GATE LYON SAINT-ÉTIENNE
UMR 5824

Bâtiment MILC
35, rue Raulin
69007 Lyon - France

Maison de l'Université, Bâtiment B
10, rue Tréfilerie
42023 Saint-Etienne cedex 02 - France

https://www.gate.cnrs.fr
gate@gate.cnrs.fr

WP 2604 – February 2026

Consistency in collective decision-making under uncertainty: an axiomatic approach

Stéphane Gonzalez, Le-Nhat-Linh Huynh

Abstract:

We study collective decision-making when individual preferences depend on the state of the world. The paper introduces an axiom, Aggregation Consistency, linking the way society aggregates utilities across individuals with the way each individual aggregates outcomes across states. The axiom requires that any alternative preferred in every realized state remains preferred before uncertainty is resolved. Combined with standard social choice and aggregation principles, it implies that the same functional form must govern both interpersonal and intrapersonal aggregation. Under familiar conditions, this yields two canonical families of solutions: generalized utilitarian rules based on quasi-arithmetic means, and Rawlsian rules based on minimum or maximum operators. The analysis unifies utilitarian and egalitarian criteria within a single axiomatic framework for collective choice under uncertainty.

Working Paper

Consistency in collective decision-making under uncertainty: an axiomatic approach

Stéphane Gonzalez[1] and Le-Nhat-Linh Huynh[1]

[1]Université Jean Monnet Saint-Etienne, CNRS, Université Lumière Lyon 2, emlyon business school, GATE, 42023 Lyon, France.
stephane.gonzalez@univ-st-etienne.fr,
le.nhat.linh.huynh@univ-st-etienne.fr

Version of February 3, 2026

Abstract

We study collective decision-making when individual preferences depend on the state of the world. The paper introduces an axiom, Aggregation Consistency, linking the way society aggregates utilities across individuals with the way each individual aggregates outcomes across states. The axiom requires that any alternative preferred in every realized state remains preferred before uncertainty is resolved. Combined with standard social choice and aggregation principles, it implies that the same functional form must govern both interpersonal and intrapersonal aggregation. Under familiar conditions, this yields two canonical families of solutions: generalized utilitarian rules based on quasi-arithmetic means, and Rawlsian rules based on minimum or maximum operators. The analysis unifies utilitarian and egalitarian criteria within a single axiomatic framework for collective choice under uncertainty.

1 Introduction

Collective decisions are made under uncertainty. Governments commit to climate policies without knowing future environmental states; health authorities allocate resources before an epidemic's trajectory is clear; firms invest in volatile markets. Even mundane choices—like friends planning a weekend contingent on the weather—reveal the same problem: preferences over outcomes depend on unknown states of the world. This state-dependence generates two distinct yet interdependent aggregation problems. At the intrapersonal level, each agent must evaluate outcomes across states; at the interpersonal level, society must aggregate these evaluations across agents to choose an alternative based on multiple people's utilities. The literature has largely addressed these problems in isolation: decision theory focuses on intrapersonal aggregation under uncertainty, while social choice theory examines interpersonal aggregation of fixed preferences. This separation may generate an inconsistency: a policy preferred by individuals in all states can nonetheless be rejected in the ex ante social evaluation. Addressing this problem requires a unified treatment of intrapersonal and interpersonal aggregation under uncertainty.

Bayesian social aggregation provides the canonical attempt to merge the two literatures, aiming for collective decisions that account simultaneously for uncertainty and democratic fairness. Within this framework, Harsanyi (1955) showed that if individuals and society both follow expected-utility rationality behind a veil of ignorance, the social welfare function must maximize the sum of individual expected utilities. In contrast, Rawls (1971) advocated the maximin principle, which gives absolute priority to the worst-off individual and corresponds to an extreme form of risk aversion. Reconciling intrapersonal (risk) aggregation with interpersonal (social) fairness—finding decision criteria that respect both expected utility and equity considerations—remains a central challenge. A key assumption in Harsanyi's theorem is that agents share the same probabilistic beliefs. Yet in many contexts, this is untenable. Savage (1954) introduced subjective expected utility, under which outcomes are evaluated using each agent's own beliefs. Disagreement is then unavoidable: rational agents may differ simply because they hold different priors. Mongin (1995) showed that Harsanyi's utilitarian conclusion collapses once such heterogeneity of beliefs is allowed. More broadly, aggregation becomes fragile once individuals diverge either in their beliefs or in their evaluation of uncertainty. To address this difficulty, some contributions have weakened the ex ante Pareto principle (corresponding to unanimity in this framework), as in Gilboa et al. (2004).

Our approach takes a different route. Rather than tolerating heterogeneous ways of aggregating across states, we posit a single, centralized procedure common to all agents. The perspective is normative: just as classical axioms guide how society should aggregate across individuals, axioms from decision theory guide how individuals should aggregate across states. What is missing in the literature is a formal link between these two levels. We provide this link through an axiom of Aggregation Consistency. Informally, Aggregation Consistency requires that if a given alternative is chosen in every possible state of the world (when evaluated after the uncertainty is resolved), then that alternative should also be chosen before the uncertainty is resolved. In other words, whenever an alternative is optimal ex post in all scenarios, it remains optimal ex ante once those state-contingent evaluations are aggregated. This axiom ensures a form of dynamic coherence or time consistency in collective choice under uncertainty: society should not reverse a decision that is unanimously favored in every realizable circumstance.

To do that, we cast the problem in the axiomatic tradition of social choice and decision theory, treating intrapersonal and interpersonal aggregation symmetrically. A solution is a pair $\psi = (\varphi, F)$, where φ is a (possibly multi-valued) selection rule, and F aggregates each agent's utilities across states. Our central axiom, Aggregation Consistency, then connects the two levels: if an alternative is selected by φ in every state-contingent profile, it must also be selected after these profiles are aggregated by F. For intuition, consider climate policy under uncertainty about future warming. Suppose that in both high- and low-warming scenarios the preferred option is to invest in adaptation. Our consistency axiom requires adaptation to remain selected once these scenarios are aggregated.

