unemployment. For both panels, the solid line refers to the model and the dotted line refers to the data. The shaded areas show confidence intervals.

**Figure 7.** Co-spectrum of labor productivity growth and unemployment for the data and the model. The solid line refers to the model and the dotted line refers to the data. The shaded area shows confidence intervals.

**Figure 8.** Correlation of forecast errors (panel a) and covariance of the updates of forecast errors (panel b) for labor productivity growth and unemployment for the data (dotted line) and the model (solid line with circles). The shaded area shows confidence intervals.

**Figure 9.** Correlation of forecast errors (panel a) and covariance of the updates of forecast errors (panel b) for labor productivity growth and unemployment for the data and
models with persistent growth rate (solid lines with stars for $\rho_\zeta = 0.00$, with circles for $\rho_\zeta = 0.40$, and with squares for $\rho_\zeta = 0.00$). The shaded area shows confidence intervals.

**Figure 10.** The IRFs of unemployment rate for three models. The solid line refers to the model with sticky prices and fair wage, the dashed line refers to the model with flexible prices and fair wage, and the dashdot line refers to the model with sticky prices and indivisible labor.

**Figure 11.** Co-spectrum of labor productivity growth and unemployment for the data and the three models. The dotted line refers to the data, the solid line refers to the model with sticky prices and fair wage, the dashed line refers to the model with flexible prices and fair wage, and the dashdot line refers to the model with sticky prices and indivisible labor. The shaded area shows confidence intervals.
Figure 3

Panel a

Panel b

[Graphs showing data over years]
Figure 4
Figure 5

Panel a

Panel b
Figure 6

Panel a

Panel b
Figure 8

Panel a

Panel b
Figure 9
Figure 11

- Fair Wage and Sticky Prices
- Fair Wage and Flexible Prices
- Indivisible Labor and Sticky Prices
- Empirical