Behavioral Equilibrium∗

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Abstract An apparently major finding from "Behavioral Economics" is that the core assumption of game theory, that human subjects always best-reply to their opponents’ moves, seems to be too strong, and fails, sometimes, to account for actual behaviors. The purpose of this paper is to define a wider theoretical framework permitting such non-perfectly rational behaviors. It is shown that if some moderate and generic conditions on behavioral rules, in general consistent with empirical evidence, holds, then an equilibrium exists for any compact and convex game and compact game with mixed strategies, as well as social games.

Keywords: procedural choice, behavioral game theory, equilibrium, behavioral economics, fixed point, behavioral equilibrium

JEL classification: C62, C72, C91, D01.

1 Introduction

Economics relies strongly on the assumption that individuals behave in a rational manner. The rationality hypothesis, jointly with the assumption of well defined preferences, is the ground on which economic theory can model human conducts, and thereby human institutions, e.g. markets, firms and administrations. But, a growing amount of evidence has accumulated during the last decades, specifically in the 'behavioral economics' field, which suggests that the rationality/preferences view of human conduct may not fit empirical data in many circumstances. These two cornerstones of rational choice theory appear to be challenged in empirical or experimental studies, in economics of course, but also in psychology, cognitive science and more recently in neuroscience.

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Indeed, in some settings, preferences seem to be inefficient or inconsistent to account for actual choices (\(?\), \(?\)), and revealed preference axioms seem violated (?). Furthermore, would human have well defined preferences, they would often be at variance with economic core hypotheses: expected utility has been for long under intense critical scrutiny (\(?\), \(?\), \(?\), \(?\)) and self-interest seems to be an oversimplifying hypothesis (?). Besides, many empirical materials suggest that human decisions can hardly be described as perfectly rational: in many cases, humans are neither able to produce optimal behaviors nor capable of reaching valid judgments (?). Advances and promising approaches in other realms, such as neuroscience or cognitive psychology lead alike to views on human decision often at odd with economic modeling (\(?\), \(?\), \(?\)).

This naturally raises concerns over the descriptive validity of game theory, viewed as a theory of human decisions in interaction. Indeed, most game theoretical results are based on the apparently shaky hypotheses that on the one hand agents have preferences properly represented by expected utility, and on the other hand they are perfectly rational. These worries are reinforced by empirical investigations into strategic interactions, which often support the view that subjects considerably deviate from game theoretic predictions (?). On a general note, it seems fair to say that even in the experimentally controlled and somewhat favorable environments, standard self-interested expected utility maximization seems to fall short of describing accurately subjects’ interactive behavior (?).

The picture seems thus extremely blurred on how human conducts can be accurately described within the standard framework: Preferences appear to only offer an approximative account for actual behaviors, and bounded rationality seems to produce behaviors often distant from optimizing ones. Hence it may be relevant for economic theory to gather all these eventualities in a unified framework, for instance in considering that almost any behavior can occur, rational self-interested ones being a special case. This implies to tackle a larger class of behaviors than the ones commonly focused on by economic theory, namely utility maximization. In particular, such an effort should especially aim at allowing three types of empirically observed phenomena: dynamic construction of preferences within sequential choice processes instead of firmly rooted ordering of alternative items; almost arbitrary motivations for agents instead of material gains alone; and last but not least, gradual adaptation through bounded rationality and learning, rather optimal reply.

As the notion of equilibrium is the very central concept of economic analysis, such an endeavor could lay the groundwork for a unified "behavioral" view by extending such a concept to many type of non-optimizing behaviors. The purpose of this paper is thus to find out some very general conditions under which an equilibrium, extended to almost any behavior, could exist. Such a result may have some theoretical significance, if comprehensive enough, as it
would generalize the existence of equilibrium in interacting situations without relying on the empirically weakened notions of preferences and (substantive) rationality.

In the next section, we will propose some minimal and moderate principles to model behaviors, which appear to be consistent with empirical findings. The central feature postulated here is some local consistency in the process of choice. The third section is dedicated to the construction of the theoretical canvas in an individual decision framework. The fourth section will show that, under the same requirements for interactive behavior, an equilibrium exists for compact and convex games, as well as for compact games with probabilistic play. The fifth part will show that such a result can be extended to general equilibrium analysis. And the sixth section will put forth the issues raised by any weakening of such hypotheses. The seventh part will show how this frame can capture a wide range of behaviors. A final section will conclude, insisting on the possible viewpoint on human behavior that such a model can be compatible with.

2 What principles for modeling behaviors?

On a general note, experimental evidence and subsequent behavioral economics models have put forth that most standard assumptions fail to account for observed conducts in some circumstances. Three main points can be highlighted here in order to summarize these findings. First, basic motivations or goals of economic agents seem at odd with mainstream assumptions: not only subjects appear not to conform to self-interested expected utility maximization (?), but they seem to exhibit a wide degree of heterogeneity in terms of motivations (?2.1). Secondly, decisions seem to be the result of some sequential process of choice, which compares various attributes of the alternatives, rather than the outcomes of some firmly rooted, immediately accessible and consistent preferences postulated by standard theory (?2.2). In addition, even if goals/preferences could be clearly defined, it seems that in a certain number of situations, subjects lack the competence necessary to act immediately and perfectly rationally (?2.3) (?§2.3). A possible conclusion is that most reasonable principles commonly assumed to describe human behaviors may fail in some situations (?2.4), hence, in order to model such conducts, hypotheses as little restrictive as possible should be upheld (?§2.5).

2.1 A variety of driving forces

Overwhelming evidence has been gathered by experimental economics, which suggests that very heterogeneous motivations are at work in human choices. Subjects may have concerns for others’ situations in a variety of ways (?2.1), for instance altruism, inequality aversion, and reciprocity. Other attitudes can also be easily thought of: competitiveness, status-seeking, (?2.1), imitation, etc. In addition, a wide spectrum of stances toward risk and uncertainty (?2.1) seem
to exist. And social norms and practices (?) as well as intrinsic motivation (?) appear to play a powerful role in human conducts. Considered merely in terms of objectives, human motives seem to be much richer than pure self-interest. Yet, it seems that in many settings, especially competitive environments or market, human behaviors appear properly described by its self-interested account (?, ?), adding even more heterogeneity of motivations in the picture.

2.2 Sequential procedures of choice rather than stable preferences

More radically, the very concept of well-behaved and defined preferences is sometimes challenged by empirical investigation. Indeed, choices appear to be the outcome of a whole sequential decision process, which by "tatonnement" or trial-and-error reaches some satisfying situation, rather than the immediate result of the rational optimization of some binary relation (?). And, due to cognitive limitations, agents may only evaluate or construct their preferences imperfectly when facing a set of alternatives, leading to inconsistencies sometimes (?). This process seems furthermore deeply influenced by many rationally irrelevant items: initial endowment, framing, labeling and context effects (?), irrelevant alternatives (?), etc. All these phenomena can be explained at least for a part by the sequential/cognitive nature of choices: processes can be influenced by some description of the situation which triggers the use of some heuristics or routines instead of some other; initial endowments may play the role of an anchor from which the procedure will start, etc. Examples of such procedural choices can be found, in the realm of gambles, in (?).

2.3 Adaptive movements rather than rational best-reply

A consequence is that agents cannot identify clearly and immediately satisfactory situations; they need to experiment, learn, evaluate, compare the various alternatives, in order to make a sound choice. And, because of well known cognitive boundaries (?), they are often myopic and in most situations only perceive some of the relevant qualities of a situation. They can of course revise their judgment, expectations, or beliefs, but this dynamic may have major consequences. For instance, two different starting points may lead to different outcomes, movements or revisions can be costly, so "satisficing" situation may be stable, etc. The core of such an approach is to assume that human subjects rarely, if ever, assess situations from an external viewpoint, but rather anchor their appraisal in some reference point (?). Consequently, an equilibrium should not be seen as the fixed point of some best-reply function, but more relevantly as a situation where no further movement/revision takes place, be it prescribed by some cognitive procedure, learning rule or preference relation.

Furthermore, bounded rationality raises very deep issues when comes the question of how to model it. One main theoretical issue, if not the crucial one, raised by the weakening of substantive rationality into bounded, adaptive
rationality, is that it implies qualitative leap: reasonable principles at work for
the former may not be appropriate for the latter even approximatively (? , ?. ?).
In general, biases or "anomalies" underscored by experimentalists cannot be
viewed as random deviations from substantively rational behaviors. This may
explain why there is no consensus about how to approach bounded rationality in
a formal way, even though it has been a recurring topic in economic literature.
Even more fundamentally, the rise of neuroscience and cognitive psychology have
lead some to cast doubts about the long term efficacy of the "folk psychology"
scheme, on which is based the rational account for human behavior (? , ?)

2.4 No general robust principle?

The existence and apparent robustness of such phenomena renders fragile the
use of the reasonable and appealing principles on which economic modeling tra-
ditionally relies. Indeed, preferences, commonly understood as transitive and
complete binary relations, cannot explain some empirical data, in the realm of
uncertainty (?) or with endowment or status quo effect (?). Relying on some
elegant and appealing principles as in standard decision theory, is made impos-
sible by actual choices from subjects since most of them fail in some settings:
independence or even some weakened version of it (?) for preferences under
risk; monotonicity in own payoffs because of reciprocity, inequality aversion
or altruism concerns (?). Moreover, in many environments, the idea that subjects
best reply falls short of describing actual conducts, which exhibit at best some
bounded and adaptive rationality (?). As a consequence, as stressed above,
bounded rationality does not rely on some generally accepted principle, and
appealing to heuristics, routines, cognitive short-cut does not seem to involve
any unifying rule, in the general case.