What is novel in our approach is that coherence is enforced without embedding a specific state-aggregation rule into the axiom itself. Most existing approaches build consistency directly on a fixed aggregator. For example, Harsanyi's mixture-preservation axiom (Harsanyi, 1955) and the more recent principles of Brandl et al. (2016); Brandl and Brandt (2019, 2024) embody the same intuition—that any alternative selected in each of several profiles must also be selected once these profiles are aggregated—but they enforce this intuition by presupposing expected utility as the aggregation rule. Similarly, Gonzalez and Pnevmatikos (2024) characterizes the Rawlsian rule by assuming from the outset that, when facing uncertainty, each agent follows the maximin criterion. By contrast, our contribution is to preserve the same powerful intuition while leaving the aggregator itself open. Standard axioms on F then determine its admissible forms, so that Aggregation Consistency becomes a bridge between across-states and across-individuals aggregation under minimal and widely accepted requirements.

A first main result shows that, once Aggregation Consistency is combined with standard axioms on the social choice rule (Nonemptiness, Anonymity, Unanimity, and Continuity) and on the aggregation rule (Continuity and Symmetry), the solution (φ, F) is such that φ selects the set of alternatives maximizing the aggregator F of individual utilities:

$$\varphi(\mathbf{u}) \;=\; \arg\max_{a\in A} F\big((u_i(a))_{i\in N}\big).$$

Classical characterizations of aggregation functions then narrow down the admissible forms of F. Only two structurally parallel families of solutions remain:

(i) Generalized utilitarian rules. If F satisfies Idempotency, Strict Monotonicity, and Bisymmetry, results by Aczél (1948) and Grabisch et al. (2009) imply that F is a *quasi-linear mean*. Formally, there exists a strictly monotone function g and a probability distribution $\omega = (\omega_1, \ldots, \omega_\ell)$ such that for any profile $(x_1, \ldots, x_\ell)$,

$$F(x_1, \ldots, x_\ell) \;=\; g^{-1}\left(\sum_{i=1}^{\ell} \omega_i \, g(x_i)\right).$$

The uniform distribution and the identity g yield the arithmetic mean, i.e., classical utilitarianism. Other transformations capture different normative attitudes: $g(x) = \sqrt{x}$ emphasizes low values (inequality aversion), while $g(x) = x^2$ amplifies high values (quadratic mean). More generally, concave g favors low values and convex g favors high values. Aggregation Consistency then requires that the *same* g govern both intrapersonal aggregation (across states for each individual) and interpersonal aggregation (across individuals for society). The resulting rule is

$$\varphi(u) \;=\; \arg\max_{a} \; g^{-1}\!\left(\sum_{i=1}^{n} g(u_i(a))\right).$$

(ii) Min/Max-based rules. If instead F satisfies Monotonicity, Idempotency, Associativity, Continuity, and admits a Neutral Element, a classical result (Czogała and Drewniak, 1984) implies that F reduces to

$$F(x_1, \ldots, x_n) \;=\; \min\{x_1, \ldots, x_n\} \quad \text{or} \quad F(x_1, \ldots, x_n) \;=\; \max\{x_1, \ldots, x_n\}.$$

Aggregation Consistency then produces the Rawlsian *maximin* rule or, symmetrically, the *maximax* rule. Although maximax is normatively less compelling, it completes the structural parallel: in both cases, intrapersonal and interpersonal aggregation coincide—alternatives are evaluated by their worst (or best) state, and society by the worst-off (or best-off) individual.

The framework thus speaks directly to the Harsanyi–Rawls debate. Utilitarian and maximin criteria arise as parallel instantiations of a single scheme, depending on the axioms imposed on F. The central message is simple: under mild and natural conditions, the way we aggregate across states must mirror the way we aggregate across individuals. Aggregation Consistency, combined with standard social choice axioms and classical properties of aggregation functions, explains why generalized utilitarianism and Rawlsian max–min emerge as the canonical solutions of a unified model.

2 Preliminaries

2.1 Definition and Notation

Let $N = \{1, \ldots, n\}$ with $n \geq 2$ be the set of agents, A a finite set of social alternatives, and $\mathcal{S}$ a (possibly infinite) set of states of the world. For any set X, let $X^{(\mathbb{N})}$ denote the set of finite sequences of elements from X.

For each state $s \in \mathcal{S}$ and each agent $i \in N$, let $u_i^s : A \to \mathcal{D}$ be the agent's utility function, where $\mathcal{D}$ is an open interval of $\mathbb{R}$. We write $\mathbf{u}^s = (u_i^s)_{i\in N}$ for the utility profile at state s, and $\mathcal{U} = (\mathcal{D}^A)^N$ for the set of all utility profiles.

We distinguish two problems:

- Selecting an alternative given a utility profile. A *selection rule* is a correspondence $\varphi : \mathcal{U} \rightrightarrows A$ that assigns to each profile $\mathbf{u} \in \mathcal{U}$ a set of alternatives $\varphi(\mathbf{u}) \subseteq A$.
- Aggregating utilities across states when the realized state is unknown. An *aggregation function* is a mapping $F : \mathbb{R}^{(\mathbb{N})} \to \mathbb{R}$ that assigns to each finite sequence $(u_i^s(a))_{s\in S}$ an aggregated utility, representing agent i's evaluation of alternative a across a finite set of states $S \subseteq \mathcal{S}$.

To simplify notation, we adopt two standard conventions.

