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Working Paper

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May 16, 2022

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1 Introduction

Since the beginning of the COVID-19 pandemic, many countries have adopted a variety of public policies to prevent the collapse of their labor markets. In this context, short-time work (noted STW hereafter, also known as "short-time compensation" or "shared-work programs") has become one of the most popular tools for limiting job destruction and preventing firm closures by allowing employers facing a temporary shock to reduce working hours instead of laying workers off.

As noted by Boeri and Bruecker (2011), until the late 2000s labor economists paid less attention to STW than to other institutions such as employment protection legislation, unions and collective bargaining, or minimum wages. During the Great Recession, some European countries succeeded in maintaining employment by putting in place generous STW programs. A prominent example is Germany,[1] considered as the gold standard of STW, where the unemployment rate fell from 8.5% (mid-2007) to 7.5% (end of 2009) while, over the same period, the unemployment rate in the US rose from 4.5% to 10% (Source: OECD). The COVID-19 pandemic has generated renewed interest in STW programs. According to the European Central Bank, several tens of millions of workers have benefited from STW since the beginning of the outbreak (Botelho et al., 2020). At the peak of the crisis, the number of beneficiaries reached 12 million in France (47% of employees), 10.6 million in Germany (26%), 8.1 million in Italy (42%), and 3.9 million in Spain (23%). In France, where the program has been used massively, 2.5 billion hours were compensated by the program in 2020 (DARES, 2021a).

Why have governments spent so much on STW since the beginning of the pandemic? Due to the outbreak, many European countries have adopted lockdown or physical distancing measures to tackle the spread of the virus. Some firms continued to operate by generalizing working from home while others were forced, by the nature of their activity, to reduce or stop production. To help companies through the crisis, the French government decided to strengthen the STW program in March 2020 thereby providing France with *"the most protective program in Europe"* (Source: Ministère du Travail, de l'Emploi, et de l'Insertion.). Compared to the pre-pandemic program, the STW program adopted in March 2020 is more generous[2] and less stringent.[3] By allowing firms to cut hours with few constraints, the STW program provides an alternative to lay-offs. It aims at significantly alleviating the burden of labor cost for firms experiencing disruption to their activity,[4] and encouraging firms to adjust at the intensive margin instead of the extensive margin. In this sense, STW allows firms to retain workers, save on hiring and training costs, and limit human capital depreciation resulting from job

[1] In Germany, the *Kurzarbeit* (a STW program) was introduced in 1910, and then developed during the 1920s' hyperinflation crisis and the Great Recession.

[2] With the pre-pandemic STW program, the employer had to pay 70% of the gross salary to the employee, and received a public aid of around 7.50 euro per hour. With the new program, the employer still has to pay 70% of the gross salary to the employee, but receives 70% of the gross salary. See Section 2 for more details.

[3] With the pre-pandemic STW program, the employer had to obtain the authorization of the administration before using STW. With the new program, the employer can use STW before approval. Moreover, the absence of response from the administration within two days is considered as implicit approval. Given the massive number of applications at the beginning of the pandemic, some firms without any economic difficulties used STW to adjust hours (Cahuc et al., 2021).

[4] The French economic plan in response to the COVID-19 pandemic also includes tax measures (reduction or deferral) government-backed loans, solidarity fund, etc. In this paper, we focus on the impact of STW.

loss, which may affect workers' careers and prolong the economic downturn (Jacobson et al., 1993; Ljungqvist and Sargent, 1998; Davis and Von Wachter, 2011; Chéron and Terriau, 2018; Costa Dias et al., 2020). However, STW also has its drawbacks. First, it is a very expensive program.[5] Second, STW may generate windfall effects: some firms may use the program to adjust hours worked for jobs that are not at risk of being destroyed (Cahuc et al., 2021). These windfall effects may be particularly high in tightly regulated labor markets, such as France, where it is difficult to cut hours and wages (Lydon et al., 2019). Consequently, STW raises many questions. Does it save jobs? What is the cost per job saved? How large are the windfall effects? What is the impact on income, consumption, and inequalities?

Surprisingly, despite its increasing use, little is known about the effects of STW. Empirical studies provide mixed results. Calavrezo et al. (2010) use a propensity score matching approach to study the relationship between STW and establishments' survival in France over the period 2000–2005. Their results suggest that STW is associated with more layoffs and lower survival of firms. Using a similar methodology, Kruppe and Scholz (2014) find no significant effect of STW on employment in Germany during the Great Recession. Boeri and Bruecker (2011), Hijzen and Venn (2011) and Cahuc and Carcillo (2011) exploit country-level data and show that STW helped save jobs during the economic downturn. Giupponi and Landais (2018), Kopp and Siegenthaler (2021), and Cahuc et al. (2021) probably provide the most compelling evidence of STW effects. They take advantage of rich firm-level and administrative data to deal with the selection of firms and show that STW saved jobs and stabilized employment during the Great Recession.

In the theoretical literature, STW has received little attention compared to other labor-market institutions. Burdett and Wright (1989) include STW in the implicit contract model of Feldstein (1976) to analyze the effects of unemployment insurance (UI) and STW on layoffs and hours per worker. They show that UI distorts the level of employment while STW distorts hours per worker. Based on a simplified version of this model, Braun and Brügemann (2017) show that STW can improve welfare compared to a system that relies exclusively on UI by mitigating the distortions induced by UI. Tilly and Niedermayer (2017) develop a life-cycle model with search frictions, aggregate and idiosyncratic shocks, and both general and firm-specific human capital. They estimate the model using German data and show that STW substantially reduced job losses and (slightly) improved welfare during the Great Recession. Cooper et al. (2017) develop a search model with heterogeneous multi-worker firms. They show that STW succeeded in keeping unemployment down in Germany in the late 2000s. They also highlight the adverse effect of STW on reallocation: STW lowers the pool of unemployed workers, which reduces the vacancy filling rate in productive firms and generates output losses. Cahuc et al. (2021) develop a model that captures between-firm and within-firm job heterogeneity. They show that STW achieved its goal of preventing layoffs in France during the Great Recession. They also point out that STW generates large windfall effects.

While interesting, all these studies investigate the effects of STW during the Great Recession. As underlined by Cahuc et al. (2021), the effects of STW depend largely on the size of the shock, the take-up rate, and the design of the program. In this sense, the COVID-19 crisis provides an interesting laboratory for investigating the effects of STW. In France, less than 4% of employees benefited from STW each year during the

[5] STW expenses amounted to nearly 26 billion euros throughout 2020 in France (DARES, 2021a)

Great Recession (Cahuc et al., 2021). At the beginning of the COVID-19 pandemic (from March to May 2020), 40% of employees benefited from STW each month. In France, STW has never been studied in the context of historically high take-up rates. Moreover, the STW scheme has been modified several times since the beginning of the COVID-19 crisis (replacement rates, government assistance rates, eligibility criteria, etc.). These changes offer a unique opportunity to analyze the response of the labor market to the design of STW.

In our paper, we investigate the impact of STW in France on the propagation of the pandemic shock. We develop a heterogeneous agents model with search frictions, human capital, and aggregate and idiosyncratic productivity shocks. Firms experiencing large shocks can respond through the intensive (via STW) or the extensive (via dismissals) margin. We calibrate our model using French data and assess its capacity to match both pre-pandemic moments and moments that belong to the adjustments of the labor market and consumption inequalities during the COVID-19 outbreak. We then simulate counterfactual scenarios to quantify what the labor market trajectory would have been under alternative STW schemes: (i) the pre-pandemic STW program, (ii) the most generous program implemented in March 2020, and (iii) a flexible but less generous program than that of March 2020 launched in June 2020. In all these experiments, we also analyze the impact of the policy on labor market stocks and flows, consumption, and inequalities. The distributional effects of the pandemic and lockdown policies have already been investigated (Chetty et al., 2020; Glover et al., 2020; Kaplan et al., 2020). However, all these studies were conducted in the US, where hardly any STW schemes are used. None of these studies analyzes the role played by STW, a policy that has been widely used in Europe. In this sense, our paper makes an important contribution to the STW literature.