2.5 Some minimal choice principles

Accordingly, models aiming at capturing such phenomena should posit very
loose constraints on agents’ conducts. For sequential/adaptive choices to be
possible, decisions have to be conceived of as processes, based on the iterative
use of some evaluation rule. Assuming \( A \) is the set of alternatives, this evaluation
procedure would be a correspondence (i.e. a multi-valued mapping), from \( A \) into
\( A \), denoted \( R : A \rightharpoonup A \). Given any \( a \in A \), its meaning is that any point in
the image of \( a \) seem, in the eye of the agent, to "fit" better than \( a \). Put another way,
this proposition suggest, proposes or requires deviations – any point in \( R(a) \) –
if the situation is \( a \).

For illustrative purpose, imagine a consumer who wants to buy a given com-
modity, and has to decide which trade off she should make between price and
quality. In a general way, it can concern any choice which take place among a
set of alternative items or situations which are liable to be assessed according
to multiple dimensions: gambles with monetary payoffs and probabilities, com-
modities with various qualities, bundle of goods with various quantities, labour
supply with leisure and income, etc. Her first evaluation comes from some prod-
uct $a$ which may not be suitable given some criterion – quality for instance – and
leads to consider some other product(s), $a'$ which may have, in the eye of the
consumer, some other deficiency, and thus brings about some new consideration,
and so on. Such an iterated process can be real or simply virtual, by the use
of imagination and counter-factual reasoning, or a mixture of both: an agent
can actually choose an item and then realize she would prefer to deviate from
it in order to reach some other alternatives, since its actual experience of this
situation reveals some issues; reversely, agents may consider some alternative
and foresee that, would it be actual, she would prefer to deviate. It is easy to
see that such kind of procedure can even encompass collective decisions, when
members of a group try to reach some sound decision\footnote{For instance, in MOVIE RRR}. A sound choice $a^*$ is
made when the evaluation procedure does not suggest any further move, that
is when $R(a^*) = \emptyset$. Such a formal frame may easily include sequential pro-
duress as well as learning process, $R(a)$ being construed as the set of learning
opportunities from $a$.

Yet, as stated above, no general principle appears to be empirically robust
enough to be essential. Hence, it seems difficult to hypothesize that a principle
such as monotonicity could direct the whole dynamics, because of the diver-
sity of motivations and cognitive processes used by subjects. A very weakened
and plausible version could be that the evaluation procedure follows some lo-
cal monotonicity, that is, in some neighborhood, the sequential rules suggest
moves in some direction ("cheaper", "safer", "better", "less risky", etc.). It can
nevertheless be easily envisioned cases where moves in two different directions
("safer" and "more comfortable" vs. "cheaper") can be prescribed. So, in order
not to be too restrictive, this condition could be defined as some local consis-
tency, stating that in some neighbourhood, directions of possible moves should
not be contradictory: in some local area, the rule should not require "cheaper"
and "better" if better necessarily implies "more expensive". More formally, if
direction is defined as the gross direction from a situation $a$ to the image set
$R(a)$, then it should hold that in some neighborhood of $a$, directions of such
movements are not opposite. Or to put it more precisely, opposite directions
should not be suggested by some procedure for two arbitrary close elements.

This property, defined formally below, will be referred to as local non-opposition
of directions. This ensures that no sudden breaks in direction can take place in
the evaluation process. Assuming some product was evaluated as too expensive,
a similar enough product for all its attributes should not imply for the consumer
to consider more expensive products. Such a property does not imply continuity
of the correspondence. Indeed, examples of discontinuous behavior can be easily
imagined, e.g. threshold triggered behaviors: once some threshold is reached –
say, the safety of use of a commodity –, moves can go in some other direction –
for instance, comfort –, as far as this direction is not contradictory, that is, it
does not require a choice that would be less safe. Similarly, two close situations can lead the agent to improve some criterion – so in the same direction –, but not with the similar magnitude: a consumer can realize once some level for some quality is reached, that it is not enough for the targeted purpose.

3 Individual decision

Such a canvas does not hitherto ensure that some actual decision can be reached, since nothing guarantees that such a sequence has no cycle. The question tackled in this section is to know whether, in an individual decision scheme, and given any evaluation procedure, agents can make a choice, and which type of choice results from such a process.

In order to model such a procedure, we will assume that given some set of alternatives $A$, a compact and convex subset of a locally convex Hausdorff topological vector space $E$, whose dual is $E'$, some evaluation procedure will take place. Some classical denotations will be used: for any $A$, $\bar{A}$ is the closure of $A$ and $˚A$ will be the interior of $A$. $\text{Gr} f$ (resp. $\text{Gr} \Phi$) is the graph of a mapping $f : E \to F$ (respectively of a correspondence $\Phi : A \rightrightarrows F$), i.e. $\{(x, f(x)), x \in A\}$ (resp. $\{(x, y), x \in A, y \in \Phi(x)\}$). Furthermore, $\langle \cdot, \cdot \rangle$ will denote inner product in $E$. As a consequence, $E$, being an inner product space, is also a metric space.

A first requirement is that if a situation is acceptable from the agent’s viewpoint, then she should not make any further choice, or should not deviate from it. Thus, either $a$ is satisfying or complies with the procedural rule, and $R(a) = \emptyset$, either not, and $a$ should not be in $R(a)$. As a consequence, $a$ is never an element of $R(a)$.

Property 1 (Irreflexivity) $\forall a \in A, a \notin R(a)$

Now, it is possible to spell out the property of local non opposition of directions. It will imply that a direction at $a$ from $R$ is not opposite to at least one move, for any $a'$ in some neighborhood of $a$.

Property 2 (Local Non-Opposition of Directions (L|NOD)) For any $a \in A$ such that $R(a) \neq \emptyset$, exists a neighborhood $V_a$ of $a$, and $p^a \in E'$, named a direction of $R$ at $a$, such that for any $a' \in V_a$, and all $y^{a'} \in R(a')$, $\langle y^{a'} - a', p^a \rangle > 0$.

Property 2 means that given some situation, exists a neighborhood and a movement direction such that, within this neighbourhood, all moves are consistent with this direction. In other words, arbitrarily close situations should necessarily lead to non-opposite moves. In more technical terms, this means that for any movement from the neighborhood of $a$, one can consider that there is a general direction, that is, a direction which is at worst not orthogonal with any $y^{a'} - a'$, with $a' \in V_a$. Note that this definition implies that $R$ is non-empty on some neighborhood of $a$ – which implies $\{a \in A | R(a) = \emptyset\}$ is closed. At
first glance, this does not seem not to raise any particular issue, in behavioral terms. It also implies the existence of a direction for any \( a \in A \) in the following sense: For any \( a \in A \) such that \( R(a) \neq \emptyset \), exists \( p^a \in E' \) which is such that for any \( y^a \in R(a) \), \( \langle p^a, y^a - a \rangle > 0 \).

If the procedure of choice follows these three principles, then it can be shown that the resulting choice correspondence \( C \) is not empty, for any compact and convex \( A \). Let \( \mathcal{A} \) be the set of compact and convex subsets from \( E \).

**Proposition 1** Let \( C : A \rightrightarrows E \), such that \( a \in C(A) \) iff \( R : A \rightrightarrows A \) is such that \( R(a) = \emptyset \). If \( R \) satisfies properties ?? and ??, then \( C(A) \) is non empty for any \( A \in \mathcal{A} \).

**Proof.** See appendix. \( \square \)

In very plain terms, this means that if an agent follow such a procedural or adaptive process in order to make a choice, then there is always a possible choice.

## 4 An Equilibrium Concept for Behavioral Game Theory

The major interest of this model of decision processes is that it fits very well with a game theoretic frame, where an equilibrium can be seen as a situation where no player has any reason to move from, within an interactive adaptive process.

### 4.1 Compact and convex games

The games here are interactions with finitely many players, set \( N \), whose cardinal is \( n \), each of them, generically referred to as \( i \), having a set of possible actions \( S_i \). Every \( S_i \) is a subset of a locally convex Hausdorff \( \mathbb{R} \)-vector topological space \( E_i \), whose dual is \( E'_i \). Any strategy chosen by player \( i \) will be referred to as \( s_i \). A profile of actually chosen actions \( s = (s_1, s_2, \ldots, s_i, \ldots, s_n) \in S \) will be called a strategy profile of the game. For convenience, this profile will be referred to as \( (s_i, s_{-i}) \), when focusing on player \( i \). Thus, \( s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) \). Similarly, \( S_i \) will be the set of all possible \( s_i \), \( S_{-i} \) is the set of all \( s_{-1} \) profiles, and eventually \( S = \times_{i \in N} S_i \) as well as \( S_{-i} \), is thus a non-empty subset of the locally convex Hausdorff vector topological space \( E = \times_{i \in N} E_i \).

A payoff device for agent \( i \) is a mapping: \( \pi_i : S \rightarrow \mathbb{R} \). The payoff "matrix" of a game is the mapping \( \pi : S \rightarrow \mathbb{R}^n \), with \( \forall s \in S, \pi(s) = (\pi_i(s))_{i \in N} \). Payoffs are to be understood here a little differently from the standard case, where
the payoff matrix is assumed to be a numerical representation of preferences. Payoffs are here only some feature of the game, for instance a material gain mechanism. They are mainly defined in order to define rational behaviors as a special case.

It is now possible to define a game: A game is a profile composed of a finite set of players, their sets of possible actions, and the corresponding set of consequences or payoffs. Formally a game $\Gamma$ can be completely described as: $(N, S, \pi)$. It is possible to distinguish different classes of games, depending on the properties of strategy spaces. A game is said to be compact if for any player, the sets of possible actions $S_i$ are compact. Note that in case the game is compact, $S$ being a product of compact subsets from metric vector spaces is a compact, thus complete, subset of a normed vector space. Similarly, a game is said to be convex if for any player, the sets of possible actions $S_i$ are convex.

On a general note, the set of compact and convex games correspond to the most studied type of games, since otherwise.

We shall hypothesize that human conducts in interactive situations follow the very general properties, postulated in the previous section. Properties are the same, but may need some adaptation to fit the interactive decision framework. Indeed, choice procedures of player $i$ for instance starts from $S$ into $S_i$ and not $S$ as it was the case in the individual decision case.