- First, for a finite sequence $(u_i^{s_1}, \dots, u_i^{s_m}) \in (\mathcal{D}^A)^{(\mathbb{N})}$, representing the utilities of agent i across states $s_1, \dots, s_m$, we write

$$F(u_i^{s_1}, \dots, u_i^{s_m})(a) := F(u_i^{s_1}(a), \dots, u_i^{s_m}(a)), \quad a \in A.$$

- Second, for a finite sequence $(\mathbf{u}^{s_1}, \dots, \mathbf{u}^{s_m}) \in \mathcal{U}^{(\mathbb{N})}$, representing profiles across states $s_1, \dots, s_m$, we write

$$[F(\mathbf{u}^{s_1}, \dots, \mathbf{u}^{s_m})]_i(a) := F(u_i^{s_1}(a), \dots, u_i^{s_m}(a)), \quad i \in N,\ a \in A.$$

A *solution* to an ex ante social choice problem is a pair $\psi = (\varphi, F)$, where φ is a selection rule and F an aggregation function.

2.2 Axioms

Subsections 2.2.1 and 2.2.2 recall standard axioms of social choice and aggregation. The three axioms for social choice, presented in Subsection 2.2.1 (Nonemptiness, Anonymity, and Unanimity) are natural and satisfied by almost all voting rules in the literature. The axioms on aggregation functions are equally classical (see Grabisch et al. (2009) for a survey) and provide the basis for standard characterizations in this field. Our main novelty, introduced in Subsection 2.2.3, is the axiom of Aggregation Consistency, which connects the two domains and yields an axiomatic characterization of solutions to ex ante social choice problems.

2.2.1 Axioms for the selection rule φ

The following axioms are standard in social choice theory. They ensure that the rule always produces (at least) one recommendation, treats all agents symmetrically, and never overrides unanimous agreement. These are extremely weak requirements, satisfied by essentially all social choice rules used in practice.

Nonemptiness. A solution $\psi = (\varphi, F)$ satisfies *Nonemptiness* if, for every $\mathbf{u} \in \mathcal{U}$,

$$\varphi(\mathbf{u}) \neq \emptyset.$$

This guarantees that the rule always yields at least one admissible alternative.

Anonymity. Let $\mathfrak{S}(N)$ denote the set of permutations of N. For any $\sigma \in \mathfrak{S}(N)$, the permuted profile $\mathbf{u}_\sigma = (u_{\sigma(i)})_{i\in N}$ reorders agents according to σ. A solution $\psi = (\varphi, F)$ satisfies *Anonymity* if, for all $\mathbf{u} \in \mathcal{U}$ and all $\sigma \in \mathfrak{S}(N)$,

$$\varphi(\mathbf{u}) = \varphi(\mathbf{u}_\sigma).$$

This axiom ensures that the rule is invariant to relabeling of agents.

Unanimity. Given $\mathbf{u} \in \mathcal{U}$, let $M(\mathbf{u})$ be the set of alternatives that are unanimously at least as good as any other:

$$M(\mathbf{u}) = \{\, a^* \in A : u_i(a^*) \geq u_i(a) \text{ for all } i \in N,\, a \in A \,\}.$$

A solution $\psi = (\varphi, F)$ satisfies *Unanimity* if, whenever $M(\mathbf{u}) \neq \emptyset$,

$$\varphi(\mathbf{u}) \subseteq M(\mathbf{u}).$$

In other words, if some alternatives are unanimously preferred, the rule selects only from this set.

2.2.2 Axioms for the aggregation function F

The following axioms describe regularity and symmetry properties of the aggregation function F. They are standard in the theory of means and aggregation operators (see, e.g., Grabisch et al., 2009). They restrict F to behave like a "well-behaved average": monotone, symmetric, continuous, and in some cases satisfying additional functional equations such as bisymmetry or associativity.

For any two finite sequences $\mathbf{x} = (x_s)_{s \in S}$ and $\mathbf{y} = (y_s)_{s \in S}$, a solution $\psi = (\varphi, F)$ satisfies:

Aggregation Monotonicity if the aggregation function F is monotonic, i.e.,

$$\mathbf{x} \leq \mathbf{y} \quad \Rightarrow \quad F(\mathbf{x}) \leq F(\mathbf{y}).$$

Thus, increasing any input value cannot decrease the output.

Aggregation Strict Monotonicity if F is strictly monotonic, that is,

$$\mathbf{x} \leq \mathbf{y} \text{ and } \mathbf{x} \neq \mathbf{y} \quad \Rightarrow \quad F(\mathbf{x}) < F(\mathbf{y}).$$

Hence, an increase in at least one input strictly increases the output.

Aggregation Unanimous Monotonicity if F is unanimously monotonic, i.e., monotonic and satisfying

$$x_s < y_s \ \forall s \in S \quad \Rightarrow \quad F(\mathbf{x}) < F(\mathbf{y}).$$

Thus, it exhibits a positive response whenever all input values strictly increase.

Aggregation Symmetry. F is *symmetric* if its value is invariant to any permutation of its arguments:

$$F((x_s)_{s \in S}) = F((x_{\pi(s)})_{s \in S}) \quad \forall \pi \in \mathfrak{S}(S),$$

where $\mathfrak{S}(S)$ is the set of permutations of S. In other words, F treats all components equally, regardless of their position.

Aggregation Bisymmetry. F is *bisymmetric* if, for any four inputs $x_1, x_2, x_3, x_4 \in \mathbb{R}$,

$$F(F(x_1, x_2),\, F(x_3, x_4)) = F(F(x_1, x_3),\, F(x_2, x_4)).$$

This property ensures coherence of aggregation under pairwise exchanges and underlies the characterization of quasi-arithmetic means.

Aggregation Associativity. F is *associative* if the order of grouping does not affect the result: for any finite sequence $(x_1, \ldots, x_m)$ and any $k < m$,

$$F(x_1, \ldots, x_m) = F\big(F(x_1, \ldots, x_k),\, F(x_{k+1}, \ldots, x_m)\big).$$

Aggregation Idempotency. F is *idempotent* if identical inputs yield the same value:

$$F(x, \ldots, x) = x \quad \text{for all } x \in \mathbb{R}.$$

Aggregation Neutral Element. F admits a *neutral element* e if, for any finite sequence $(x_1, \ldots, x_m)$,

$$F(x_1, \ldots, x_{k-1}, e, x_{k+1}, \ldots, x_m) = F(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_m) \quad \text{for all } k.$$

Adding the neutral element does not affect the aggregate.