Our contribution is fourfold. First, we build a structural model that includes several novelties in a tractable manner: an accurate representation of STW policies, rare disaster shocks, and state-dependent parameters to capture the specific supply and demand effects of the lockdown policies. Second, our model is able to replicate several features of the labor market before and during the pandemic. It matches long-run wealth inequalities, as well as labor market stocks and flows. Taking advantage of a recent study of household consumption by the French National Institute for Statistics and Economic Studies (INSEE), we replicate the heterogeneity in the dynamic profile of consumption by income decile during the pandemic. Third, we highlight the role played by rare disaster shocks and the change in the design of STW policies on the precautionary saving motive and the distortion in assets distribution. Both rare disaster shocks and the more generous STW policies lead to an increase in the precautionary motive and wealth dispersion. Fourth, our counterfactual experiments illustrate the stabilizing virtue of STW and its success in saving jobs. However, as documented previously, the policy has also generated substantial windfall effects characterized by an excessive reduction in hours worked. For instance, if the STW program launched in June 2020 was implemented during the strict lockdown in March 2020, it would have had the same effect on unemployment as the more generous program launched in March 2020, despite involving less of a cut in hours and in household incomes.

The rest of the paper is organized as follows: Section 2 presents the policy responses to the COVID-19 crisis. Section 3 presents the model. Section 4 describes the procedure used to estimate the structural parameters of the model and compares the moments generated by the model to their empirical counterparts. Section 5 explores the effects of STW on employment, labor market transitions, consumption, and inequalities. A final section concludes.

2 The evolution of STW in France

STW is a public program that allows firms experiencing economic difficulties to temporarily reduce the number of hours worked. Employers must compensate workers for the cut in hours, and they receive state support to cover all or part of this compensation. In France, STW (or *"Chômage partiel"* in French) was first introduced back in 1919. It came to be widely used in July 2009, in response to the 2008 financial crisis. The STW scheme was then reformed in 2013 and remained unchanged up to the outbreak of the pandemic.

Before the pandemic, all firms and all employees were eligible for STW, but the firm had to justify the use of STW by one of the following reasons: the economic situation; difficulties in the supply of raw materials or energy; disaster or bad weather of an exceptional nature; the transformation, restructuring, or modernization of the company; any other exceptional circumstances. Unlike what happens in other countries, there is no legal obligation associated with public aid.[6] To benefit from STW, employers have to follow the procedure described in Table 1. Under the initial STW program (from July 2013 to February 2020), the employer was required to consult the work council or–in the absence of a work council–the staff representatives before submitting its STW application. The employer then had to wait for approval from the Administration to implement STW. In the absence of a response within 15 days, the application was deemed approved.

In March 2020, in response to the spread of COVID-19, governments resorted to non-pharmaceutical measures to limit the transmission of the virus. On March 16, 2020, President Emmanuel Macron announced a nationwide lockdown to take effect the next day. On March 25, 2020, the STW program was strengthened to limit the economic impact of the lockdown. The new scheme (noted STW_2) allowed employers to implement STW immediately, **before consulting the work council or staff representatives**, and **before obtaining the Administration's authorization**. Moreover, under this new system, the absence of an answer from the Administration within **2 days** was considered as approval (compared to 15 days previously). The compensation paid by employers to employees was maintained at 70% of the gross salary, but the compensation paid by the State to employers was raised. Before March 2020, the hourly allowance reimbursed to employers was set at 7.74 euros in firms with up to 250 employees and 7.23 euros in firms with more than 250 employees. In March 2020, the government decided to **cover 100% of the compensation paid by the employers to the employees**, within the limit of 4.5 times the French minimum wage (8.03 euros in 2020), regardless of the firm's size. Given the generosity of this program,[7] STW was used massively. Two weeks after the announcement of the new STW scheme, almost half of the employees in the private sector were put on STW.

As underlined by France's Cour des Comptes (2021) (national audit office), this un-

[6] In some countries, public aid is conditional on the implementation of training programs (Portugal, Netherlands, Hungary) or recovery plans (Italy, Spain). In France, there are no such requirements.

[7] Some other countries, such as Germany or Italy, have already implemented STW programs covering 100% of the compensation paid by the employer to the employee, but the upper limit was much lower.

precedented volume of applications generated a large number of frauds. During the first weeks, the Administration was unable to examine the applications manually. Consequently, most applications were accepted automatically, without being inspected. This generated large windfall effects. Based on VAT declarations, *France Stratégie* estimates that around 8% of firms benefited from STW without having economic difficulties.[8] Together with the end of the first wave of COVID-19, it led the Government to slightly reduce the generosity of the STW program in June 2020 (See SWT_3). First, **the rate of subsidy of STW was reduced from 70% to 60% of gross wages**. This program was maintained throughout the year 2020, in particular to help companies through the second lockdown, implemented at the end of October 2020. Second, the government decided in October 2020 to **restore the 15-day period** after which the absence of response from the Administration was considered as approval.

Figure 1 shows the remainder of short-time compensation, for non-worked hours, paid by the firm under the three types of STW programs. From the workers' side, compensation remains unchanged and comes to 70% of their gross salary per reduced hour since the beginning of the pandemic. Under SWT1, the fraction of the wage rate per reduced hour charged to the firm is around zero for workers on the minimum wage and immediately increases above that. Under SWT2 and SWT3, the fraction is constant up to 4.5 times the hourly minimum wage. It is zero in the former case and 10% in the latter case. This means that, for a 1-hour reduction, employees receive 70% compensation of their hourly wage. If the hourly wage is less than 4.5 times the minimum wage, 100% of the compensation is financed by the government under SWT2 and 85% (60%/70%) under SWT3. As for SWT1, the bill footed by the firm is higher if the hourly wage exceeds 4.5 times the minimum wage. The design of the STW programs can be summarized by three main parameters: (i) the fraction of the hourly wage received by the employee, (ii) the fraction charged to the firm, and (iii) the threshold above which the fraction charged to the firm increases. The three STW programs have different values for (ii) and (iii), which provides an interesting environment in which to quantify the impact of the policy for various degrees of generosity.

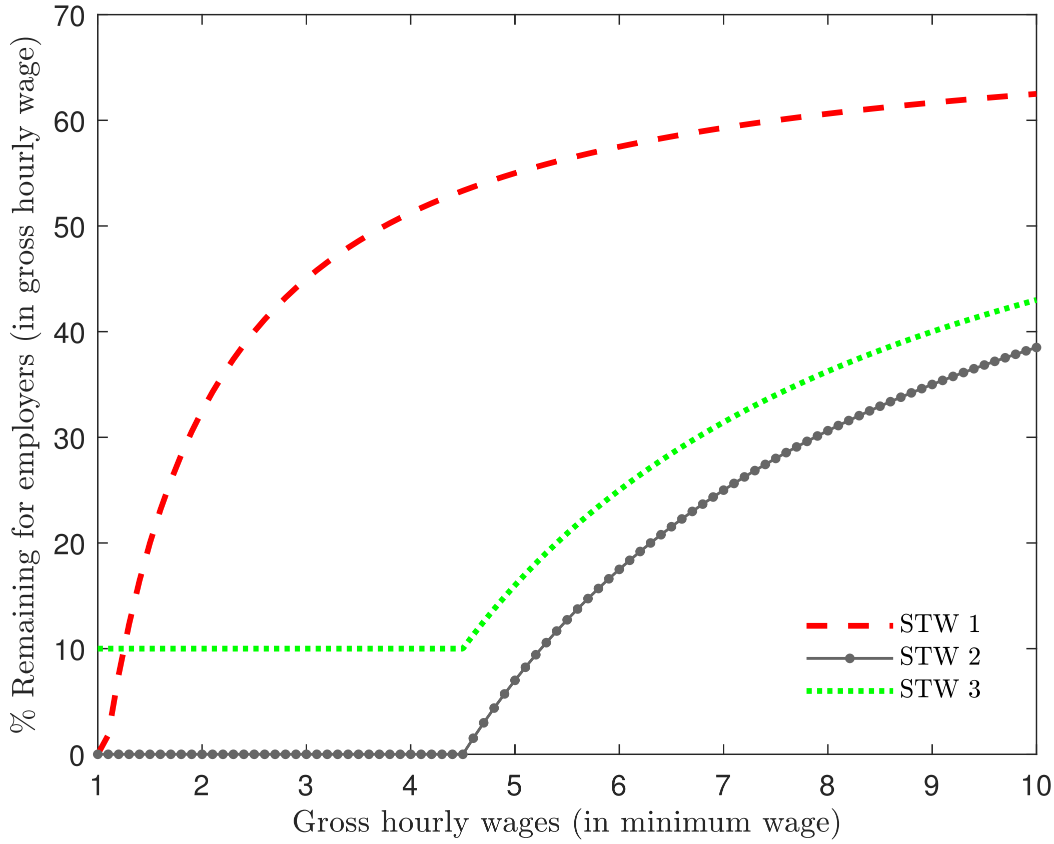
[8] In France, labor law is stringent and it is difficult to adjust hours worked downward. Some firms used STW to circumvent the legislation. Moreover, some firms benefited from STW while continuing their activity informally.