**Property 3 (Interactive Irreflexivity)** $\forall s = (s_i, s_{-i}) \in S$, $s_i \not\in R_i(s)$

Similarly, for any $i \in N$, $R_i : S \Rightarrow S_i$. local non-opposition of directions can be adapted in the following way:

**Property 4 (LNOD within $S_i$)** For any $s \in S$ such that $R_i(s) \neq \emptyset$, exists a neighborhood $V_s$ of $s$ and a $p^*_i \in E'$, such that for any $s' \in V_s$ and any $s'^*_i \in R_i(s')$ with $\langle p^*_i, s'^*_i - s_i \rangle > 0$, with $s = (s_i, s_{-i})$ and $s' = (s'_i, s'_{-i})$.

These properties of procedural choice being set, it is possible to define the resulting correspondence product: The correspondence product is the correspondence from $S$ to $S$, such that $R = (R_1, R_2, \ldots, R_i, \ldots, R_n)$. In other words, for $s \in S$, $R(s) = \times_{i \in N} R_i(s)$. Then, an behavioral equilibrium of a game can be defined:

**Definition 1 (Equilibrium of a game)** An equilibrium of a given game $\Gamma$ is a $s \in S$ with $R(s) = \emptyset$, i.e $s \in S$ such that $\forall i \in N$, $R_i(s) = \emptyset$.

This allows to give the following existence result for behavioral equilibrium.

**Proposition 2** Given any compact and convex game $\Gamma = (N, S, \pi)$, with players’ individual choice processes satisfying properties ?? and ??, then exists an equilibrium of the game.

**Proof.** See Appendix.
4.2 Compact Games with Probabilistic play

Many economic applications use compact but not convex games (for instance finite games), and it is hence interesting to focus on the meaning of Proposition ?? for such a game when mixed strategies are allowed (i.e. probabilistic play). A compact game will be a game where the sets of possible actions for any player are compact (\(S_i\) are compact subsets of normed \(\mathbb{R}\)-vector spaces), but may not be convex. All other assumptions from the previous section remains true. As the sets of possible actions are not convex, we shall extend in a very classical manner the strategy spaces from pure strategies to mixed strategies, i.e. probabilities on the set of possible actions. For any player \(i\), \(\Sigma_i\) will be defined as the set of probability measures on \(S_i\), which is assumed to be equipped with the borelian \(\sigma\)-algebra, denoted \(\mathcal{B}(S_i)\). In order to ‘convexify’ strategy spaces, we shall use a simplified version of the concept of "mixture", defined by \(m_a\): the "mixture" of two probabilities is an operation denoted \(m_a\), \(\forall a \in [0,1]\), such that \(\forall \sigma_i, \sigma'_i \in \Sigma_i\), \(m_a(\sigma_i, \sigma'_i) = \sigma^*_i\) is such that \(\forall A \in \mathcal{B}(S_i), \sigma^*_i(A) = a\sigma_i(A) + (1-a)\sigma'_i(A)\). It is easy to check that for all \(\sigma_i, \sigma'_i \in \Sigma_i\), \(m_a(\sigma_i, \sigma'_i)\) is a probability measure on \((S_i, \mathcal{B}(S_i))\), and is thus an element from \(\Sigma_i\).

\((\sigma_1, \sigma_2, \ldots, \sigma_i, \ldots, \sigma_n) \in \Sigma\) is a mixed strategy profile of the game. Similarly as in the previous paragraph, this profile will be referred to as \((\sigma_i, \sigma_{-i})\), when focusing on player \(i\) and the corresponding sets of possible strategies are the following ones: \(\Sigma\) is the set of all possible \(\sigma\), \(\Sigma_i\) the one of all possible \(\sigma_i\), and eventually \(\Sigma_{-i}\) is the set of profiles whose components are \(\sigma_j\) with \(j \neq i\) (i.e. \(\times_{j \in N - \{i\}} \Sigma_j\)). Individual response definition needs to be slightly modified in the following way: For any individual \(i\), the choice procedure correspondence is defined as \(R_i : \Sigma \Rightarrow \Sigma_i\). Consequently, all properties of behaviors should hold on \(\Sigma\) and not only on \(S\). In this case, the existence of a mixed strategy equilibrium can be shown.

**Proposition 3** Given any compact game with mixed strategies, if, for any player \(i\), the choice procedure correspondence satisfies properties ?? and ?? on \(\Sigma_i\), then exists a "behavioral" mixed strategy equilibrium.

**Proof.** For any \(i\), \(\Sigma_i\) equipped with the narrow topology is compact as \(S_i\) is compact and \(E\) is a metric space, being an inner product vector space, by ?, th. 60). Furthermore, \(\Sigma_i\) is convex. So a compact game extended with mixed strategy is a specific case of compact and convex game, hence proposition ?? applies.

\(\square\)

As a consequence, any finite game has a behavioral equilibrium in mixed strategy, provided that players are filled with decision patterns that follow properties ?? and ???. In this context, bounded rationality processes seem especially suitable because of the overwhelming evidence indicating that human subjects handle probabilities, in terms of preferences over lotteries or in terms of belief updating, only very imperfectly (??). Similarly, it may especially suit complicated
games, where immediate rational behavior seems unlikely. Thus an application can be envisioned in general equilibrium theory, although this would require to extend the proposition to social games.

5 Social Games and General Behavioral Equilibrium

Social games (?) (also called abstract economies) are games where the strategy is not held constant with others’ choices. More precisely, the correspondence determining the available subset of strategy space from the players’ choices is continuous and compact and convex valued. It is thus possible to show that the existence of an equilibrium in this case. Using the denotations as in proposition ??, and assuming that the constraint correspondence is $C_i$ for agent $i$ with $C_i : S \rightarrow S_i$, we have the following result:

**Proposition 4 (Equilibrium in a Social Game)** If $C_i$ correspondences are convex, compact valued and upper hemi-continuous, and $R_i$ correspondences satisfy property ?? and ??, then exists an equilibrium of the game.

**Proof.** See Appendix.

5.1 General Equilibrium with relative prices

Such a result can then be applied to a general equilibrium model. In order to do so, some additional denotations are needed. A population $I$ of consumers, with $\#I = n$, whose individuals will be indexed by $i$, interact with a population of suppliers $J$, with $\#J = m$, from which a generic individual will be referred to as $j$, around a set $L$ of commodities, $\#L = l$, indexed by $h$. Endowments are denoted $w^h_i$, which all belongs to the positive real line. Individual demand from agent $i$ in commodity $h$ is denoted $q^h_i \in Q^h_i \subset \mathbb{R}$. Very classically, $q^h = \sum_{i \in L} q^h_i$, and $w^h = \sum_{i \in L} w^h_i$. $q_i$ and $w_i$ will be the following vector of quantities of agent $i$. $X = \times_{i \in I} X_i$. For supplier $j$, $Y_j$ will be its production set, a subset of $\mathbb{R}^l$, with $Y = \times_{j \in J} Y_j$. $\alpha^i_j$ represents the share of consumer $i$ in the profits of supplier $j$.

In all these denotations, $s$ will refer to supplies from the population $j$ similarly as $q$ refers to demands for $I$ (for instance $s^h_j$ refers to supply by $j$ in the $h$th commodity).

Prices will be considered as exogenous here, as it is generally the case in general equilibrium theory. They are referred to as the following profile $p = (p_h)_{h \in L}$. Since only relative prices matter in this section, $p$ will be, in a very classical way, normalized, i.e. $\sum_{h \in L} p_h = 1$. The set of normalized prices is thus $\Delta_l = \{p \in \mathbb{R}^l \mid \sum_{h \in L} p_h = 1\}$

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2 This does not rule out the possibility of endowment effect, status quo effect, etc... but they are supposed to happen on relative prices.
Behaviors of agents will be modeled by correspondences depending on (others’ as own) individual demands or supplies, and prices. For any consumer \(i\), \(R_i : X \times Y \times \Delta_i \Rightarrow X_i\). Similarly, for supplier \(j\), \(R_j : X \times Y \times \Delta_j \Rightarrow Y_i\). Note that \(E’\) will be here the dual of \(\mathbb{R}^{ln} \times \mathbb{R}^{lm} \Delta_l\). As in the behavioral equilibrium frame, \(R_i\) (resp. \(R_j\)) will indicate which moves are suggested or requested for agent \(i\) (resp. \(j\)) when in the situation \(q \in X\), \(p \in \Delta\), \(s \in Y\). It can thus be the case that agents’ consumption choices depend on others’ as well as prices, allowing phenomena such as social influence, imitation, status-seeking, endogenous social norms, conspicuous or positional consumption (?), etc. It is also the case that individual demand can depend on previous situations, as initial endowment, thus this frame also allows phenomena such as endowment or status quo effect, as referred to in the previous chapter. Symetrically, it does not require that suppliers maximize profits or behave only in taking prices into account, they can indeed decide which level of production by reference to level of consumptions, competitors’ supply, etc. In addition, they can have some non-standard preferences such as maximizing market shares, sales, etc instead of profits.

This allows to describe the feasibility constraints. For consumers, it is linked to prices and profits through incomes, and can be summed up for agent \(i\) with the following inequality \(p.q_i \leq p.w_i + \sum_{j \in J} \alpha_{ij} p.y_j\). Note that \(p.y_j\) is simply the profit made by some supplier. So behaviors necessarily satisfy some individual feasibility conditions by the income inequality. \(F_i(q, p, s) = \{q_i \in X_i : q_i.p \leq p.w_i + \sum_{j \in J} \alpha_{ij} p.y_j\}\). Since suppliers do not necessarily maximize their profits, a feasibility constraint is also necessary, which states that they should have positive profits, which in other words means that the value of their product should be at least equal to the value of their costs. Formally, for any supplier \(j\), \(F_j : \Delta_l \times X \times Y_i \Rightarrow Y_j\), \(F_j(q, p, s) = \{s_j \in Y_j : p.y_j \geq 0\}\).