Continuity. A solution $\psi = (\varphi, F)$ satisfies *Continuity* if both F and φ are continuous.

a) F is continuous if, for every finite sequence $(x_s)_{s \in S}$,

$$\lim_{x_s \to x_s^*} F((x_s)_{s \in S}) = F((x_s^*)_{s \in S}).$$

Small perturbations in inputs produce small changes in the aggregate.

b) φ is continuous if, for each alternative $a \in A$, the set

$$\mathcal{U}_a = \{\mathbf{u} \in \mathcal{U} : a \in \varphi(\mathbf{u})\}$$

is closed in $\mathcal{U}$. Thus, if a sequence of profiles $(\mathbf{u}^n)$ with $a \in \varphi(\mathbf{u}^n)$ converges to $\mathbf{u}^*$, then $a \in \varphi(\mathbf{u}^*)$.

2.2.3 Consistency axiom

Unlike the preceding axioms, which are standard in the literatures on social choice and aggregation theory, the next one is novel. It constitutes the central axiom of the paper and provides the missing link between two distinct domains of analysis: the aggregation of individual preferences across states (intrapersonal) and the aggregation of individual preferences across agents (interpersonal). By imposing a single coherence condition across these two dimensions, it captures the idea that the logic guiding choice under uncertainty and the logic guiding collective choice should coincide.

Aggregation Consistency. A solution $\psi = (\varphi, F)$ satisfies *Aggregation Consistency* if, for every finite sequence of state-contingent profiles $(\mathbf{u}^s)_{s \in S}$ with $S \subseteq \mathcal{S}$, the following implication holds:

$$\Big[\ \varphi(\mathbf{u}^s) = \varphi(\mathbf{u}^{s'}) \text{ for all } s, s' \in S\ \Big] \quad \Longrightarrow \quad \varphi\big(F((\mathbf{u}^s)_{s \in S})\big) = \varphi(\mathbf{u}^s) \text{ for all } s \in S.$$

That is, whenever the same set of alternatives is selected in every state-contingent profile, the same alternatives must also be selected once these profiles are aggregated across states. This requirement expresses a dynamic or "cross-domain" coherence between state-by-state and ex ante collective choice.

In particular, Aggregation Consistency rules out the following kind of inconsistency: society might select policy a in every possible state once uncertainty is resolved, yet reject a in favor of another policy b when evaluating the ex ante problem. The axiom requires that such "sure winners" remain winners even after utilities have been aggregated across states by F. As we show below, this mild requirement is enough, together with standard properties, to force a tight alignment between intrapersonal and interpersonal aggregation.

3 Main results

3.1 Axiomatic characterization of symmetric aggregation functions

The next theorem provides the first main result of the paper. It shows that, once *Aggregation Consistency* is combined with standard axioms, the social choice rule φ must take a simple and familiar form: it selects the alternatives maximizing an aggregated evaluation F of individual utilities. In other words, coherence between intrapersonal and interpersonal aggregation implies that collective choice can be represented by the maximization of a single symmetric aggregation function.

Theorem 1. A solution $\psi = (\varphi, F)$ satisfies Nonemptiness, Anonymity, Unanimity, Continuity, Aggregation Consistency, Aggregation Unanimous Monotonicity and Aggregation Symmetry if and only if, for every profile $\mathbf{u} \in \mathcal{U}$,

$$\varphi(\mathbf{u}) = \arg\max_{a\in A} F((u_i(a))_{i\in N}).$$

Proof. Let $\psi = (\varphi, F)$ be a solution satisfying the axioms of the statement of the Theorem.

Step 1: *Let's prove that*

$$\varphi(\mathbf{u}) \subseteq M_F(\mathbf{u}) := \arg\max_{a\in A} F((u_i(a))_{i\in N}).$$

Let $\sigma \in \mathfrak{S}(N)$ denote the cyclic permutation on N defined by

$$\sigma(1) = 2,\ \sigma(2) = 3,\ \ldots,\ \sigma(n-1) = n,\ \text{and}\ \sigma(n) = 1.$$

Let σ^k represent the k-fold composition of σ.
By **Anonymity**, we have

$$\varphi(\mathbf{u}) = \varphi(\mathbf{u}_{\sigma^1}) = \varphi(\mathbf{u}_{\sigma^2}) = \cdots = \varphi(\mathbf{u}_{\sigma^n}).$$

Then, by **Aggregation Consistency**, we obtain

$$\varphi(\mathbf{u}) = \varphi(F((\mathbf{u}_{\sigma^k})_{k\in N}))\,. \tag{1}$$

Let $\mathbf{v}$ be the utility profile defined by $\mathbf{v} = F((\mathbf{u}_{\sigma^k})_{k\in N})$. For each $a \in A$ and for all individuals $i, j \in N$, we have

$$v_i(a) = F((u_{\sigma^k(i)}(a))_{k\in N}) = F((u_i(a))_{i\in N}) = v_j(a), \tag{2}$$

where the last two equalities follow from **Aggregation Symmetry**.
Since A is finite, there exists $a^* \in A$ such that $v_i(a^*) \geq v_i(a)$ for all $a \in A$. Hence, the set

$$M(\mathbf{v}) = \{a^* \in A : \forall i \in N, \forall a \in A,\ v_i(a^*) \geq v_i(a)\}$$

is nonempty. By **Unanimity**, we have

$$\varphi(\mathbf{v}) \subseteq M(\mathbf{v}). \tag{3}$$

Now observe that

$$\begin{aligned} M(\mathbf{v}) &= \{a^* \in A : \forall i \in N, \forall a \in A,\ v_i(a^*) \geq v_i(a)\} \\ &= \{a^* \in A : \forall i \in N, \forall a \in A,\ F((u_i(a^*))_{i\in N}) \geq F((u_i(a))_{i\in N})\} && \text{(by (2))} \\ &= M_F(\mathbf{u}). \end{aligned}$$

Since $M(\mathbf{v}) = M_F(\mathbf{u})$ combining (1) and (3) we conclude that

$$\varphi(\mathbf{u}) \subseteq M_F(\mathbf{u}) = \arg\max_{a \in A} F((u_i(a))_{i \in N}).$$

Step 2: *We now prove that* $M_F(\mathbf{u}) \subseteq \varphi(\mathbf{u})$.