Table 1: Evolution of the STW scheme

	STW_1	STW_2	STW_3
	From July 2013 to February 2020	From March 2020 to May 2020	From June 2020 to December 2020
Conditions	Economic situation; Difficulties in the supply of raw materials or energy; Disaster or bad weather of exceptional nature; Transformation, restructuring or modernization of the company; Any other exceptional circumstances.		
Procedure	STEP 1: The employer consults the work council or the staff representatives.	STEP 1: The employer can implement STW immediately. The employer then must consult the work council or the staff representatives within 2 months.	
	STEP 2: The employer seeks authorization from the Administration	STEP 2: The employer can implement STW immediately. The employer then must then seek authorization from the Administration within 30 days.	
	STEP 3: The Administration acknowledges the application.		
	STEP 4: The Administration examines the application.		
	STEP 5: The Administration issues its decision. If the Administration fails to answer within 15 days, it is deemed to accept the employer's application.	STEP 5: The Administration issues its decision. If the Administration fails to answer within 2 days [from March 2020 to October 2020] - 15 days [since October 2020], it is deemed to accept the employer's request.	
	STEP 6: The employer can implement STW.	STEP 6: The employer can implement STW prior to Administration's approval.	
Duration	The maximum duration for STW cannot exceed 6 months.	The maximum duration for STW cannot exceed 12 months.	
Compensation paid by the employer to the employee	70% of gross salary (floor: net minimum wage; cap: none)		
Compensation paid by the State to the employer	7.74 (7.23) euros per hour in firms with less (more) than 250 employees.	70% of gross wage (floor: net minimum wage; cap: 70% $\times$ 4.5 gross minimum wages)	60% of gross wage (floor: net minimum wage; cap: 60% $\times$ 4.5 gross minimum wages)

Sources: Circulaire DGEFP n° 2013-12, Décret n° 2020-325, Ordonnance n° 2020-770

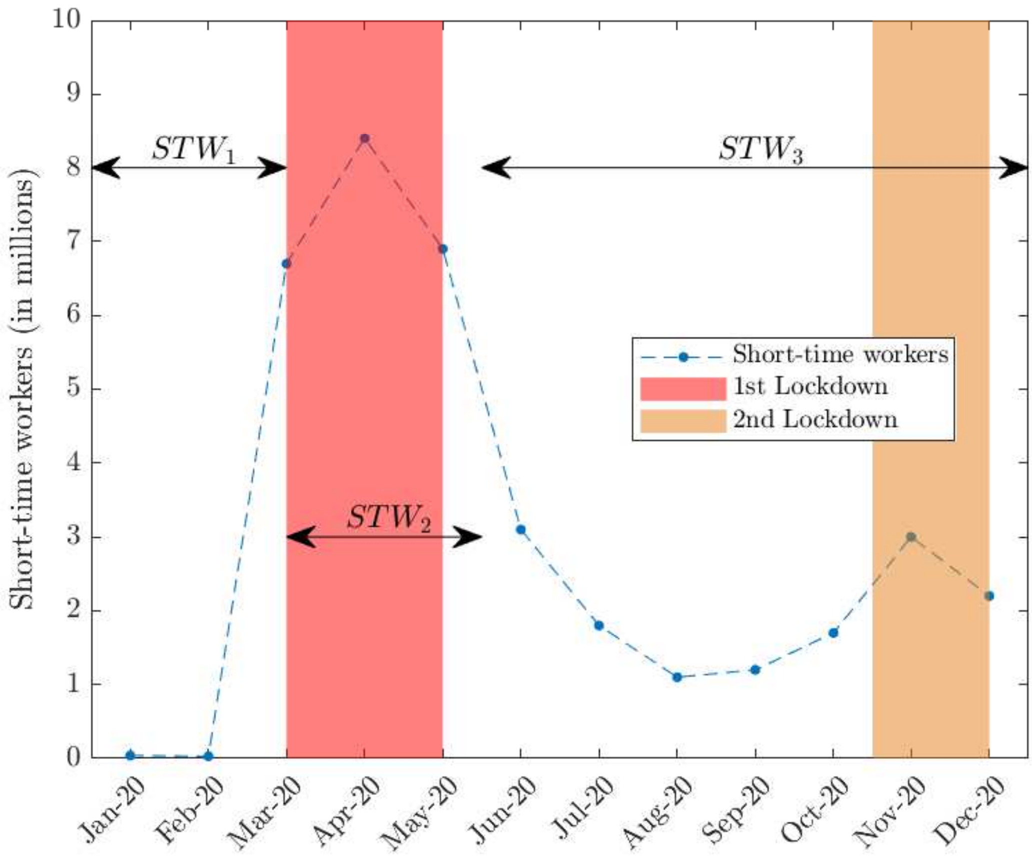
Figure 1: Remainder of short-time compensation to be covered by firms after government subsidies



Sources: Circulaire DGEFP nº 2013-12, Décret nº 2020-325, Ordonnance nº 2020-770.
All replacement rates and thresholds are given in Tables 1 and 3

Figure 2 shows the change in the number of short-time workers since the beginning of the pandemic. During the 2010s, on average, fewer than 50,000 workers benefited from STW each month (DARES, 2021b). In March 2020, 6.7 million workers were placed on STW. The number of short-time workers peaked at 8.4 million in April 2020 and then decreased as a result of the easing of the first lockdown. In September 2020, France was hit by the second epidemic wave. At the end of October 2020, France entered its second lockdown. As a result, the number of short-time workers rose rapidly and declined at the end of December when the government announced the easing of the second lockdown.

Figure 2: Number of short-time workers during the COVID-19 pandemic



Source: DARES

3 Model

We develop a heterogeneous agents model similar to those of Aiyagari (1994), Huggett (1996), and Krusell and Smith (1998), in which individuals face income risks due to endogenous hirings and separations in the labor market. The model includes search and matching frictions in the spirit of Mortensen and Pissarides (1994). As in Ljungqvist and Sargent (1998, 2008), a worker's general human capital appreciates when employed and depreciates when unemployed. Adjustment of employment occurs along the extensive margin (separation) and the intensive margin (STW). As in Bils et al. (2011), we consider a small open economy in which the real interest rate is determined exogenously to fluctuations. We underline that our analysis is conducted in a partial equilibrium setup. As we are interested in the short-term effects of lockdown, we ignore the feedback effects of a closing of the model and the question of the financing of the economic policy measures is left aside. The economy is subject both to standard business cycle-type aggregate shocks that impact job productivity and to rare disaster shocks that affect different components of aggregate supply and aggregate demand. Rare disaster shocks correspond to the lockdown policies imposed by the government.[9]

3.1 Heterogeneities

Wealth. Let $a \in [a_{\min}, a_{\max}]$ be household wealth. Due to incomplete markets, agents cannot perfectly insure against idiosyncratic shocks, which create heterogeneities in asset accumulation.

Human capital. We consider human capital accumulation as a learning-by-doing process as in Ljungqvist and Sargent (1998, 2008). Human capital is general,[10] i.e. skills are portable across jobs. There is a discrete support of human capital level $h = \{h_1, ..., h_H\}$. Human capital appreciates with probability ψ_n when the individual is employed, and depreciates with probability ψ_u when unemployed. As in Lalé (2018), the evolution of human capital is governed by a Markov process. The probability of moving from human capital h to h' is written:

$$\mu_n(h,h') = \begin{cases} 1-\psi_n & \text{if} \quad h < h_H \quad \text{and} \quad h' = h, \\ \psi_n & \text{if} \quad h < h_H \quad \text{and} \quad h' = h+1. \end{cases} \tag{1}$$

$$\mu_u(h,h') = \begin{cases} 1-\psi_u & \text{if} \quad h > h_1 \quad \text{and} \quad h' = h, \\ \psi_u & \text{if} \quad h > h_1 \quad \text{and} \quad h' = h-1. \end{cases} \tag{2}$$

We assume that under the STW program, human capital h still appreciates as if the worker was fully employed.[11]

Idiosyncratic productivity shocks Each firm has only one job that is either filled or vacant. Each filled job is characterized by an idiosyncratic productivity ε that evolves according to a Markov Process. Every period, a new value ε' is drawn from the conditional distribution $G(\varepsilon'|\varepsilon)$. For a new match, a new idiosyncratic productivity ε' is drawn from the unconditional distribution $G_0(\varepsilon')$.