It is also possible, using classical assumptions, to show that the set of attainable states have the following properties, denoting for any set \(E\) by \(AE\) its asymptotic cone, that is, in the case of closed convex sets (see ?), the sets of all limits from sequences of the form \(\{\lambda_n x^n\}\), where each \(x^n\) is in \(E\) and \(\lambda_n\) tends to 0, and \(Y_j\) being the production sets of supplier \(j\) with \(Y = \times_{j \in J} Y_j\). Moreover, we will denote the set of attainable states \(F\), i.e. the set of \((q, y)\) with \(y \in Y\) such that for all \(q - y - w = 0\), and the projection of \(F\) on some \(X_i\) or \(Y_j\), \(\tilde{X}_i\) or \(\tilde{Y}_j\). Then the following result is a classical statement:

**Lemma 1** (?). If the economy is such that, for all agents:

1. For any \(i\), consumption sets \(X_i\) are closed, convex and bounded from below, and \(w_i \in X_i\)

\(^3\)It may be useful to postulate that feasibility does not only depend from prices, for instance in taking into account excess or shortages.
2. For any \( j \), Production sets \( Y_j \) are closed and convex, and \( 0 \in Y_j \).

3. \( AY \cap \mathbb{R}_n^+ = \{0\} \)

4. \( Y \cap -Y = \{0\} \)

Then, the \( F \) is compact and non empty and for any \( j \), \( 0 \in \hat{Y}_j \).

**Proof.** See (\( ? \), p. 77-78, or (\( ? \), p. 97).

In order to extend the set of possible behaviors, properties of reaction correspondences can be easily rephrased from the behavioral equilibrium canvas to the general equilibrium one:

**Property 5 (General Equilibrium Irreflexivity)** For any \( i \in I \) and \( j \in J \), \( \forall p, q, s \in \Delta_l \times X \times Y \), \( q_i \notin R_i(q, p, s) \), and \( s_j \notin R_j(q, p, s) \)

The local non-opposition of direction can be formulated in terms of individual demand given some prices:

**Property 6 (LNOD within Individual Demand Procedure)** \( p, s \) being set, for any \( q \in X \) such that \( R_i(q, p, s) \neq \emptyset \), exists a neighborhood \( V_q \) of \( q \) and a direction \( d^q_i \in E' \), such that for any \( q' \in V_q \) and any \( q'^* \in R_i(p, q', s) \) with \( \langle d^q_i, q'^* - q' \rangle > 0 \).

The same can be put forward for suppliers’ behaviors:

**Property 7 (LNOD within Individual Supply Procedure)** \( p, q \) being set, for any \( s \in Y \) such that \( R_j(q, p, s) \neq \emptyset \), exists a neighborhood \( V_s \) of \( s \) and a direction \( d^s_j \in E' \), such that for any \( s' \in V_s \) and any \( s'^* \in R_j(p, q, s') \) with \( \langle d^s_j, s'^* - s' \rangle > 0 \).

As a consequence, actual behaviors will depend on both the actual choice correspondence \( R_k \) and the feasibility one \( F_k \), for any \( k \), being a consumer or a supplier. \( R_k \) will simply be the procedure used to make a decision, would the agent have no constraint, whereas \( F_k \) is simply the feasibility correspondence, taking into account budget constraints for consumers of production sets for suppliers. A sound individual choice – demand or supply – at price \( p \) will thus be a \( q_i \) (resp. a \( s_j \)) such that \( R_i(q, p, s) \cap F_i(q, p, s) = \emptyset \) (resp. \( R_j(q, p, s) \cap F_j(q, p, s) = \emptyset \)). By a slight abuse of notation the correspondence defined for any \( (p, q, s) \) by \( R_i(p, q, s) \cap F_i(q, p, s) \) will be referred to as \( R_i \cap F_i \) (the same holds for the set of suppliers). A general equilibrium will be a situation \( (q, p, s) \in X \times \Delta_l \times Y \) such that for all \( k \in I \cup J \), \( R_k(q, p, s) \cap F_k(q, p, s) = \emptyset \).

\[4\text{Technically it could be referred to as a free disposal equilibrium, but it does not make much sense since agents are not necessarily equipped with monotonic preferences.}\]
Proposition 5 (Existence of a Behavioral General Equilibrium) If the conditions of lemma ?? hold and $R_k$ satisfies, for all agents, properties ?? and ?? or ??, then exists an equilibrium of the economy.

Proof. See Appendix.

□

This equilibrium concept does not require much from agents behavior – consumers as well as suppliers – since no preference, as weak as they can be in some models, no rationality, or no general theory of decision making is explicitly or implicitly postulated. Thus, this model can easily grasp many behaviors which are documented in the literature: endowment of status quo effect on the basis of initial endowments, social influence in consumption, various heuristics and routines (for instance maximization of market shares instead of profits, or to give some crude example maximization of the executive team earnings)... Such a model has also the interest that it does not require agents to have perfect information, for instance about products. They may assess wrongly the quality of some products, or they may have only a very fuzzy beliefs about this qualities, and for instance, they can react to prices, or other consumers’ consumption as a signal for quality (perhaps wrongly). In general, the set of information necessary for agents to take a decision is incredibly lower. Agents merely need to adapt in some way to the evolution of prices, others consumption and supply, which seems to be the least such a model can ask for.

5.2 Behavioral general equilibrium with nominal prices

One of the main limit of the previous model is the fact that it deals with relative prices, in order to allow their normalization. Whereas this assumption is comfortable in the modeler’s viewpoint, it is equivalent to hypothesize some rationality about prices from the agents, since this means they do not rely on cognitive short cuts in this matter. It has been argued for long and in a very classical manner that in many situations agents evaluate economic decisions in nominal terms, for instance worker with their salaries (?). ? have recently revived this idea in setting up an experiment to test such an effect. Their conclusion does support the idea that subjects mostly address economic decisions in nominal terms. These are good reasons to try to consider a model of general equilibrium, concerned by almost arbitrary behaviors as in the previous section, but this time with prices playing their roles on the nominal basis.

So, on the contrary to assumptions commonly made in the relevant literature and in the previous section, prices $p$ should here belong to $\mathbb{R}^l$. This assumption departs from standard assumptions in this kind of models: prices are not normalized in order to allow money illusion, which implies that nominal prices may play a role. For instance $\lambda p$ may imply for a given agent different individual demands than $p$ (first degree homogeneity is thus abandoned). This allows behaviors, including suppliers’ one, to depend on absolute level of prices.
But appealing to \(\mathbb{R}^l\) raises many issues in formal terms, especially the one that the set of possible prices is not compact anymore. A possible assumption to go beyond this hindrance can be reached by assuming that some finite level of absolute price cannot be passed for any commodity, for instance \(P\). A reason for such a phenomenon could be that too high nominal values bring about high cognitive or transaction costs. Thus, such an assumption can be justified by some hypothesis that there exists a scalar \(\lambda\) large enough such that given any price vector \(p\) in \(\mathbb{R}^l\), the behaviors of all agents remain the same if prices become \(\frac{1}{\lambda}p \in [0, P]^l\). This is not to say that only relative prices matter, but that exists some divider – presumably a round number such as 100, 1000 etc. – such that it does not affect behaviors\(^5\). This implies that given a price vector in \(\mathbb{R}^l\) with some given behaviors, there exists a price vector in \([0, P]^l\) such that the behaviors are the same. As a conclusion, to restrict oneself to \([0, P]\) does no imply that actual prices cannot belong to \(\mathbb{R}^l\).

The same argument can be put forward in order to justify the fact that no price below some threshold should be used, for instance \(\frac{1}{100}P\) if \(P\) is large enough, if indeed exists such a divider so that agents’ behaviors are not affected by the division of nominal prices, then it is possible to restrict the analysis to \([\frac{1}{100}P, P]^l\). This does not influence the fact that some equilibrium can exists outside this interval, but it is much more comfortable for the formal analysis of the model\(^6\). And it seems that such hypotheses can be seen as realistic in the sense that under some minimal price transaction costs overpass the price of the commodity and thus render irrelevant such an exchange and as suggested previously since handling too high figures generate cognitive costs that are often dealt with by changing the currency unit.

Hence since \([\frac{1}{100}P, P]^l\) has the necessary property to apply a proof similar as the one of proposition \(\text{??}\), the following proposition is straightforward:

**Proposition 6 (Existence of a Behavioral General Equilibrium with nominal prices)**

If the conditions of lemma \(\text{??}\) hold and \(R_0\) satisfies, for all agents, properties \(\text{??}\) and \(\text{??}\) or \(\text{??}\) and \(\text{??}\) or \(\text{??}\) and \(p \in [0, 1]^l\), then exists an equilibrium of the economy.

**Proof.** Straightforward, with the better reply correspondence of the auctioneer being slightly modified into: \(R_0(p, q, s) = \{p' \in \Delta_l | \frac{1}{|p'^l|}p'^l (q - s - w) > \frac{1}{|p|^l} p(q - s - w)\}\).

\(\Box\)

\(^5\)An example can be given by the changeover from old francs to new francs in France in 1960, or the fact that in Italy the currency unit was not \textit{de facto}, before euro was issued, lira but thousands of them.

\(^6\)On the one hand, \(P\) as a maximum price solves the question of the compacity of the strategy set of the auctioneer, and on the other hand, setting a minimum price at \(\frac{1}{100}P\) prevents some non-well behaved behaviors of the auctioneer as well as the agents when prices are null.
This results substantially weaken, in $\mathbb{R}^n$ at least, the behavioral conditions to have an equilibrium which can be summed up in the two assumptions: behaviors have to be feasible, and should be decided according to some locally consistent procedure. Nothing can be said about welfare in this frame as preferences might not be defined for some, if not all, agents.

6 Possible Extensions and Relaxations of Hypotheses

On a general note, it seems quite difficult to weaken the properties hypothesized for the choice procedure correspondence. The three of them are necessary for the existence in the general case: Examples of non-existence of equilibrium can be found once one of the properties is dropped.

Proposition 7 Properties ?? and ?? (resp. ?? and ??, or ?? and ??) are necessary for the existence result of Proposition ?? (resp. Propositions ?? and ?? and ?? or ??).