Let $a \in M_F(\mathbf{u})$. By definition of $M_F(\mathbf{u})$, we have

$$F((u_i(a))_{i \in N}) \geq F((u_i(b))_{i \in N}) \quad \forall b \in A. \tag{4}$$

Let $\epsilon > 0$ sufficiently small and $(\mathbf{w}^n)_{n \in \mathbb{N}^*}$ be a sequence of utility profiles defined, for each $i \in N$ and $n \in \mathbb{N}^*$, by

$$\begin{cases} w_i(a) = u_i(a) + \dfrac{\epsilon}{n}, \\ w_i(b) = u_i(b), \quad \text{when } b \neq a. \end{cases}$$

Hence, $w_i(a) > u_i(a)$ for all $i \in N$.
Since F satisfies **Aggregation Unanimous Monotonicity**, it follows that for every $n \in \mathbb{N}^*$,

$$F((w_i(a))_{i \in N}) > F((u_i(a))_{i \in N}).$$

Then, by (4), we have

$$F((w_i(a))_{i \in N}) > F((w_i(b))_{i \in N}) \quad \text{for all } b \neq a.$$

Hence, $M_F(\mathbf{w}^n) = \{a\}$.

By **Step 1**, for each $n \in \mathbb{N}^*$,

$$\varphi(\mathbf{w}^n) \subseteq M_F(\mathbf{w}^n) = \{a\}.$$

By **Nonemptiness**, it follows that

$$\varphi(\mathbf{w}^n) = \{a\}.$$

Since $(\mathbf{w}^n)_{n \in \mathbb{N}^*}$ converges to $\mathbf{u}$ as $n \to \infty$, the **Continuity of** φ implies that

$$a \in \varphi(\mathbf{u}).$$

Therefore,

$$M_F(\mathbf{u}) \subseteq \varphi(\mathbf{u}).$$

Combining **Step 1** and **Step 2**, we conclude that

$$\varphi(\mathbf{u}) = M_F(\mathbf{u}).$$

■

In the remainder of this section we investigate which aggregation functions F are compatible with this representation under alternative, classical properties coming from the theory of aggregation.

3.2 Axiomatic characterization of generalized utilitarian rules

In this subsection, we focus on the case where F satisfies Idempotency, Strict Monotonicity, and Bisymmetry. These conditions are standard in the characterization of quasi-arithmetic means and, when combined with Aggregation Consistency, imply that the same functional form must govern both intrapersonal aggregation and interpersonal aggregation. This yields the family of generalized utilitarian rules.

Theorem 2. A solution $\psi = (\varphi, F)$ satisfies Nonemptiness, Anonymity, Unanimity, Continuity, Aggregation Consistency, Aggregation Idempotency, Aggregation Strict Monotonicity, and Aggregation Bisymmetry if and only if the following holds.

There exists a continuous and strictly monotone function $g : \mathcal{D} \to \mathbb{R}$ such that:

(i) (*Intrapersonal aggregation*) For every finite set of states $S \subseteq \mathcal{S}$, every agent $i \in N$, and every alternative $a \in A$,

$$F\big((u_i^s(a))_{s\in S}\big) \;=\; g^{-1}\left(\sum_{s\in S} \omega_s\, g(u_i^s(a))\right),$$

for some weights $(\omega_s)_{s\in S}$ satisfying $\omega_s > 0$ and $\sum_{s\in S}\omega_s = 1$.

(ii) (*Interpersonal aggregation / social choice*) For every profile $\mathbf{u} \in \mathcal{U}$,

$$\varphi(\mathbf{u}) = \arg\max_{a\in A} g^{-1}\left(\sum_{i\in N} g(u_i(a))\right)$$

In particular, the same generator g governs the aggregation of utilities across states for each agent and the aggregation of utilities across individuals for society.

Proof. We prove only the sufficiency part of the theorem and leave the necessity part to the reader. The proof is structured in four steps. The first step corresponds to point (a) of the Theorem.

Step 0: *A solution $\psi = (\varphi, F)$ satisfies Continuity, Aggregation Idempotency, Aggregation Strict Monotonicity, and Aggregation Bisymmetry if and only if there exists a continuous and strictly monotonic function $g : \mathcal{D} \to \mathbb{R}$ and weights $\omega_s > 0$ satisfying $\sum_{s\in S} \omega_s = 1$ such that*

$$F\left((u_i^s)_{s\in S}\right) = g^{-1}\left(\sum_{s\in S} \omega_s g(u_i^s)\right).$$

This result follows directly from the theorem in Section 3, page 399 of Aczél (1948)[1]

Step 1: *Assume that $\psi = (\varphi, F)$ satisfies Aggregation Consistency and Continuity, where the aggregation function F follows the quasi-arithmetic means form as described in **Step 0**. If there exists a subset $B \subseteq A$, an integer $m \geq 2$, and utility profiles $\mathbf{u}_{s_1}, \mathbf{u}_{s_2}, \ldots, \mathbf{u}_{s_m} \in \mathcal{U}$ such that*

$$\varphi(\mathbf{u}_{s_1}) = \varphi(\mathbf{u}_{s_2}) = \cdots = \varphi(\mathbf{u}_{s_m}) = B,$$

then it follows that

$$B \subseteq \varphi\left(g^{-1}\left[\frac{1}{m}\sum_{i=1}^{m} g(\mathbf{u}_{s_i})\right]\right).$$

[1]See also Grabisch et al. (2009) for a detailed proof using the notation of aggregation functions.