Aggregate shocks We consider one aggregate shock that translates into the different components of the model (supply and demand). We denote by z the aggregate state of the economy. It follows a stochastic process. In normal times, we adopt a standard AR(1) representation with persistence ρ_z and in which innovations are normally distributed $\epsilon^z \sim \mathcal{N}(0, \sigma_z^2)$. We use the Rouwenhorst technique to discretize the AR(1) process into a finite Markov chain with transition matrix $Q_0(z'|z)$.

We assume that the COVID-19 pandemic corresponds to a rare disaster see Barro (2006)). As a consequence, the occurrence of this event does not fit with a standard

[9] In this paper, we take the lockdown policies as given. When the COVID-19 outbreak occurs, we assume that it forces the government to impose the lockdown. We thus refer to a lockdown shock as if it were purely exogenous. We do not investigate what would have happened if the government decided not to impose restrictions on individual mobilities.

[10] We do not consider the existence of firm-specific human capital. In this sense, the effect of job loss on human capital and labor market trajectories quantified in our paper can be viewed as a lower bound of the true effect (Tilly and Niedermayer, 2017).

[11] As a complement to the STW program, the government set up a training subsidy (*"FNE-Formation"*) for individuals benefiting from STW. The subsidy rate is equal to 100% of the training cost for firms with fewer than 300 employees, and 70% for firms with 300 employees or more. To proxy this program in a simple way, we consider that human capital continues to appreciate during STW at no cost.

symmetrical distribution of innovations. To keep things simple and tractable, we assume there is a probability λ of a pandemic occurring and resulting in a lockdown. If a lockdown does occur, it may be **strict**[12] ($z = z_s$), with probability ϖ, or **light**[13] ($z = z_l$), with probability $1 - \varpi$. z_s and z_l are thus two states characterizing different types of lockdown. The probability of remaining in lockdown is φ.

To sum up, if the economy is initially in normal times, the probability of a rare disaster occurring is λ. With a probability $1 - \lambda$ the aggregate state evolves according to a standard discretized AR(1) process. If the economy is initially in lockdown, the probability of it returning to a normal situation is $1 - \varphi$. We denote by $P(z'|z)$ the full transition matrix of the aggregate state combining normal business cycle states $z = \{z_1, z_2, ..., z_n\}$ and rare disasters[14] $z = \{z_s, z_l\}$.

Regime-switching policy To model the STW policy, we adopt a regime-switching approach. Let $\boldsymbol{s} = \{1, 2, 3\}$ be the regime associated with a particular design of the STW policy (See Table 1). $\boldsymbol{s} = 1$ corresponds to the pre-COVID STW (STW_1), $\boldsymbol{s} = 2$ is the STW applying from March to June 2020 (STW_2), and $\boldsymbol{s} = 3$ corresponds to the STW implemented from June to December 2020 (STW_3). From a timeline perspective agents did not anticipate the switch from regime 1 to 2 nor the switch from regime 2 to 3. However, once STW_2 or STW_3 are introduced, they take into account that the economy may move between regime 2 and 1 or between regime 3 and 1. For instance, before the first lockdown agents only believe that STW_1 applies and do not anticipate any switch to STW_2 or STW_3. During the first lockdown, agents take into account that the emergency STW program allowing firms to adjust hours worked freely is likely to end. This means that agents form new expectations that include the switch between lockdown episodes for which STW_2 is allowed and normal times during which only STW_1 is available. When the government adopts STW_3, agents form new expectations based on potential regime-switches between STW_3 and STW_1.

Regime switching is conditional on the aggregate state and, in particular, on whether the economy is in lockdown or not. We adopt a Markovian representation to represent state switching. Let $S(\boldsymbol{s}'|\boldsymbol{s}; z)$ be the transition matrix of moving from state $\boldsymbol{s}$ to state $\boldsymbol{s}'$ given the aggregate state z. We capture the persistence in the STW regime through the parameter ν that defines the lag between the decision to end the emergency STW program since the end of the lockdown. Appendix B.2 describes how the matrix is constructed. From now on, we simply assume that agents form their decisions given the probabilities of moving from one regime to another.

3.2 Search and matching

The model is characterized by matching frictions. Hirings depend on the number of unemployed workers u and the number of vacancies v. The two inputs are combined in a matching function. As in Menzio et al. (2016), we consider that searching is directed. Unemployed workers with human capital h only search vacant jobs characterized by the same human capital level. The matching function is written:

$$m(h, z, \boldsymbol{s}) \quad = \quad m(\chi(z), u(h, z, \boldsymbol{s}), v(h, z, \boldsymbol{s})). \tag{3}$$

[12] As in France in March 2020 where strict restriction were imposed on household mobility.

[13] As in France in November 2020 where mobility was restricted but not as much as in March 2020.

[14] Appendix B.1 describes how we build $P(z'|z)$.

where $\chi(z)$ captures the switch in matching efficiency that occurred during the COVID-19 lockdowns. The matching function (3) is increasing and concave in its two arguments and exhibits constant returns to scale. The probability of an unemployed worker finding a job and the probability of the firm filling a job vacancy are given respectively by:

$$f(h,z,\mathbf{s}) = \frac{m(u(h,z,\mathbf{s}),v(h,z,\mathbf{s}))}{u(h,z,\mathbf{s})}, \tag{4}$$

$$q(h,z,\mathbf{s}) = \frac{m(u(h,z,\mathbf{s}),v(h,z,\mathbf{s}))}{v(h,z,\mathbf{s})}, \tag{5}$$

3.3 Productivity and income

Let $\ell \in [\underline{\ell}, \overline{\ell}]$ be the individual hours worked chosen by the firm under the STW program. $\underline{\ell}$ is the lowest number of hours worked allowed from STW and $\overline{\ell}$ corresponds to the number of hours worked in a full-time position[15] (without STW). The productivity of a job is given by the following function:

$$y(\varepsilon,h,\ell,z) = (zh\varepsilon\ell)^{\alpha} \tag{6}$$

Following Burdett et al. (2011), Bagger et al. (2014), and Blundell et al. (2016), we consider the following rule for hourly wages:

$$w(\varepsilon,h) = \max(w_{\min},(\varepsilon h)^{\gamma}) \tag{7}$$

where γ governs the curvature of the hourly wage with respect to the idiosyncratic productivity ε and human capital h. $w_{\min}$ is the minimum hourly wage. This rule implies that the hourly wage does not respond to aggregate productivity shocks.[16] We deem it realistic as we focus on a relatively short-time window. This assumption is also in line with Tilly and Niedermayer (2017) who find that the labor market response in Germany during the Great Recession was in hours worked (through STW), not in hourly wages.

3.4 Bellman equations

Optimal continuation and acceptance decisions. We denote by W, U, J and V, the value functions for an employed worker, an unemployed worker, a filled job, and a vacant job, respectively. The optimal decisions are given by:

$$\Omega(a,\varepsilon,h,z,\mathbf{s}) = \max[W(a,\varepsilon,h,z,\mathbf{s}),U(a,h,z,\mathbf{s})], \tag{8}$$

$$\Lambda^{n}(\varepsilon,h,z,\mathbf{s}) = \max[J(\varepsilon,h,z,\mathbf{s}),V(h,z,\mathbf{s})], \tag{9}$$

$$\Lambda^{o}(\varepsilon,h,z,\mathbf{s}) = \max[J(\varepsilon,h,z,\mathbf{s}),-F(w\overline{\ell})], \tag{10}$$

[15] For the sake of simplicity, we consider that there are no part-time jobs in the economy (in reality, part-time jobs represent around 15% of total employment in France (Fontaine et al., 2018)). Moreover, we consider that, in normal times, firms' capacity to reduce working hours is limited. This reflects the difficulty that firms have in adjusting hours worked downward in France in the absence of STW, due to stringent legislation.

[16] As a robustness test, we simulate our model by considering that wages respond to aggregate productivity shocks. The results, provided in Appendix A.3, are almost the same.

where $\Omega(a,\varepsilon,h,z,\boldsymbol{s})$ is the optimal acceptance of a match and continuation employment decision for a worker. $\Lambda^n(\varepsilon,h,z,\boldsymbol{s})$ denotes the firm's acceptance decision of a new match. $\Lambda^o(\varepsilon,h,z,\boldsymbol{s})$ stands for the optimal employment continuation decision for the firm. As in Mortensen and Pissarides (1999), the firm's acceptance and continuation decisions differ due to firing costs $F(w\bar{\ell})$. They are charged to the firm if an existing employment relation ceases but not if a new match fails to become an employment relation. Note that as the wage does not depend on the asset level, the value of the firm is independent of the worker's wealth.