Proof. Counter-examples will be given in the interactive frame, but it is easy to adapt the examples to individual decision frame, for instance in considering that both strategy spaces correspond to two different dimension on some individual choice. Not assuming irreflexivity, may lead to the fact that there is no satisfactory response to some situation. It is formally possible to give a counter example, even though it has no economic sense.

Example 1 Within a range of possible actions between $[0, 1]$, defining $R_i(s) = \{s_i\}$ for all $s$ guarantees that there is no $R_i(s) = \emptyset$. Clearly property ?? holds and no equilibrium exists.

Similarly, it is easy to find a counter-example if property Local Non-Opposition of Directions (properties ??, ??, ??) is dropped.

Example 2 In a 2-player game, whose strategy spaces are $[0, 1]$, first agent play according to the following correspondence:

- for all $s \in S$ with $s_2 \leq \frac{1}{2}$ and $s_1 \neq 1$, $R_1(s) = \{1\}$
- for all $s \in S$ with $s_2 > \frac{1}{2}$ and $s_1 \notin \{0\}$, $R_1(s) = \{0\}$
- $R_1(s) = \emptyset$ otherwise

Taking $R_2$ defined by $R_2(s) = \{s_1\}$ for any $s \in S$ with $s = (s_1, s_2)$ and $s_1 \neq s_2$; $R_2(s) = \emptyset$ otherwise. Response correspondences are irreflexive. Equilibrium clearly does not exist.

\[\square\]

Yet, there is no reason why generalization to more general spaces should not occur, as far as $E$ is a locally convex Hausdorff etc... space with inner product (thus a metric space). See ?? for some generalization.
Yet, in order to encompass some additional and documented cases, one can relax properties ??, ??, ?? by simply requiring that the procedure used by agents contains a subprocedure that satisfy local non-opposition of direction. Indeed, it weakens fundamentally this requirement by only demanding locally consistent procedural path to be possible and not compulsory. This can be formulated in the following terms, using the denotations of the individual decision frame:

**Property 8 (Weak LNOD)** A procedure of choice $R: A \Rightarrow E$ is said to be weakly LNOD if exists $\hat{R}: A \Rightarrow E$ with $\hat{R}$ is LNOD as in property ?? for any $a \in A$ with $R(a) \neq \emptyset$ and $\hat{R}(a) \subset R(a)$ and $\hat{R}(a) \neq \emptyset$ in $a$.

This allows us the following extended result:

**Proposition 8 (Decision with weak LNOD)** If $R$ satisfies properties ?? and ??, then $C$ as in Proposition ?? is non empty-valued.

*Proof.* Proof is straightforward, in defining $\hat{R}': A \Rightarrow E$ with $\hat{R}'(a) = \hat{R}(a) \cap R(a)$. $\hat{R}'$ has thus an equilibrium, using the same proof as in proposition 1, and so has $R$ evidently.

The adaptation to the strategic context as well as general equilibrium one is straightforward:

**Proposition 9 (Equilibrium with weak LNOD)** If $R_i$ correspondences satisfy properties ?? (or ??) and ??, then exists an equilibrium of the game, be it for a compact and convex game, a compact game with mixed strategies or a social game, as defined in Propositions ??, ??, ?? and ??.

**Corollary 1 (Equilibrium in a Social Game)** If $C_i$ correspondences are convex, compact valued and upper hemi-continuous, and $R_i$ correspondences satisfy property ?? and ??, then exists an equilibrium of the game.

*Proof.* Same proof as the one of proposition ??.

An immediate extension to this proposition is to assume that instead of LNOD, only weak LNOD holds. This would give us the following proposition, which may be useful when addressing some specific behaviors:

**Corollary 2 (Existence of a Behavioral General Equilibrium with Weak LNOD)** If the conditions of lemma ?? hold and $R_k$ satisfy, for all agents, properties ?? and ?? for individual demands and supplies, then exists a free disposal equilibrium of the exchange economy.

*Proof.* The proof is straightforward from the proof of proposition ??.
Similarly, the following corollary can be spelled out:

**Corollary 3 (Existence of a Behavioral General Equilibrium with Weak LONDON in nominal prices)**

If the conditions of lemma ?? hold and $R_k$ satisfy, for all agents, properties ?? and ?? for individual demands and supplies and $p \in [0, 1]$, then exists an equilibrium of the economy.

The following corollary ensues from such an extension:

**Corollary 4** For any compact and convex set of alternatives (resp. convex and compact game, compact game with probabilistic play, social game), if at least one of following conditions on the behavior of the agent (resp. any agent $i$) holds for any $a$ such that $R(a) \neq \emptyset$ (resp. for any $s$ with $R_i(s) \neq \emptyset$), in addition with irreflexivity, then the choice function is non-empty (resp. exists a behavioral equilibrium):

1. $R_i$ is a continuous mapping.
2. $R_i$ is a closed and convex valued lower hemi-continuous correspondence\(^8\).
3. $R_i$ is a closed and convex valued upper hemi-continuous\(^9\) correspondence.
4. $R_i$ is convex and closed valued, and its lower inverse correspondence is open for any $y \in A$\(^10\).

**Proof.** See Appendix.

Such a corollary is interesting for economics in the sense that it allows formal heterogeneity between different players, not only in terms of preferences, and that such a diversity of players has no effect on the existence of the equilibrium. Examples of such a the possible heterogeneity of behaviors in such a frame is given in section 8.

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\(^8\)A lower hemi-continuous (lhc) correspondence $F$ is such that for any $x$ on its domain, for any open set $V$ that intersects $F(x)$ (i.e. $V \cap F(x) \neq \emptyset$), there exists a neighborhood $U$ of $x$ such that $F(x') \cap V \neq \emptyset$ if $x' \in U$.

\(^9\)For a definition of this very standard concept, see ?? or ??) for definitions in relation with economic applications or more technically ??). Let us just recall the general definition: An upper hemi-continuous (uec) correspondence $F$ is such that for any $x$ on its domain, for any open set $O$ that contains $F(x)$, there exists a neighborhood $U$ of $x$ such that $F(x') \subseteq O$ if $x' \in U$. Upper hemi-continuous correspondence are extremely important in economics for, by Berge’s maximum theorem, the maximization of some continuous function results in upper hemi-continuous choices, in many formal environments.

\(^10\) $R_i^+ : A \rightrightarrows A$ with for any $A'$ subset $A$, $P^-(A') = \{x \in A : A' \cap R_i(x) \neq \emptyset\}$ is open
7 Behavioral correspondence equivalence

Indeed, propositions ??, ?? and ?? can be rephrased in more standard game theoretical terms, i.e. using the definition of equilibrium as a fixed point of some behavioral correspondence. Indeed, given some procedure $R_i$, it is easy to construct a correspondence $B_i : S \Rightarrow S$ that would be a response correspondence, not necessarily rational, and find back a more game theoretical denotation of equilibrium, which would be a play $s^*$ with $s^* \in \times_{i \in N} B_i(s^*)$. Indeed, this can be achieved by taking $B_i(s) = R_i(s)$ if $R_i(s) \neq \emptyset$, and $B_i(s) = \{s' \in S_i \mid R_i(s') = \emptyset\}$ otherwise. This gives us the following result:

Remark 1 If $s^*$ is such that $R(s^*) = \emptyset$ then $s^* \in B(s^*) = \times_{i \in N} B_i(s^*)$. Properties ?? and ?? on $R_i$ imply non-empty and closed values, and local non-opposition of directions for all $s \in S$ with $s_i \notin B_i(s)$, for $B_i$.

Proof. The first proposition of the remark is by construction. Non-empty value for $B_i$ is obtained using an argument similar to the proof of Proposition ??, using the correspondence $R_i'$ when some $s_{-i}$ is fixed. Indeed, for $R_i'(\cdot, s_{-i}) : S_i \Rightarrow S_i$, exists a $s^*_i \in S_i$ which is such that $R_i'(s^*_i) = \emptyset$ – same reasoning as at the end of the proof of proposition ?? . Closed-valuedness is immediate due to the fact that $\{s \in S \mid R_i(s) = \emptyset\}$ is closed (by Property ??), as well as $S_i$. Local non opposition of directions is also straightforward, due to the fact that the conditions only hold for $s_i \notin B_i(s)$.

\[ \square \]

Reversely, one can construct from any behavioral response $B_i$, a corresponding procedure $R_i$, by taking $R_i(s) = \emptyset$ if $s_i \in B_i(s)$, and $R_i(s) = B_i(s)$ otherwise.

Remark 2 If $B_i$ is non-empty, has a closed graph, and LNOD holds for all $s \in S$ with $s_i \notin B_i(s)$, then $R_i$ satisfies properties ?? and ?? .

Proof. Irreflexivity is by construction. Existence of a direction and local non-contradiction of directions are straightforward from the definition, once again due to the fact that the direction conditions do not hold if $s_i \in B_i(s)$ neither if $R_i(s_i) = \emptyset$. The closed graph of $B_i$ means that $\{(s_i, s) \in S \times S_i \mid s_i \in B_i(s)\}$ is closed, so is $\{s \in S \mid s_i \in B_i(s)\} = \{s \in S \mid R_i(s) = \emptyset\}$. That is required to have Property ?? properly defined.

\[ \square \]

An immediate interpretation of these two remarks is that approaching conducts in terms of choice procedures or in terms of behavioral response is almost\textsuperscript{11} equivalent. Thus, the procedure choice canvas can virtually describe any behaviors for which LNOD holds. This possibility may be a crucial feature since

\textsuperscript{11}The requirement of a closed graph instead of closed values in remark ?? precludes complete equivalence. From an empirical or practical point of view, this difference does not seem essential though.

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it allows to work on cognitive procedure of choices as well as on behavioral patterns. This canvas is thus neutral on the question of the nature of human decisions or choices. For instance, it allows instinctive or practical unconscious rules of thumb, as well as sophisticated cognitive procedures. For properties are almost equivalent in both cases, it also permit to test the given properties on behaviors, which are observable by nature, and to infer whether these relevant properties hold for procedures of choice.