By **Aggregation Consistency** and step 0, there exist weights $\omega_{s_1}, \omega_{s_2}, \ldots, \omega_{s_m} > 0$ with $\sum_{i=1}^{m} \omega_{s_i} = 1$ such that

$$B = \varphi(\mathbf{u}_{s_1}) = \ldots = \varphi(\mathbf{u}_{s_m}) = \varphi\left(g^{-1}\left(\sum_{i=1}^{m} \omega_{s_i}\, g(\mathbf{u}_{s_i})\right)\right). \tag{5}$$

Reversing the roles of $\mathbf{u}_{s_1}, \ldots, \mathbf{u}_{s_m}$ cyclically and applying **Aggregation Consistency** again, we obtain:

$$\begin{cases} B = \varphi\left(g^{-1}\left(\omega_{s_m}\, g(\mathbf{u}_{s_1}) + \omega_{s_1}\, g(\mathbf{u}_{s_2}) + \ldots + \omega_{s_{m-1}}\, g(\mathbf{u}_{s_m})\right)\right), \\ B = \varphi\left(g^{-1}\left(\omega_{s_{m-1}}\, g(\mathbf{u}_{s_1}) + \omega_{s_m}\, g(\mathbf{u}_{s_2}) + \ldots + \omega_{s_{m-2}}\, g(\mathbf{u}_{s_m})\right)\right), \\ \vdots \\ B = \varphi\left(g^{-1}\left(\omega_{s_2}\, g(\mathbf{u}_{s_1}) + \omega_{s_3}\, g(\mathbf{u}_{s_2}) + \ldots + \omega_{s_1}\, g(\mathbf{u}_{s_m})\right)\right). \end{cases} \tag{6}$$

Repeating the steps in (5) and (6) iteratively, we define the recursive sequence

$$\begin{cases} \mathbf{v}_\ell^0 = \mathbf{u}_{s_\ell}, \quad \ell = 1, \ldots, m, \\ \mathbf{v}_\ell^k = g^{-1}\left(\sum_{j=1}^{m} M_{\ell j}\, g(\mathbf{v}_j^{k-1})\right), \quad k \geq 1, \end{cases} \tag{7}$$

where the matrix M is given by

$$M = \begin{pmatrix} \omega_{s_1} & \omega_{s_2} & \ldots & \omega_{s_m} \\ \omega_{s_m} & \omega_{s_1} & \ldots & \omega_{s_{m-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{s_2} & \omega_{s_3} & \ldots & \omega_{s_1} \end{pmatrix}, \quad \text{with } \sum_{i=1}^{m} \omega_{s_i} = 1.$$

Applying g to both sides in (7) yields

$$g(\mathbf{v}_\ell^k) = \sum_{j=1}^{m} M_{\ell j}\, g(\mathbf{v}_j^{k-1}), \quad k \geq 1,$$

which in matrix form reads

$$\begin{pmatrix} g(\mathbf{v}_1^k) \\ \vdots \\ g(\mathbf{v}_m^k) \end{pmatrix} = M \begin{pmatrix} g(\mathbf{v}_1^{k-1}) \\ \vdots \\ g(\mathbf{v}_m^{k-1}) \end{pmatrix}. \tag{8}$$

By induction, we have

$$\begin{pmatrix} g(\mathbf{v}_1^k) \\ \vdots \\ g(\mathbf{v}_m^k) \end{pmatrix} = M^k \begin{pmatrix} g(\mathbf{u}_{s_1}) \\ \vdots \\ g(\mathbf{u}_{s_m}) \end{pmatrix}.$$

We note that M is a **doubly stochastic matrix**, i.e., the sum of each row and each column equals 1. In particular, M defines the transition matrix of a Markov chain with m states whose stationary distribution is uniform[2]. Hence, the stationary distribution satisfies

$$\pi_\ell = \frac{1}{m}, \quad \ell = 1, \ldots, m,$$

[2]See Chapter 12 in Ibe (2014).

which implies

$$\lim_{k\to\infty}(M^k)_{ij} = \pi_i = \frac{1}{m}, \quad \forall i,j.$$

Consider

$$\mathbf{v}_\ell^k = g^{-1}\left(\sum_{j=1}^{m} M_{\ell j}^k\, g(\mathbf{u}_{s_j})\right), \quad k \geq 1.$$

Since g is continuous and strictly monotone on the interval $\mathcal{D}$, the inverse function g^{-1} is also continuous and strictly monotone on the image of $\mathcal{D}$. It follows that

$$\begin{aligned}\lim_{k\to\infty}\mathbf{v}_\ell^k &= g^{-1}\left(\sum_{j=1}^{m}\lim_{k\to\infty}(M^k)_{\ell j}\, g(\mathbf{u}_{s_j})\right)\\ &= g^{-1}\left(\frac{1}{m}\sum_{j=1}^{m} g(\mathbf{u}_{s_j})\right).\end{aligned}$$

Therefore, the coefficients of $\mathbf{u}_{s_1},\dots,\mathbf{u}_{s_m}$ in $\mathbf{u}_{s_1}^k,\dots,\mathbf{u}_{s_m}^k$ converge to $\frac{1}{m}$ as $k\to\infty$.
By Aggregation Consistency applied at each iteration we have

$$B = \varphi(\mathbf{u}_{s_1}) = \cdots = \varphi(\mathbf{u}_{s_m}) = \varphi\left(\mathbf{v}_\ell^k\right), \quad \ell = 1,\dots,m \text{ and } k \geq 1.$$

By continuity of φ, $\mathbf{v}_\ell^k$ converges to $g^{-1}\left(\frac{1}{m}\sum_{j=1}^{m} g(\mathbf{u}_{s_j})\right)$ as $k\to\infty$, we concluded that

$$B \subseteq \varphi\left(g^{-1}\left(\frac{1}{m}\sum_{i=1}^{m} g(\mathbf{u}_{s_i})\right)\right).$$