3.5 Firms with a filled job

The value of a firm is written:[17]

$$\begin{aligned} J(\varepsilon,h,z,\boldsymbol{s}) &= y - w\ell(1+\tau(w)) - \Psi(w)w\max\left(\bar{\ell}-\ell,0\right) \\ &+ \beta_f(1-\delta)\sum_{h'}\mu_n(h,h')\int\int\int\Lambda^o(\varepsilon',h',z',\boldsymbol{s}')dG(\varepsilon'|\varepsilon)dS(\boldsymbol{s}'|\boldsymbol{s};z')dP(z'|z) \end{aligned} \quad (11)$$

where $\tau(w)$ is the payroll tax rate defined later and δ the exogenous separation rate. $\Psi(w)$ is the remainder of short-term compensation for non-worked hours paid by the firm. Absent any STW policy,[18] hours are fixed and equal to $\ell = \bar{\ell}$. Under the STW policy, the firm is allowed to decide the level of individual hours unilaterally. The optimal level of hours worked chosen by the firm is $\ell^\star$, which is given by:

$$\ell^\star = \arg\max_{\ell\in[\underline{\ell},\bar{\ell}]} y - w\ell(1+\tau(w)) - \Psi(w)w\max\left(\bar{\ell}-\ell,0\right) - Q^{stw}(\boldsymbol{s})\mathbb{1}_{\ell<\bar{\ell}} \quad (12)$$

$\mathbb{1}_{\ell<\bar{\ell}}$ corresponds to an indicator variable taking the value 1 if $\ell < \bar{\ell}$. Q^{stw} represents an administrative cost of applying to STW (delays, proof of disruptions to economic activity, disasters, etc). It is paid only if the firm aims at reduction in hours worked through the STW program. The STW approval process was modified during the pandemic (See Table 1). Before March 2020, firms had to provide proof of economic difficulties to benefit from STW. The refusal rate was significant (about 10%), and the procedure was stringent and costly (Cahuc et al., 2018). During the pandemic, the procedure was eased somewhat. Applications were accepted automatically and without strict scrutiny (Cour des Comptes, 2021), with the result that the refusal rate fell to 0 (UNEDIC, 2020). To take these changes into account, we consider that the cost is regime-dependent.

Beyond administrative costs, the wage bill charged to the firm under each STW policy regime is defined as:

$$\Psi(w) = \begin{cases} \rho_F - \rho_X\frac{w_{\min}}{w} & \text{if} \quad \boldsymbol{s}=1 \\ \rho_F\max\left(1-4.5\times\frac{w_{min}}{w},0\right) & \text{if} \quad \boldsymbol{s}=2 \\ \left(\rho_F-\rho_G\min\left(1,4.5\times\frac{w_{min}}{w}\right)\right) & \text{if} \quad \boldsymbol{s}=3 \end{cases}$$

Under the pre-COVID STW program (STW_1, $\boldsymbol{s}=1$), a fraction ρ_F of the wage bill associated with the hours reduction is charged to the firm. The firm receives a fixed

[17] To simplify the exposition of the model we remove indices ε,h,ℓ,z for productivity and indices ε,h for wages.

[18] Firms made little use of STW before the pandemic. During the 2000s and 2010s, less than 1% of employees were on STW on average, with a peak of 4% during the Great Recession (Cahuc et al., 2021).

amount as a subsidy corresponding to a fraction ρ_X of the minimum wage. Under the March STW program triggered during the first lockdown ($STW_2, \boldsymbol{s} = 2$), the wage bill is zero if the hourly wage rate is below 4.5 times the minimum wage. Above that threshold, the wage bill increases. If $w \longrightarrow +\infty$ the wage bill is passed on to the firm at a rate ρ_F. Under the June STW program ($STW_3, \boldsymbol{s} = 3$) the firm is charged a fraction $\rho_F - \rho_G$ when the hourly wage ranges from $w_{\min}$ to $4.5 \times w_{\min}$. Above this level, the fraction of the wage bill passed on to the firms follows the same shape as for the March STW (see Figure 2).

The second term on the right-hand-side of Equation (11) corresponds to the expected value of the filled job, discounted at rate β_f. The job may be exogenously destroyed with probability δ. With probability $\mu_n(h, h')$ the level of human capital changes from h to h'. The individual and the aggregate productivity changes following the stochastic processes $G(\varepsilon'|\varepsilon)$ and $P(z'|z)$ respectively. In the event of a change in aggregate state z', the probabilities governing the regime-switching are updated in accordance with the state-contingent transition matrix $S(\boldsymbol{s}'|\boldsymbol{s}; z')$.

3.6 Firms with a vacant job

The value of a vacant job is written:

$$\begin{aligned} V(h,z,\boldsymbol{s}) &= -\kappa \\ &+\beta_f \int\int\int \left[\begin{array}{c} (1-q(h,z,\boldsymbol{s})) \max\limits_{h} V(h,z',\boldsymbol{s}') \\ +q(h,z,\boldsymbol{s})\Lambda^n(\varepsilon',h,z',\boldsymbol{s}') \end{array} \right] dG_0(\varepsilon')dS(\boldsymbol{s}'|\boldsymbol{s};z')dP(z'|z) \end{aligned} \tag{13}$$

Each vacant job is directed toward a worker with human capital h. The cost of posting a vacancy is κ. In case of a successful match, the job becomes filled with probability q.

3.7 Bellman equations workers

Employed workers Employed workers choose their consumption-saving plan to maximize their expected utility. The Bellman equation is written:

$$\begin{aligned} W(a,\varepsilon,h,z,\boldsymbol{s}) &= \max_{c,a'} \Bigg\{ u(c) \\ &+ \beta_w(z) \sum_{h'} \mu_n(h,h') \int\int\int \left[\begin{array}{c} (1-\delta)\ \Omega(a',\varepsilon',h',z',\boldsymbol{s}') \\ +\delta\ U(a',h',\boldsymbol{s}') \end{array} \right] dG(\varepsilon'|\varepsilon)dS(\boldsymbol{s}'|\boldsymbol{s};z')dP(z'|z). \Bigg\} \\ s.t.& \\ & c + a' \leq (1+r)a + w\left(\min\left(\ell,\bar{\ell}\right)(1-\tau^s) + \rho_F \max\left(\bar{\ell}-\ell, 0\right)\right), \\ & a' \geq \underline{a}. \end{aligned} \tag{14}$$

The agent derives utility from consumption c following a standard CRRA utility function $u(c)$. In order to replicate the impact of lockdowns on consumption, we impose that at any time $c < \bar{c}(z)$, we explain this constraint in section 3.8. The flow of utility from future consumption is discounted at rate $\beta_w(z)$. r is the real interest rate and τ^s is the social security contribution rate for employees. In the model, a worker may receive STW compensation. The benefit corresponds to a fraction ρ_F of the worker's regular earnings (70% of the gross salary a worker receives short-time, whatever the period considered). Note that benefits under STW are not subject to the tax rate τ^s. With a probability $\mu_u(h,h')$ the employed worker acquires greater human capital. In the event of an exogenous or endogenous job separation, she becomes unemployed.

Unemployed workers Unemployed workers decide on their consumption-saving plan so as to maximize their expected utility. The Bellman equation is given by:

$$\begin{aligned} U(a,h,z,\mathbf{s}) &= \max_{c,a'} \Big\{ u(c) \\ &+ \beta_w(z) \sum_{h'} \mu_u(h,h') \int\int\int \left[\begin{array}{c} (1-f(h,z,\mathbf{s}))U(a',h',z',\mathbf{s}') \\ +f(h,z,\mathbf{s})\Omega(a',\varepsilon',h',z',\mathbf{s}') \end{array} \right] dG_0(\varepsilon')dS(\mathbf{s}'|\mathbf{s};z')dP(z'|z) \Big\} \\ s.t. & \\ & c+a' \leq (1+r)a+b \\ & a' \geq \underline{a} \end{aligned} \tag{15}$$

Unemployed workers derive their consumption-saving plan given their budget constraint. b is the income when unemployed. Unemployed workers receive a contact with a vacant job with probability $f(h)$, and lose their human capital at rate $\mu_u(h,h')$.