8 A wide range of possible Behaviors

In the following part, we will only consider (except when explicitly told otherwise) correspondence in terms of acceptable behaviors given some play. In order to cover some standard cases of economic theory, we need to focus on the specific cases of continuous mappings, as well as hemi-continuity – lower as well as upper. This is the topic of the following corollaries.

The classical preference optimization framework can be seen as a special case of such a procedure: given any situation \(a\), agents choose among the set of best/maximal elements. Similarly better-reply procedure can be immediately described in this canvas: given any situation \(a\), agents choose among the set of element preferred to \(a\). More generally, the image set from the procedure can simply be seen as the set of situations preferred to the current one. This is in line with preference models by \(\) and \(\), who have laid out a general canvas, where neither intransitivity nor completeness are required. There, preferences are described using a correspondence with some particular properties. Given some compact and convex set of alternatives \(A\), agent prefers \(x\) to \(y\) if \(y \in P_x(x)\). Following their canvas, it is possible to define a class of intransitive and/or incomplete preferences, through the better-reply correspondence.

**Definition 2 (Better Reply Correspondences)** \(P\) is a correspondence with \(P : A \rightrightarrows A\) with \(\forall a \in A, a \not\in P(a)\).

**Definition 3 (Weak LNOD Better-Reply)** \(P\) is a weak LNOD better-reply correspondence if \(P\) is a better-reply correspondence and Weak Local Non-Opposition of Directions hold (Property ???).

**Corollary 5** Agents with better-reply correspondences such that properties ?? and ?? have a non-empty choice function, for any compact and convex non-empty set of alternatives. This includes slightly modified ??? better-reply correspondence\(^{12}\), and hence complete, transitive, convex and continuous binary relations (“ordered preferences”).

**Proof.**

\(^{12}\)Which are such that (a) \(P(x)\) is convex for all \(x \in A\), and (b) the lower inverse correspondence \(P^-(\{y\})\) is open for all \(y \in A\), i.e. \(P^- : A \rightrightarrows A\) with for any \(E\) subset \(A\), \(P^-(E) = \{x \in A : E \cap P(x) \neq \emptyset\}\) is open. We add the following technical property – in order to avoid some pathological cases: (c) \(P(a)\) is either closed or open.
The first part of the corollary results from the direct application of corollary ??, 4, assuming \( \forall a, R(a) = P(a) \).

For complete, transitive, convex and continuous binary relation, the result ensues from Gale and Mas-Colell better-reply correspondences which are a generalization of such preferences as binary relations (?).

\[ \square \]

As a conclusion, a large set of very different behaviors/procedures of choice can be grasped within this model. It is worth noting that many of such behaviors seem to be observed in some situations.

**Remark 3** In the case of a compact game with mixed strategies RRR, corollary 2 RRR includes, among others, behaviors such as:

1. other regarding preferences represented by some continuous social (expected) utility function as in (?) and some adapted version of (?)

2. generalized expected utility maximizers where preferences satisfy betweenness such as in weighted utility theory (?), in implicit expected utility or implicit weighted utility theories (?), in regret theory (?), in disappointment theory (?), or in skew-symmetric bilinear preferences (?).

3. behaviors based on non-binary choice rules as in ?)

4. almost rational player as in (?)

5. procedure based players such as in (?)

6. patterned behaviors, as for instance bias and heuristics based choice, or emotionally perturbed behaviors.

**Proof.** Continuous maximization of utility implies that behavior correspondence has a closed graph by Berge’s theorem. For (i) expected utility implies convex-valuedness as in ?). Betweenness (see definition in ?)) also implies convex-valuedness straightforwardly. So (i) and (ii) are specific cases of Corollary ??, (iii). (iv) and (v) imply continuous mapping behaviors as shown in the relevant articles, and are thus specific cases of Corollary ??, (i). (iii) implies uhc and convex correspondences as proved in ?), and are hence covered by Corollary ??, (iii).

(iv) and (v) follow from (iii): ?) maximum theorem implies upper hemi-continuity and closed-valuedness of the correspondence, the definition of quasi-concavity implies its convex-valuedness. The same corollary holds in the case of compact games with mixed strategies, if in (4) player \( i \) maximizes expected payoffs, and in (5) the utility function is quasi-concave in mixed strategies.

\[ \square \]

Generally speaking, this frame captures a broad set of possible behaviors, which admits many different determinations, rational optimization being a special case.

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9 Necessary conditions

Although it has not been demonstrated that properties ?? and ?? (or ?? and ?? or ?? and ??) could not be weakened with the existence results still holding, it seems from corollary ?? that many possible directions are dead-ends. Indeed, the reference to the major fixed point theorems such as ?, ?, or ? does not bring any further generality: upper-hemi continuous and closed and convex-valued correspondence are a special case of propositions ?? and ??, as well as lower hemi-continuous and "Browder-continuous" closed and convex valued ones.

More fundamentally, it is possible to show that if weak local non-opposition of directions do not hold for some correspondence \( R \) at some point, then it is possible to build other correspondence equivalent to \( R \) near \( a \) and corresponding to a standard case for the rest of the domain, which does not lead to any decision (or any equilibrium). This means that if we require that any correspondence constructed by components of acceptable correspondences should be such, then the set of acceptable correspondence is the weakly LNOD one. To state this, more formally, we can define component based behaviors. The idea is to say that if a set of correspondences are possible, then any correspondence based on components should be also possible. A component-based correspondence is defined as such:

**Definition 4 (Component based behavior)** Given a set of possible correspondences \( C \), from \( \Sigma \) into itself, then a component based correspondence \( \phi \) is a correspondence such as exists a finite cover of \( \Sigma \) by a family of closed sets, say \( \{ N_k \}_{k \leq K} \), such that for any such \( k \), exists a correspondence \( \phi_k \in C \) such that for all \( x \in N_k \), \( \phi(x) = \phi_k(x) \).

This means that, finitely, the \( \phi \) correspondence can be constructed by component of acceptable correspondence defined on some closed sets. For instance it can mean that if on some closed local area a behavior is acceptable in some correspondence, there is no reason for it not to be, if the correspondence is defined in another manner on the rest of its domain. Because of the formal equivalence between behavioral rules and process, we will restrict our attention to behavioral rules and show that there is no fixed point.

**Proposition 10** If the set of acceptable correspondences contains a subset with weakly LNOD ones, and one which is not and is non-empty and closed-valued and which is such that where it is not weakly LNOD, no fixed point is in the closure of its graph\(^{13} \), then exists a component based correspondence for which no equilibrium exists.

**Proof.**

\(^{13}\)This means in plain terms, that on some neighbourhood of a where this correspondence is not weakly LNOD, then the image set is not arbitrarily close to a fixed point.
For the sake of simplicity, we will only focus on correspondence from \([0, 1]\) to itself, but the demonstration can be straightforwardly extended. Any \(f\) not being weakly LNOD implies that exists \(a\), such that exists \(a'\) in any neighborhood of \(a\), such that there is no \(p \in E'\) with for some \(a^* \in f(a)\) and \(a^{**} \in f(a')\), \(\langle p, a^* - a \rangle > 0\) and \(\langle p, a^{**} - a' \rangle > 0\). Considering the closure of the graph of \(f\) and define \(\bar{f}\) the corresponding correspondence. We have by definition \(Grf \subset Gr\bar{f}\).

Thus, it implies, since we are in a metric space being in a inner product vector space, then exists some \(\bar{a}^{**} \subset \bar{f}(\bar{a})\). Since we restricted ourselves to \([0, 1]\) and as there are only two possible directions, it implies \(\langle a^* - a, \bar{a}^{**} - a' \rangle < 0\). It is thus easy to construct, outside \(\{a\}\) a mapping \(\hat{f}\) continuous on \([0, 1] - \{a\}\) with no point such that \(x_i \in \hat{f}(x)\), taking for instance \(\hat{f}(x) = a^*\) on \([0, a]\) and \(\hat{f}(x) = a^{**}\) on \([a, 1]\) if \(a^* < a^{**}\) (or the reverse if the contrary holds). In order to conclude, assume that there is another player whose behavioral correspondence is simply \(g(x) = x\), this evidently satisfies weakly LNOD, then there is no equilibrium.

PNM of the definition of strategic interaction of the correspondences here since \(i\) and \(-i\) are involved.

Consequently, relaxing the weak LNOD condition renders possible the construction of some standard case – continuous mapping, uhc and convex and closed valued correspondence – except near \(x\), such that there the choice correspondence is empty valued (or no equilibrium exists). Thus, if it is required that any correspondence made of components of other acceptable correspondence should be acceptable, then weak LNOD appears to be necessary.

**Corollary 6** The only set of acceptable correspondences which is closed under component composition and which is such that for any such correspondences there exists a non empty choice function or an equilibrium in a game canvas, is the set of all irreflexive and weakly LNOD correspondences.

## 10 Conclusion

This theoretical canvas seems to have many interesting features, especially when focusing on its compliance with empirical findings, especially within the growing subfield of “behavioral economics”. Indeed, it is consistent with the idea that behaviors may be the results of some sequential processes, rather than simply being an immediate best-reply, fitting well the idea that human subjects adapt very progressively and gradually to situations. It also allows a wide spectrum of different driving forces of behaviors, a phenomenon which appears to be well documented, and do not presume these motivations should follow some \textit{a priori} pattern, as for instance in the case of preferences. Besides, this model permits to apprehend interactions of formally heterogeneous agents: hyper-rational ones with weakly or slowly adaptive ones, sequential decision-makers with best-responders, other-regarding ones with purely self-interested subjects, agents with preferences-based motivations with players driven by heuristics,
social norms, emotions, etc. All these types of agents can be captured in the model and thus do not influence the existence of an equilibrium, as far as they exhibit the minimal consistency required in the assumptions of the model.