Step 2: *Based on the result in Step 1, we now prove that*

$$\varphi(\mathbf{u}) \subseteq B_g(\mathbf{u}) := \arg\max_{a\in A}\left\{g^{-1}\left(\sum_{i\in N} g(u_i(a))\right)\right\}.$$

Define the cyclic permutation $\sigma \in \mathfrak{S}(N)$ by:

$$\sigma(1) = 2, \quad \sigma(2) = 3, \quad \dots, \quad \sigma(n-1) = n, \quad \sigma(n) = 1.$$

For each $k \in \{1,\dots,n\}$, let σ^k denote the permutation obtained by applying σ k times.
By the **Anonymity** axiom, for each $k \in \{1,\dots,n\}$, it follows that

$$\varphi(\mathbf{u}) = \varphi(\mathbf{u}_{\sigma^1}) = \cdots = \varphi(\mathbf{u}_{\sigma^k}) = \ldots = \varphi(\mathbf{u}_{\sigma^n}).$$

By **Step 1** yields:

$$\varphi(\mathbf{u}) = \varphi(\mathbf{u}_{\sigma^1}) = \cdots = \varphi(\mathbf{u}_{\sigma^n}) \subseteq \varphi\left(g^{-1}\left(\frac{1}{n}\sum_{k=1}^{n} g(\mathbf{u}_{\sigma(k)})\right)\right). \tag{9}$$

Let $\mathbf{v}$ be the utility profile defined as:

$$\mathbf{v} = g^{-1}\left(\frac{1}{n}\sum_{k=1}^{n} g(\mathbf{u}_{\sigma(k)})\right).$$

Then for each $a \in A$ and each $i \in N$, we have

$$v_i(a) = g^{-1}\left(\frac{1}{n}\sum_{j \in N} g(u_j(a))\right).$$

Hence, $v_i(a) = v_j(a)$ for all $i, j \in N$, so we deduce that

$$\begin{aligned} M(\mathbf{v}) &= \{a^* \in A : \forall i \in N, \forall a \in A,\ v_i(a^*) \geq v_i(a)\} \\ &= \left\{a^* \in A : \forall a \in A,\ g^{-1}\left(\frac{1}{n}\sum_{i \in N} g(u_i(a^*))\right) \geq g^{-1}\left(\frac{1}{n}\sum_{i \in N} g(u_i(a))\right)\right\} \\ &= B_g(\mathbf{u}) \neq \emptyset. \end{aligned}$$

By the **Unanimity** axiom, it follows that:

$$\varphi\left(g^{-1}\left(\frac{1}{n}\sum_{k=1}^{n} g(\mathbf{u}_{\sigma(k)})\right)\right) \subseteq B_g(\mathbf{u}). \tag{10}$$

Thus, combining (9) and (10), we conclude:

$$\varphi(\mathbf{u}) \subseteq B_g(\mathbf{u}).$$

Step 3: *Proof that* $B_g(\mathbf{u}) \subseteq \varphi(\mathbf{u})$ Let $a \in B_g(\mathbf{u})$. By definition of $B_g(\mathbf{u})$, we have

$$g^{-1}\left(\sum_{i=1}^{n} g(u_i(a))\right) \geq g^{-1}\left(\sum_{i=1}^{n} g(u_i(b))\right) \quad \forall b \in A. \tag{11}$$

Let $\epsilon > 0$ sufficiently small and $(\mathbf{w}^n)_{n \in \mathbb{N}^*}$ be a sequence of utility profiles defined, for each $i \in N$ and $n \in \mathbb{N}^*$, by

$$\begin{cases} w_i(a) = u_i(a) + \dfrac{\epsilon}{n}, \\ w_i(b) = u_i(b), \quad \text{when } b \neq a. \end{cases}$$

Hence, $w_i(a) > u_i(a)$ for all $i \in N$.
Since $g^{-1}\left(\sum_{i=1}^{n} g(u_i(a))\right)$ satisfies **Aggregation Unanimous Monotonicity**, it follows that for every $n \in \mathbb{N}^*$,

$$g^{-1}\left(\sum_{i=1}^{n} g(w_i(a))\right) > g^{-1}\left(\sum_{i=1}^{n} g(u_i(a))\right)$$

Then, by (11), we have

$$g^{-1}\left(\sum_{i=1}^{n} g(w_i(a))\right) > g^{-1}\left(\sum_{i=1}^{n} g(w_i(b))\right) \quad \text{for all } b \neq a.$$

Hence, $B_g(\mathbf{w}^n) = \{a\}$.

By **Step 2**, for each $n \in \mathbb{N}^*$,

$$\varphi(\mathbf{w}^n) \subseteq B_g(\mathbf{w}^n) = \{a\}.$$

By **Nonemptiness**, it follows that

$$\varphi(\mathbf{w}^n) = \{a\}.$$

Since $(\mathbf{w}^n)_{n \in \mathbb{N}^*}$ converges to $\mathbf{u}$ as $n \to \infty$, the **Continuity of** φ implies that

$$a \in \varphi(\mathbf{u}).$$

Therefore,

$$B_g(\mathbf{u}) \subseteq \varphi(\mathbf{u}).$$

Combining **Step 2** and **Step 3**, we conclude that

$$\varphi(\mathbf{u}) = B_g(\mathbf{u}).$$

■

3.3 Axiomatic characterization of social choice rules based on Min and Max aggregation

Theorem 3 is the analogue of Theorem 2 for min- and max-based aggregation functions, which evaluate alternatives by their worst or best component. Under the usual axioms for associative, idempotent, and monotone aggregation with a neutral element, classical results show that F must be either the minimum or the maximum operator. Aggregation Consistency then transforms these intrapersonal rules into their interpersonal counterparts: the Rawlsian maximin rule and its dual maximax rule.