3.8 Aggregate shock propagation

We assume that the aggregate shock propagates into four different variables:

- **The productivity** $y(\varepsilon,h,\ell,z,\mathbf{s})$. Equation (6) captures the decline in productivity outside a pandemic (business cycle) and during the pandemic thanks to the special lockdown states: z_l for a light lockdown and z_s for the strict lockdown. This parameter may reflect either problems in the supply of raw materials or the productivity loss associated with social distancing or remote working (Bloom et al., 2020).

- **The workers' discount factor** $\beta_w(z)$. As highlighted by De Nardi et al. (2017) and Albertini et al. (2021b), the time preference can be affected by a health shock. It is likely that a lockdown will alter the preference for the present. $\beta_w(z)$ captures changes in time preferences that could be associated to the pandemic and thus mimic the demand effects of the lockdown. An increase in $\beta_w(z)$ causes a decline in consumption which occurs during the first and the second lockdown. We adopt the following representation:

$$\beta_w(z) = \begin{cases} \tilde{\beta} & \text{if} \quad z \in \{z_l, z_s\} \\ \beta & \text{otherwise} \end{cases}$$

  Note that we consider only changes in the worker's discount factor. The firm's discount factor is always equal to $\beta_f = \beta$

- **Consumption limit** $\bar{c}(z)$. The consumption limit means that consumption has an upper bound during a strict lockdown period because the lockdown severely restricts individuals' movement. As this upper bound is expressed as a level, it mainly impacts the wealthiest households. This allows us to take into account the fact that lockdown periods have greatly reduced the consumption of leisure and luxury goods (hotels, restaurants, travel, etc.), whose share in total consumption increases with income level[19] (Anguis, 2006) during lockdowns. We adopt the following formulation:

$$\begin{aligned}\bar{c}(z) &= \begin{cases} \tilde{c} & \text{if} \quad z = z_s \\ +\infty & \text{otherwise} \end{cases} \\ \text{and, } c &< \bar{c}(z)\end{aligned}$$

- **Matching efficiency** $\chi(z)$**.** Changes in $\chi(z_t)$ reflect the impact of COVID-19 on matching efficiency. At some points during the pandemic, the number of job vacancies increased relative to the number of unemployed workers, while the number of filled jobs remained relatively stable (DARES, 2021a). This implies that the outbreak reduced the matching efficiency. On the one hand, some Human Resources (HR) managers and unemployed workers were affected directly (by being infected by the virus) or indirectly (by being put on STW). This reduced the effective amount of time devoted to the matching process. On the other hand, the implementation of lockdown/social distancing measures and the development of working from home strongly affected HR practices and altered the hiring process (Hamouche, 2021). To capture the switch in the matching efficiency, we consider the following functional form:

$$\chi(z) = \begin{cases} \tilde{\chi} & \text{if} \quad z = z_s \\ 1 & \text{otherwise} \end{cases}$$

3.9 Job creation and job destruction

Job creation The job creation condition is given when the profit opportunity of a vacant job is exhausted, that is when $V(h, z, \mathbf{s}) = 0$. From Equation (13), one has:

$$\frac{c}{q(h, z, \mathbf{s})} = \beta_f \int \int \int \Lambda^n(\varepsilon', h, z', \mathbf{s}') dG_0(\varepsilon') dS(\mathbf{s}'|\mathbf{s}; z') dP(z'|z), \tag{16}$$

which shows that the expected cost of posting a vacancy is equal to the expected gains.

Job Separation Let $\mathbb{1}_\varrho(a, \varepsilon, h, z, \mathbf{s})$ be an indicator function defining the optimal separation decision for a firm (ϱ = firm) and for a worker (ϱ = worker). It is defined as:

$$\mathbb{1}_\varrho(a, \varepsilon, h, z, \mathbf{s}) = \begin{cases} 0 & \text{if} \quad \begin{array}{lll} \varrho = firm & \text{and} & J(\varepsilon, h, z, \mathbf{s}) \leq -F(w\ell) \\ \varrho = worker & \text{and} & W(a, \varepsilon, h, z, \mathbf{s}) \leq U(a, h, z, \mathbf{s}) \end{array} \\ 1 & \text{otherwise} \end{cases} \tag{17}$$

[19] An alternative solution would be to consider a basket of goods. This would require determining the relative prices in the economy. To keep the model tractable, we introduce a cap on consumption during lockdowns to mirror the differences in the composition of the basket of goods by income level.

Since there is no bargaining over the surplus to set wages or hours in our model, this decision is not necessarily mutually beneficial to both parties. The effective optimal separation and match acceptance decision is the product of the two, that is:

$$\mathbb{1}(a,\varepsilon,h,z,\boldsymbol{s}) \quad = \quad \mathbb{1}_{\text{firm}}(\varepsilon,h,z,\boldsymbol{s}) \times \mathbb{1}_{\text{worker}}(a,\varepsilon,h,z,\boldsymbol{s}) \tag{18}$$

The match is destroyed if either the firm or the worker decides to break the employment relation.

3.10 Stationary distribution

An employed worker and an unemployed worker are characterized by the individual states (a,ε,h) and (a,h), respectively. The aggregate state is $(z,\boldsymbol{s})$. $n(a,\varepsilon,h;z,\boldsymbol{s})$ is the number of employed workers in state $(a,\varepsilon,h;z,\boldsymbol{s})$. Likewise, $u(a,h;z,\boldsymbol{s})$ is the number of unemployed workers in state $(a,h;z,\boldsymbol{s})$. If the optimal a' is selected in the grid $\mathcal{A}=\{a_1,...,a_{N_a}\}$, the agent decision rules are of the form $a'=g_n(a,\varepsilon,h;z,\boldsymbol{s})$ and $a'=g_u(a,h;z,\boldsymbol{s})$ for employed workers and unemployed workers respectively.

The stationary distribution of the workers (consider the aggregate state $(z,\boldsymbol{s})$ as given) is given by:

$$\begin{aligned}
n(a',\varepsilon',h';z,\boldsymbol{s}) &= (1-\delta)\sum_{a\in\mathcal{A}}\sum_{\varepsilon\in\mathcal{E}}\sum_{h\in\mathcal{H}}\mathbb{1}(a'=\phi_n(a,\varepsilon,h,z,\boldsymbol{s}))G(\varepsilon'|\varepsilon)\mu_n(h,h')I^o(a',\varepsilon',h',z,\boldsymbol{s})n(a,\varepsilon,h;z,\boldsymbol{s}) \\
&+ \sum_{a\in\mathcal{A}}\sum_{h\in\mathcal{H}} f(h,z,\boldsymbol{s})\mathbb{1}(a'=\phi_u(a,h,z,\boldsymbol{s}))G_0(\varepsilon')\mu_u(h,h')I^n(a',\varepsilon',h',z,\boldsymbol{s})u(a,h;z,\boldsymbol{s}) \qquad (19)\\
u(a',h';z,\boldsymbol{s}) &= \sum_{a\in\mathcal{A}}\sum_{\varepsilon\in\mathcal{E}}\sum_{h\in\mathcal{H}}\mathbb{1}(a'=\phi_n(a,\varepsilon,h,z,\boldsymbol{s}))G(\varepsilon'|\varepsilon)\mu_n(h,h')(\delta+(1-\delta)(1-I^o(a',\varepsilon',h',z,\boldsymbol{s}))n(a,\varepsilon,h;z,\boldsymbol{s}) \\
&+ \sum_{a\in\mathcal{A}}\sum_{h\in\mathcal{H}}\mathbb{1}(a'=\phi_u(a,h,z,\boldsymbol{s}))G_0(\varepsilon')\mu_u(h,h')(1-f(h,z,\boldsymbol{s})+f(h,z,\boldsymbol{s})(1-I^n(a',\varepsilon',h',z,\boldsymbol{s}))u(a,h;z,\boldsymbol{s}) \qquad (20)
\end{aligned}$$

with

$$\begin{aligned}
I^o(a',\varepsilon',h',z,\boldsymbol{s}) &= \mathbb{1}(W(a',\varepsilon',h',z,\boldsymbol{s}) \geq U(a',h',z,\boldsymbol{s})) \times \mathbb{1}(J(\varepsilon',h',z,\boldsymbol{s}) \geq -F(\varepsilon',h')) \\
I^n(a',\varepsilon',h',z,\boldsymbol{s}) &= \mathbb{1}(W(a',\varepsilon',h',z,\boldsymbol{s}) \geq U(a',h',z,\boldsymbol{s})) \times \mathbb{1}(J(\varepsilon',h',z,\boldsymbol{s}) \geq 0)
\end{aligned}$$