This heterogeneity does not only concerns some aspects of the agents (utility functions) but the very core of their behavior determination and shape. It thus seems to be a relevant tool to address the general issue of studying interactions of radically heterogeneous agents, which appears to be the case in most markets (firms/consumers), labor relations (workers/firms), or stock exchange (agents with various levels of rationality).

Besides, this canvas can be seen as the formal translation of a very general view of human beings based on the idea that humans are mainly characterized by their behavioral as well as cognitive plasticity. The very simple hypothesis is to assume that human beings are "blank slate" (?), and may learn any behavioral pattern, any cognitive heuristics, any moral principle or any preference. These individual properties can be the result of idiosyncratic processes as well as social determinations. This view find echoes in neuroscience development, which suggests that one of the main properties of brain is its plasticity (?), and especially in connectionist models for human decision.

Moreover, human beings’ decision may, within individuals, be characterized by a plurality of driving forces depending on the circumstances: an agent may try to maximize its incomes in some situations, while taking into account some moral principles in some other, she may have concerns about others’ comfort in some interactions (simple games with few players), while not even seeing them in some other (market situations); or she may be extremely rational in some very important one-shot choice whereas she may rely most of the time on not so relevant routines in usual situations, and so on. Heterogeneity of conducts can thus be found within individuals, requiring a frame that can grasp such a variety of technical properties.

But, generalizing acceptable behaviors to almost anything with some minimal consistency reduces the predictive power of the theory or at least its precision. Nevertheless, it might be misleading to believe that the standard framework of utility maximization is a prediction-oriented tool, as (?) pointed out. It is indeed always possible to render some behavior consistent with some preference relation and/or some beliefs. And, most of the efficacy of such theoretical tools comes from auxiliary assumptions: preferences are convex, monotonic, self-interested, for instance, not even mentioning assumptions on beliefs, priors and common knowledge of rationality. Similarly, it should be possible to ‘fill up’ or to specify behaviors, in order to have more accurate and precise predictions, but this is mainly an empirical matter. The question to know how agents specifically behave, on which situations and contexts, as well as the explaining factors for these specific conducts, is then an empirical issue to tackle with appropriate means.
And, measuring behaviors requires less auxiliary assumptions than measuring preferences and/or beliefs, at least in an experimental context.

11 Appendix

11.1 Proof of Proposition ??

Proof.

The proof relies on a result which is a slight extension of a fixed point theorem by ?, which states that if a correspondence \( \phi \) whose directions compose a correspondence, say \( P \), is compact valued and upper hemi-continuous on some neighborhood of any point \( x \) with \( x \notin \phi(x) \), then \( \phi \) has a fixed point. This result is here slightly extended in the sense that it is only required that \( P \) has a subcorrespondence that is compact valued and upper hemi-continuous on some neighborhood. As a reminder, a subcorrespondence \( \hat{\phi} \) of any correspondence \( \phi \) is a non-empty correspondence on the same domain such that for any element \( x \) of this domain, \( \hat{\phi}(x) \subset \phi(x) \). The proof of such an extension is almost similar to ? one, which is recalled since not classical.

Lemma 2 (Extended ?)) Let \( X \) be a non-empty compact convex subset of a locally convex Hausdorff topological vector space \( E \) having topological dual \( E' \). Let \( \phi: X \rightrightarrows X \) be a non-empty valued correspondence. If \( \phi \)’s direction correspondence has a non-empty compact valued upper hemi-continuous subcorrespondence near \( x \) for each \( x \) with \( x \notin \phi(x) \), then \( \phi \) has a fixed point.

Proof. (Adapted from ?)

Assume first that \( \phi \) has no fixed point. By definition, as there is no \( x \in \phi(x) \), there is a neighborhood \( U_x \) for every \( x \) such that exists a direction subcorrespondence which is compact valued and upper hemi-continuous on \( U_x \). Since \( X \) is compact, exists a finite subcover of \( X = \bigcap_{x \in X} U_x \). That is, exists \( n \) points \( x_1, \ldots, x_n \) and their neighborhoods \( U_{x_1}, \ldots, U_{x_n} \) such that \( \bigcap_{t=1}^{n} U_{x_t} = X \). For each \( t \), exists a non-empty compact valued upper hemi-continuous correspondence \( p^t: U(x^t) \rightrightarrows E' \) with \( \forall z \in U(x^t), \forall q \in p^t, \forall v \in \phi(z), (q, v - z) > 0 \), by hypothesis – this \( p^t \) correspondence being the subcorrespondence of the general direction correspondence. Furthermore, since \( X \) is compact, exists a partition of unity of \( X \) subordinate to \( \{U(x^1), \ldots, U(x^n)\} \), say \( \beta_t: X \mapsto [0, 1] \) with \( t = 1, \ldots, n \). For any \( z \), let \( q(z) \) be a subset of \( E' \) such that \( q(z) = \{ \sum_{t=1}^{n} \beta_t(z)p | p \in p^t(z) \} \). By construction \( q(z) \) is non-empty and compact, since \( p^t \) is such. And, \( q: X \rightrightarrows E' \) is clearly upper hemi-continuous, as \( p^t \)'s are and \( \beta_t \)'s are continuous.

Let \( \Phi \) be a correspondence from \( X \) into itself, defined for any \( z \in X \) by \( \Phi(z) = \{ y \in X | \langle q, y - z \rangle > 0, q \in q(z) \} \). By construction this correspondence
has no fixed point, otherwise \( \langle q, y - z \rangle \) would be null. Since, given some \( z \), for any \( t \) with \( z \in U(x^t) \), and any \( y \in \phi(z) \), \( \langle q^t, y - z \rangle > 0 \) for any \( q^t \in p^x(z) \), we have \( \sum_{t=1}^n \beta(z) \langle q^t, y - z \rangle > 0 \), and thus \( \sum_{t=1}^n \langle \beta(z)q^t, y - z \rangle > 0 \) due to basic properties of inner product. As a consequence, for any \( y \in \phi(z) \), \( \langle q, y - z \rangle > 0 \) for each \( q \in q(z) \). Thus, for any \( z \), \( \phi(z) \subset \Phi(z) \), hence \( \Phi \) is a non-empty correspondence, and it has no fixed point by construction.

On the contrary, \( \Phi \) is convex valued. Assume indeed that \( y_1 \) and \( y_2 \) belongs to \( \Phi(z) \), then for any \( q \in q(z) \), \( a \in [0, 1] \), \( \langle q, ay_1 + (1-a)y_2 - z \rangle = a \langle q, y_1 - z \rangle + (1-a) \langle q, y_2 - z \rangle > 0 \) by definition of inner product. In addition, given any \( x \in X \) and any \( y \in \phi(x) \), since \( y \in \Phi(x) \), we have \( \langle q, y - x \rangle > 0 \) for any \( q \in q(x) \). As \( q(x) \) is compact, exists a neighborhood \( N \) of \( x \) and \( M \) of \( q(x) \) such that \( z \in N \) and \( q \in M \) implies \( \langle q, y - x \rangle > 0 \) by the definition of the topology of compact convergence (a sequence of mappings converges uniformly to some mapping on any compact subset of its domain). In addition, due to semi-continuity of \( q(x) \), exists a neighborhood of \( x \), \( V_x \subset N \) such that \( \forall z \in V_x \), \( q(z) \subset M \). As a consequence, for any \( x \in X \) and any \( y \in \phi(x) \), exists an open neighborhood \( V_x \) of \( x \) such that \( \forall z \in V_x \), \( y \in \Phi(z) \). Thus, \( \Phi^-(\{y\}) = \{x \in X \mid y \in \Phi(x)\} \) is an open set for any \( x \) in \( X \). Hence, \( \Phi \), which is also non-empty convex valued, satisfies Browder’s theorem and has a fixed point. A contradiction.

\[ \square \]

Define \( Y \) as the subset of \( A \) such that \( Y = \{a \in A \mid R(a) \neq \emptyset\} \). This set is open from property ??. In \( A \), define the following correspondence:

- for all \( a \in Y \), \( R'(a) = R(a) \)
- for all \( a \notin Y \), \( R'(a) = A \).

On \( Y \), Property ?? ensures that the direction correspondence for \( R' \) is not empty. And also, for \( Y \), and any \( y \in Y \), exists from Property ?? a direction \( p^y \) and a neighborhood \( V_y \) where exist and for any \( y' \in V_y \), \( R(y') \) with \( \langle p^y, y' - y \rangle > 0 \). Moreover, on \( A - Y \), \( a \in R'(a) \). So \( R' \) satisfies the conditions for applying Lemma ??, since \( p^y \) composes a constant direction correspondence. Hence, \( R' \) has a fixed point, say \( a^* \). This fixed point cannot be in \( Y \) as it would require, by construction, that \( a^* \in R(a^*) \) which cannot be the case by irreflexivity (property ??). Necessarily, \( a^* \) belongs to \( A - Y \) and \( a^* \) is such that \( R(a^*) \) is empty, hence \( a^* \) is necessarily an equilibrium of the game.

\[ \square \]

11.2 Proof of Proposition ??

\textit{Proof.}

This proof is similar to the proof of Proposition ??, but applied to the correspondence product, made of individual correspondences \( R_i \).
We then follow the lines of the proof of proposition 27. Define $Y_i$ as the subset of $S$ such that $Y_i = \{s \in S \mid R_i(s) \neq \emptyset\}$. By property 27, this set is open. In $S$, define the following correspondences $R'_i$, for any $i$:

- for all $s \in Y_i$, $R'_i(s) = R_i(s)$
- for all $s \not\in Y_i$, $R'_i(s) = S_i$.

A needed lemma consists of ensuring that if correspondences satisfy lemma 27, then their product correspondence also does.