Theorem 3. A solution $\psi = (\varphi, F)$ satisfies Nonemptiness, Anonymity, Unanimity, Continuity, Aggregation Consistency, Aggregation Monotonicity, Aggregation Idempotency, Aggregation Associativity and Aggregation Neutral Element if and only if, for each $S \in \mathcal{S}^{(\mathbb{N})}$, $u \in \mathcal{U}$, and each $i \in N$,

a) The aggregation function takes either a **Min** or **Max** form:

$$F\left((\mathbf{u}_i^s)_{s\in S}\right) = \bigwedge_{s\in S} \mathbf{u}_i^s, \quad \text{or} \quad F\left((\mathbf{u}_i^s)_{s\in S}\right) = \bigvee_{s\in S} \mathbf{u}_i^s.$$

b) The selection rule follows either of the forms:

$$\varphi(\mathbf{u}) = \arg\max_{a\in A} \bigwedge_{i=1}^{n} u_i(a),$$

or

$$\varphi(\mathbf{u}) = \arg\max_{a\in A} \bigvee_{i=1}^{n} u_i(a).$$

Proof. By Theorem 3 and Corollary 3 of Czogała and Drewniak (1984), any aggregation function satisfying Monotonicity, Idempotency, Associativity and admitting a neutral element must be either min or max.

Both operators are symmetric and unanimously monotonic. Hence Theorem 1 applies and yields the stated forms of the social choice rule. ■

Remark. In contrast with Theorem 2, whose proof requires a nontrivial iterative argument, Theorem 3 follows immediately from Theorem 1. Once classical axioms imply that F must be either min or max, Aggregation Consistency directly yields the corresponding ex ante social choice rule. More generally, Theorem 1 provides a methodological tool: for any aggregation operator satisfying Symmetry and Unanimous Monotonicity, once its functional form is characterized, the associated social choice rule follows automatically by applying Theorem 1.

4 Conclusion

This article introduced Aggregation Consistency as a bridge between the axioms governing across-state aggregation and those governing across-individual aggregation. The axiom requires that any alternative selected in every state-contingent problem remain selected once the corresponding profiles are aggregated ex ante. Combined with standard assumptions on the selection rule and on the aggregation function, this yields a simple representation: ex ante collective choices maximize a single symmetric aggregator of individual utilities.

When F satisfies Idempotency, Strict Monotonicity and Bisymmetry, classical results imply that F is a quasi-arithmetic mean. Aggregation Consistency then forces the same generator g to govern both intrapersonal aggregation across states and interpersonal aggregation across individuals, leading to an axiomatization of generalized utilitarian rules. By contrast, replacing these axioms with Monotonicity, Associativity and the existence of a Neutral element pins down the min/max aggregation functions and, via Aggregation Consistency, the Rawlsian maximin rule and its maximax dual.

Despite their philosophical opposition, these solution concepts share the same basic social-choice requirements — Nonemptiness, Anonymity, Unanimity and Continuity — and their characterizations follow parallel lines of proof. Our results therefore show how different axioms on F naturally give rise to distinct yet systematically related ex ante social choice rules, providing a unified axiomatic framework for utilitarian and Rawlsian approaches to collective decision-making under uncertainty.

Acknowledgments

This research was funded by the French National Research Agency (ANR) under the Citizens project "ANR-22-CE26-0019-01" and the Condorcet project "ANR-24-EXMA-0001". Financial support of MODMAD is also gratefully acknowledged. This article contributes to the doctoral research of Le Nhat Linh Huynh, who gratefully acknowledges the guidance of her PhD advisory committee, and in particular the valuable advice of Vassili Vergopoulos. We would also like to thank the audience of the “3rd Amsterdam / Saint-Etienne Workshop on Social Choice” and the participants of the SING 20 conference for their valuable comments. All remaining errors are our own.

References

J. Aczél. On mean values. *Bulletin of the American Mathematical Society*, 54(4):392 – 400, 1948.

Florian Brandl and Felix Brandt. Justifying optimal play via consistency. *Theoretical Economics*, 14(4):1185–1201, 2019.

Florian Brandl and Felix Brandt. An axiomatic characterization of nash equilibrium. *Theoretical Economics*, 19(4):1473–1504, 2024.

Florian Brandl, Felix Brandt, and Hans Georg Seedig. Consistent probabilistic social choice. *Econometrica*, 84(5):1839–1880, 2016.

Ernest Czogała and Józef Drewniak. Associative monotonic operations in fuzzy set theory. *Fuzzy Sets and Systems*, 12:249–269, 4 1984. ISSN 0165-0114.

Itzhak Gilboa, Dov Samet, and David Schmeidler. Utilitarian aggregation of beliefs and tastes. *Journal of Political Economy*, 112(4):932–938, 2004.

Stéphane Gonzalez and Nikolaos Pnevmatikos. A story of consistency: bridging the gap between bentham and rawls foundations. *Synthese*, 203, 06 2024.

Michel Grabisch, Jean-Luc Marichal, Radko Mesiar, and Endre Pap. *Aggregation Functions*. Encyclopedia of Mathematics and its Applications. Cambridge University Press, 2009.

John C. Harsanyi. Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. *Journal of Political Economy*, 63(4):309–321, 1955. ISSN 00223808, 1537534X.

Oliver C. Ibe. Chapter 12 - special random processes. In Oliver C. Ibe, editor, *Fundamentals of Applied Probability and Random Processes (Second Edition)*, pages 369–425. Academic Press, Boston, second edition edition, 2014. ISBN 978-0-12-800852-2.

Philippe Mongin. Consistent bayesian aggregation. *Journal of Economic Theory*, 66(2):313–351, 1995.

John Rawls. *A Theory of Justice*. Harvard University Press, Cambridge, MA, 1971.

Leonard J. Savage. *The Foundations of Statistics*. Wiley, 1954.