3.11 Equilibrium

DEFINITION 1. *Given exogenous processes for human capital h, aggregate productivity z, idiosyncratic productivity ε, and STW regime $\boldsymbol{s}$; the equilibrium is a list of (i) quantities $m(h,z,\boldsymbol{s})$, and $v(h,z,\boldsymbol{s})$; (ii) probabilities $f(h,z,\boldsymbol{s})$, $q(h,z,\boldsymbol{s})$ (iii) the price $w(\varepsilon,h)$ and the productivity $y(\varepsilon,h,\ell,z,\boldsymbol{s})$; (iv) value functions $J(\varepsilon,h,z,\boldsymbol{s})$, $V(h,z,\boldsymbol{s})$, $W(a,\varepsilon,h,z,\boldsymbol{s})$, and $U(a,h,z,\boldsymbol{s})$; (v) optimal hours decision $\ell^\star(\varepsilon,h,z,\boldsymbol{s})$; (vi) optimal separation decision $\mathbb{1}(a,\varepsilon,h,z,\boldsymbol{s})$; (vii) stationary distributions of employment $n(a,\varepsilon,h,z,\boldsymbol{s})$ and non employment $u(a,h,z,\boldsymbol{s})$; satisfying the following conditions:*

(i) *$m(h,z,\boldsymbol{s})$, $f(h,z,\boldsymbol{s})$, $q(h,z,\boldsymbol{s})$ and $v(h,z,\boldsymbol{s})$ are solutions of the matching function (3), the job finding rate (4), the vacancy filling rate (5) and the job creation condition (16), respectively;*

(ii) *Prices $y(\varepsilon,h,\ell,z)$ and $w(\varepsilon,h)$ satisfy equations (6) and (7);*

(iii) *Value functions $J(\varepsilon,h,z,\boldsymbol{s})$, $V(h,z,\boldsymbol{s})$, $W(a,\varepsilon,h,z,\boldsymbol{s})$, and $U(a,h,z,\boldsymbol{s})$ are solutions of the system that combines (11), (13), (14) and (15);*

(iv) *The optimal hours worked decision $\ell^\star(\varepsilon, h, z, \boldsymbol{s})$ solves* (12);

(iv) *The optimal separation decision $\mathbb{1}(a, \varepsilon, h, z, \boldsymbol{s})$ is derived from* (18);

(vii) *The distributions $n(a, \varepsilon, h, z, \boldsymbol{s})$ and $u(a, h, z, \boldsymbol{s})$ solve the law of motion described by* (19) *and* (20).

3.12 Functional forms

- The utility function takes the form of a CRRA function:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

- Payroll tax exemption:

$$\tau(w) = \min\left(\left[\tau - \frac{T_1}{T_2}\left(\underline{\tau}_1 \frac{w_{min}}{w} - 1\right)\right], \tau\right) - CICE\mathbb{1}_{\frac{w}{w_{min}} \leq \underline{\tau}_2} - PACTE\mathbb{1}_{\frac{w}{w_{min}} \leq \underline{\tau}_3}$$

  where τ is the baseline payroll tax rate, $T_1$and T_2 are two tax rebate parameters. $\underline{\tau}_j$, $j = 1, 2, 3$ are the tax rebate threshold parameters. CICE and PACTE are two fiscal reliefs on employers' payroll tax.[20]

- Matching function:

$$m(\chi(z), u(h, z, \boldsymbol{s}), v(h, z, \boldsymbol{s})) = \chi(z)\left(u(h, z, \boldsymbol{s})^{-\rho} + v(h, z, \boldsymbol{s})^{-\rho}\right)^{-1/\rho}$$

  where ρ is the CES parameter of the matching function.

- Short-time work administrative cost:

$$Q^{stw}(\boldsymbol{s}) = \begin{cases} \bar{Q}_1^{stw} & \text{if} \quad \boldsymbol{s} = 1 \\ \bar{Q}_2^{stw} & \text{if} \quad \boldsymbol{s} = 2 \\ \bar{Q}_3^{stw} & \text{if} \quad \boldsymbol{s} = 3 \end{cases}$$

- Firing costs:

$$F(w\bar{\ell}) = \psi_F w\bar{\ell}$$

4 Calibration

We calibrate some parameters using external information and use a minimization procedure to estimate the remaining parameters. We provide an online appendix describing in detail the solution method and the estimation procedure. All calibrated parameter can be found here (Tables 3 and 4).

4.1 Data

In order to match moments from the data we use a broad variety of time series from different sources.

[20]See Service-public.fr

Labor market We use several sources of information related to the labor market. We use OECD data to determine the steady-state unemployment rate. We take advantage of a survey conducted by DARES during the COVID-19 crisis ("Enquête Acemo-COVID-19") to analyze the changes in STW during the pandemic. This survey provides monthly information on STW, at the extensive (proportion of workers) and the intensive margin (number of hours per beneficiary). This dataset allows us to determine the dynamics of STW, as reported in Figure 5.

Wealth We use the Household Wealth Survey ("Enquête Patrimoine") provided by INSEE, which contains information on households' assets and liabilities, to determine the wealth distribution. This dataset allows us to compute the inter-decile wealth ratios D9/D1, D9/D5, and D5/D1, and the Mean/Median ratio presented in Table 5.

Consumption In France, individual data related to consumption are relatively scarce and are generally available at an annual frequency. Such data are unsuitable for analyzing the consumption response of heterogeneous agents during the pandemic. In this context, bank data are a valuable source of information for tracking consumer behavior in real-time (Bounie et al., 2020). We take advantage of a unique database, provided by INSEE,[21] in association with "Crédit Mutuel Alliance Fédérale" (Bonnet et al., 2021). This bank data allows us to analyze the evolution of consumption during the crisis for different types of agents. For confidentiality reasons, we can only use and report aggregate data (not individual data). This dataset allows us to compute the evolution of consumption by income decile during the pandemic, as reported in Figure 5.

4.2 Parameters set externally

Preferences and technology. The model period is assumed to be one month. We took the standard value for the relative risk aversion σ of 1.5 in line with RBC literature. The monthly discount factor is set at $\beta = 0.9967$, which gives us 0.99 at the quarterly rate and the real interest rate r is 4 percent at the annual rate.

Labor market institutions. According to INSEE,[22] the baseline payroll tax τ is roughly 41 percent of the gross wage and the baseline employee's social contribution τ^s is nearly 12%. We calibrate the (payroll) tax rebates using the rule given by the French administration.[23] The maximum rebate coefficient T_1 is equal to 0.32. The three thresholds $\underline{\tau_1}$, $\underline{\tau_2}$, and $\underline{\tau_3}$ are set respectively at 1.6, 2.5, and 3.5 times the minimum wage. The $\underline{CICE}$ rebate rate is 0.1 and the $PACTE$ one is 0.018. We assume a value of one month gross wage for the dismissal cost.

STW policies. The STW replacement rate for workers ρ_F has been 0.7 since 2013 according to the French legislation.[24] We calibrate ρ_F, ρ_X, and ρ_G using information from Table 1. Before March 2020, the French government refunded firms up to 80% of the

[21] We thank Odran Bonnet and the French National Institute for Statistics and Economic Studies (INSEE) for providing monthly data on household consumption during the pandemic.
[22] INSEE - Employees' and employers' social contributions
[23] General reduction in employer contributions
[24] Circulaire DGEFP n° 2013-12

French minimum wage, so $\rho_X = 0.8$. After June 2020, the refund was approximately 85% of the gross wage (barring exceptions) which gives us a STW replacement rate by the government ρ_G of 0.6. Before March 2020, the maximum time per worker allowed in STW was 2/3 of the previous working time. This means that $\underline{\ell} = 1/3\ \overline{\ell}$. After March 2020, the French government allowed $\underline{\ell}$ to be 0.

Lockdown policy. We follow the literature on rare disasters to calibrate the probability of entering lockdown. Barro (2006) estimates the probability of the real bills rate falling dramatically to 0.017, which gives $\lambda = 0.0014$ at the monthly rate. The probability of staying in lockdown is obtained by taking the rare disaster duration in Barro (2006). He considers that it lasts at least one year. While COVID-19 fits the definition of a rare disaster, government imposes a lockdown of a shorter duration. Lockdowns range from one month to up to 9 months. We take an agnostic view and consider a value of 6 months. When entering lockdown, the probability of it being strict or light is assumed to be equal so $\varpi = 0.5$. Lastly, we consider that $\nu = 0.5$ which involves that the STW duration outside the lockdown is two months.