**Lemma 3** Let $\phi$ be a correspondence product of $\phi_i$ on $S$ to itself, with $i \in N$. If for any $i \in N$, any $s \in S$ with $s \not\in \phi_i(s)$, exists a direction $p_i^s$ of $\phi_i$ at $s$, and a neighborhood of $s$, $V_i$, such that for any $s' \in V_i$ and any $y_i^s \in \phi_i(s')$, then $\langle p_i^s, y_i^s - s' \rangle > 0$, then exists for any $s$ with $s \not\in \phi(s)$, a direction $p^s$ of $\phi$ at $s$ and a neighborhood of $s$, $V_s$, such that for any $x' \in V_s$, and for any $y \in \phi(s')$, then $\langle p^s, y^s - s' \rangle > 0$.

**Proof.** For all $s \in S$ such that there is at least one $i$ with $R_i(s) \neq \emptyset$, take $V(s) = \cap_{i \in N} V_i(s)$ and then define $h_i : S \rightarrow E'$ in the following way:

- $h_i(s) = p_i^s$ if $p_i^s$ is a direction of $R_i$ at $s$
- $h_i(s) = 0$ if there is no direction at $s$ for $R_i$

Then, take $p^s = (h_i(s))_{i \in N}$, and $V(s) = \cap_{i \in N} V_i(s)$. This is enough to ensure the result. In case all $i$ are such that $R_i(s) = \emptyset$, then $R(s) = \emptyset$, and the former condition has not to apply.

Clearly, for $Y_i$, conditions for applying lemma 27 to $R$ are ensured, since LNOD holds. Similarly, on $S - Y_i$, since $s \in R'_i(s)$, these conditions also hold. As a consequence, Lemma 27 applies and $R'$ has a fixed point, say $s^*$. This fixed point cannot be in any $Y_i$ as it would require that exists some $i$ with $s \in R_i(s)$, which cannot be the case by irreflexivity. Necessarily, $s^*$ belongs to $X - \cap_{i \in N} Y_i$ and $s^*$ is such that $R(s^*)$ is empty by construction, and $s^*$ is an equilibrium of the game.

**11.3 Proof of Corollary 27**

**Proof.**

For 3, it can be shown that any convex valued correspondence $R$ is such that $R$ has a non-empty direction correspondence at all $x$ with $R(x) \neq \emptyset$, as far as $R(x)$ is either open and closed.

**Lemma 4 (Convex Values imply non-empty direction)** Convex values imply the existence of a direction for all $x$ with $\phi(x) \neq \emptyset$, if $\phi(x)$ is either open or closed.

**Proof.**
First, assume \( \phi(x) \) is closed. Considering \{x\} and \( \phi(x) \), which is necessarily compact being a closed subset of a compact space, as convex sets, it is possible to apply the separating hyperplane theorem. Hence exists a non-null vector \( p \) and \( c \in \mathbb{R} \) such that any \( \bar{x} \in \phi(x) \) is such that \( \langle p, \bar{x} \rangle < c \). Thus, \( \langle p, \bar{x} - x \rangle + \langle p, x \rangle < c \), and so \( \langle p, \bar{x} - x \rangle < d \) with \( d = c - \langle p, x \rangle \). Thus, either \( d \leq 0 \) and \( \langle -p, \bar{x} - x \rangle < 0 \) and \(-p\) is a direction for any \( \bar{x} \in \phi(x) \), either \( d \geq 0 \) and \( p \) is such a direction. Thus a direction exists for \( \phi(x) \) from \( x \).

In case \( \phi(x) \) is open, then considering \( \phi(x) \) and \{x\} and using the weak separating hyperplane theorem, it is possible to separate in to different open half spaces \( \phi(x) \) and \{x\}. The same argument as in the former paragraph can be put forward: exists a non-null vector \( p \) and \( c \in \mathbb{R} \) such that any \( \bar{x} \in \phi(x) \) is such that \( \langle p, \bar{x} \rangle \leq c \). Thus, \( \langle p, \bar{x} - x \rangle + \langle p, x \rangle \leq c \), and so \( \langle p, \bar{x} - x \rangle \leq d \) with \( d < c - \langle p, x \rangle \). Thus, either \( d \leq 0 \) and \( \langle -p, \bar{x} - x \rangle < 0 \) and \(-p\) is a direction for any \( \bar{x} \in \phi(x) \), either \( d \geq 0 \) and \( p \) is such a direction.

□

By lemma ??, a non-empty direction correspondence exists at \( s \). Besides, it is easy to show, from the definition of upper hemi-continuity that at any \( s \) where \( R_i \) is upper hemi-continuous, then if exists a direction at \( s \), the same direction also exists for a neighborhood of \( s \). Indeed, \( R_i \) is such that for any \( s \), for any neighborhood \( V \) that contains \( R_i(s) \), there exists a neighborhood \( U \) of \( s \) such that \( R_i(s') \subset V \) if \( s' \in U \). As exists a direction \( p \) for \( R_i \) at \( s \), thus for some neighborhood \( U \) of \( R_i(s) \), then as exists \( V \) such that any \( s' \in V \) implies \( R_i(s') \in U \), \( p \) is also a direction of \( R_i(s') \). As a consequence, a closed and convex valued UHC correspondence is LNOD. Corollary ??, 3. ensues.

1. is immediate since any continuous mapping is a special case of 3.

Similarly, 2. is straightforward with ??) Lemma which states the existence of a continuous selection mapping for any convex and closed valued lower hemi-continuous correspondence from a paracompact topological space to a Banach space. Hence, such a correspondence has a LNOD subprocedure and proposition ?? ensures the result. The same reasoning can be made for 4., given ??) theorem.

□

11.4 Proof of Proposition ??

Proof.
From upper hemi-continuity of $C_i$ and property ?? on $R_i$, ensues LNODE for $R_i \cap C_i$. Indeed, for any $s \in S$, such that $R_i(s) \neq \emptyset$, exists a neighborhood of $s$, $V_s$, and a direction $p^*_i$, such that for any $s' \in V_s$, and any $s^*_i \in R_i(s')$ with $\langle p^*_i, s^*_i - s'_i \rangle > 0$, with $s'_i = (s'_i, s'_i)$. Thus, since the inner product is strictly positive, exists an open set $O^*$ such that $\cup_{s \in V_s} R(s') \subset O^*$. Then from the definition of upper hemi-continuity – for any open set $O$ that contains $C_i(s)$, exists a neighborhood $U$ of $s$ such that $C_i(s') \subset O$ if $s' \in U$, exists $U^*$ such that $C_i(s') \subset O^*$ if $s' \in U^*$. Thus taking $V_s \cap U^*$ as the relevant neighborhood insures that for any $s' \in V_s \cap U^*$, exists $p^*_i$ such that for any $s^*_i \in R_i(s')$ with $\langle p^*_i, s^*_i - s_i \rangle > 0$. Hence LNODE as in property ?? holds for $R_i \cap C_i$.

Applying the proof of proposition ?? in the previous chapter to $R_i \cap C_i$ instead of $R_i$ guarantees an equilibrium exists.

$\square$

11.5 Proof of Proposition ??

Proof.

Most of this proof is adapted from classical results. We thus recall some lemmata: first on the behavior of the Walrasian auctioneer, referred to as agent 0, whose behavior will be defined, following CITE Debreu 1959,1982, as the following better-reply correspondence: $R_0 : \Delta_l \times \mathbb{R}^{ln} \times \mathbb{R}^{ln} \rightrightarrows \Delta_l$ and $R_0(p, q, s) = \{ p' \in \Delta_l | p'.(q - s - w) > p.(q - s - w) \}$.

Lemma 5 (? ) $R_0$ has open graph, convex upper contour sets\(^{14}\), and is irreflexive.

Thus by ?) theorem, its best-reply correspondence is closed valued (being compact in a closed set) and upper semi-continuous. Thus the behavior of the auctioneer can be spelled out, following the equivalence result of the previous chapter, by $R_0(p, q, s) = R_0(p, q, s)$ if $(p, q, s) \notin R_0(p, q, s)$ and the empty set otherwise. There is no feasibility constraint for the auctioneer since its strategy space is constant. Note that the strategy set of the auctioneer is compact and convex.

For consumers, the relevant strategy space will be $\bar{X}_i$, which is compact since $F$ is compact – given lemma ??, It is worth noting that it is also convex, since $F$ is also such by construction. Furthermore, it is easy to show that for any $p, q, s$, $\tilde{F}_i(p, q, s) = F_i(p, q, s) \cap \bar{X}_i$ is compact valued, since $\bar{X}_i$. It is also immediate, with a little algebra, that it is convex-valued for any price since $F_i$ as well as $\bar{X}_i$ are. Moreover, $F_i$ is upper hemi-continuous quite straightforwardly from its definition, and so is $\tilde{F}_i$ as the intersection of two such correspondences, by considering the constant ”correspondence” $\bar{X}_i$. In addition, $\tilde{F}_i$ is non empty\(^{14}\).
since \( w_i \) is both in \( \bar{X}_i \) – considering the possible case where production is null – and in any value of \( F_i \) by definition. As a conclusion, \( \bar{F}_i \) is compact, convex and non-empty valued, as well as upper hemi-continuous, and hence conditions for applying proposition ?? are satisfied for consumers.

Similarly, \( \bar{Y}_i \) is compact. Defining the feasibility constraint as composed by individual supply choices such that profits \( p.y_j(p, q, s) \geq 0 \), this feasibility constraint intersected with \( \bar{Y}_i \), denote it \( F_j \) (same demonstration as in case of the population of consumers) is upper hemi-continuous as well as compact, non-empty and convex valued. Thus condition for applying proposition ?? also hold for suppliers.

Thus exists a behavioral equilibrium of the social game whose strategy sets are \( \Delta_i \times \times \bar{X}_i \times \times \bar{Y}_i \). This means, given the behavior of the auctioneer, that exists some price \( p^* \) and individual demands and supplies – such that any other price \( p \) is such that \( p^*(q^* - s^* - w) > p.(q^* - s^* - w) \). Since by Walras’ law, \( p^*(q^* - s^* - w) \leq 0 \) (which is respected due to feasibility constraints of every individual), for any \( p \) “positive”, \( p.(q^* - s^* - w) \leq 0 \), hence \( q^* - s^* - w \leq 0 \).

□

References