Short time work policies. Asset ranges from $a_{\min} = -5.7$ to $a_{\max} = 300$. $a_{\min}$ involves a borrowing constraint equal to three months of the average wage, a value for which few households reach this limit. $a_{\max}$ is set to cover the ergodic set of assets level.

4.3 Parameters set internally

There are 23 remaining parameters, presented in Tables 3 and 4 (marked with an asterisk). They are internally calibrated:

$$\Theta = \{\alpha, \delta, \gamma, \rho, \psi_n, \psi_u, w_{\min}, b, \bar{Q}^{stw}(\boldsymbol{s}), \sigma_z, \rho_z, \sigma_\varepsilon, \rho_\varepsilon, z_s, z_l, \tilde{c}, \tilde{\beta}, \tilde{\chi}, \overline{\ell}, h_1, h_H\}$$

Our goal is to calibrate these parameters to match the 23 targets displayed in Table 2.

Table 2: CALIBRATION TARGETS

Numbers	Targets
1	D9/D1 wealth ratio
2	D9/D5 wealth ratio
3	Mean to median wealth ratio
4	Proportion of worker on the minimum wage
5	Unemployment rate in 2019
6-7	The job finding rate and the job separation rate in 2019
8-13	Consumption of D1, D5 and D9 by income decile during the two lockdowns.
14-15	Unemployment variation during the two lockdowns
16-18	Hours worked for STW employees before and during the two lockdowns
19-21	Percentage of employment in a STW program before during the two lockdowns
22-23	Persistence and autocorrelation of output (before the pandemic).

Labor market. We obtain a curvature for productivity equal to 0.9, which involves a roughly linear production function. This is the assumption often considered in matching models without capital. The implied exogenous separation rate is about 1% while the total separation rate is 1.1%, a value close to the one used in Langot and Pizzo (2019). Given this total separation rate, δ accounts for the largest fraction of separation, consistent with den Haan et al. (2000)'s calibration in which exogenous separations are twice as large as endogenous ones. The estimated value of $\gamma = 0.40$ involves a strong wage curvature as a function of productivity. The value of ρ that best fits the empirical moments is 0.35, which is fairly low compared to that found in den Haan et al. (2000) (1.27 at quarterly frequencies) but roughly similar to that used in Hagedorn and Manovskii (2008) (0.34 at weekly frequencies). The probability of human capital increasing during an employment spell is well below the probability of it decreasing during an unemployment spell. This result is consistent with Lalé (2018) who finds a higher rate of depreciation than appreciation.

Labor market institutions. We obtain a minimum hourly wage rate of 1.0 and the utility derived from unemployment is $b = 0.2$, which amounts to 17% of the minimum wage. This value seems reasonable given that it concerns individuals who receive unemployment benefits and individuals who do not. The administrative cost of short-time work $Q_1^{stw} = 0.3$ involves a modest use of STW of around 1% in normal times. The value obtained for the cost $Q_2^{stw} = 0.005$ is almost zero and so is the cost $Q_3^{stw} = 0.01$.

Shocks. Regarding the shock processes, we obtain an estimation for the autocorrelation of the aggregate productivity shock equal to 0.96 and a standard deviation of 0.003. These values are well within the range of those used in the RBC literature.[25] The idiosyncratic productivity shock involves a substantial persistence of 0.98 and a large standard deviation of $\sigma_\varepsilon = 0.09$. Although it is difficult to compare them with other studies, the values are fairly close to those from Fujita and Moscarini (2017) for the US, once corrected for the frequency.

Lockdown. The lockdown involves a large decline in productivity $z_s = 0.9$ and $z_l = 0.96$. Given a value of $z_m = 1$ (no shock), these values imply a decline in z of 10% and 4% respectively. The decline in consumption calls for a consumption limit equal to 55% of average consumption. In addition, the estimated value of the discount factor that proxies the additional demand effects of the lockdown is $\beta = 1.02$. Although $\beta > 1$, in this case, the model still admits a solution because of the extremely low probability of such an event occurring ($\lambda = 0.14\%$). We obtain a decline in matching efficiency of 70%. This major disruption to adjustments at the extensive margin is in line with Auray and Eyquem (2020) who use a separation rate shock to proxy the macroeconomic effects of lockdown policies.

Range of hours and human capital. We obtain an upper bound of hours worked of $\bar{\ell} = 0.8$. The lower bounds are externally calibrated with a two-thirds reduction in hours worked under the pre-COVID-19 STW programs, and a potential full reduction during the COVID-19 STW programs. Lastly, the five levels of human capital ($H = 5$) lie between $h(1) = 0.9$ and $h(5) = 4$.

[25] At quarterly frequencies the estimation of the Solow residual leads to $\rho_z = 0.95$ and $\sigma_z = 0.007$. Converted into monthly frequencies, one has $\rho_z^{1/3} = 0.98$ and $\sqrt{\sigma_z^2 / \sum_{j=0}^{n-1} \rho_z^{2\times j/3}} = 0.0041$.

Table 3: CALIBRATED PARAMETERS I

Parameter	Symbol	Value
PREFERENCE		
Discount factor	β	0.9967
Risk aversion	σ	1.5
Interest rate	r	0.004
Curvature production function*	α	0.9
LABOR MARKET		
Exogenous separation rate*	δ	0.01
Curvature wage function*	γ	0.39
CES matching function param.*	ρ	0.35
Prob. switch human cap.*	ψ_n	0.002
Prob. switch human cap.*	ψ_u	0.100
LABOR MARKET INSTITUTIONS		
Baseline payroll tax	τ	0.41
Maximal tax rebate	T_1	0.32
Coefficient rebate	T_2	0.32
1st Threshold tax rebate	$\underline{\tau}_1$	1.6
2nd Threshold tax rebate	$\underline{\tau}_2$	2.5
3rd Threshold tax rebate	$\underline{\tau}_3$	3.5
Tax relief	$CICE$	0.06
Tax relief	$PACTE$	0.018
Minimum wage*	$w_{\min}$	1.0
Home production*	b	0.20
Social contributions rate (workers)	τ^s	0.12
Dismissal costs to wage ratio	ψ_F	1.0
SHORT TIME WORK		
Administrative cost of STW_1*	$\bar{Q}_1^{stw}$	0.30
Administrative cost of STW_2*	$\bar{Q}_2^{stw}$	0.005
Administrative cost of STW_3*	$\bar{Q}^{stw}$	0.01
Replacement rate pre-Covid	ρ_X	0.8
Replacement rate March	ρ_F	0.7
Replacement rate June	ρ_G	0.6

*Note: Parameters with a superscript * are internally calibrated. The others are set using external sources of information.*

Table 4: Calibrated parameters II

Parameter	Symbol	Value
PRODUCTIVITY SHOCK PROCESSES		
Persistence aggregate shock*	ρ_z	0.96
Standard deviation aggregate shock*	σ_z	0.003
Persistence idiosyncratic shock*	ρ_ε	0.98
Standard deviation idiosyncratic shock*	σ_ε	0.09
LOCKDOWN		
Probability of going into a lockdown	λ	0.0014
Probability of a strict lockdown	ϖ	0.5
Probability of remaining in lockdown	φ	0.83
Probability of leaving STW outside a lockdown	ν	0.50
Productivity strict lockdown*	z_s	0.90
Productivity light lockdown*	z_l	0.96
Consumption ceilling (average consumption)	$\tilde{c}/\text{mean}(c)$	0.55
Discount factor in lockdowns*	$\tilde{\beta}$	1.02
Matching efficiency in strong lockdown*	$\tilde{\chi}$	0.30
STATE SPACE		
Asset range	$[a_{\min}, a_{\max}]$	[-5.7 300.0]
Hours worked STW range* $\boldsymbol{s} = 1$	$[\underline{\ell}, \overline{\ell}]$	[0.00 0.80]
Hours worked STW range* $\boldsymbol{s} > 1$	$[\underline{\ell}, \overline{\ell}]$	[0.27 0.80]
Human capital range*	$[h_1, h_H]$	[0.90 4.00]

*Note: Parameters with a superscript * are internally calibrated. The others are set using external sources of information.*

4.4 Model vs data

The first objective of our paper is to assess whether the model is able to reproduce several features of the French labor market, and in particular the response of the labor market during the COVID-19 crisis. To this end, we consider a regime-switching approach that allows us to mimic the government's decisions regarding change in STW during the COVID-19 crisis. For moments calculated using the dynamic of the model, we simulate the model given an aggregate state z and the regime switching s. The timeline, displayed in Figure 3, is as follows:

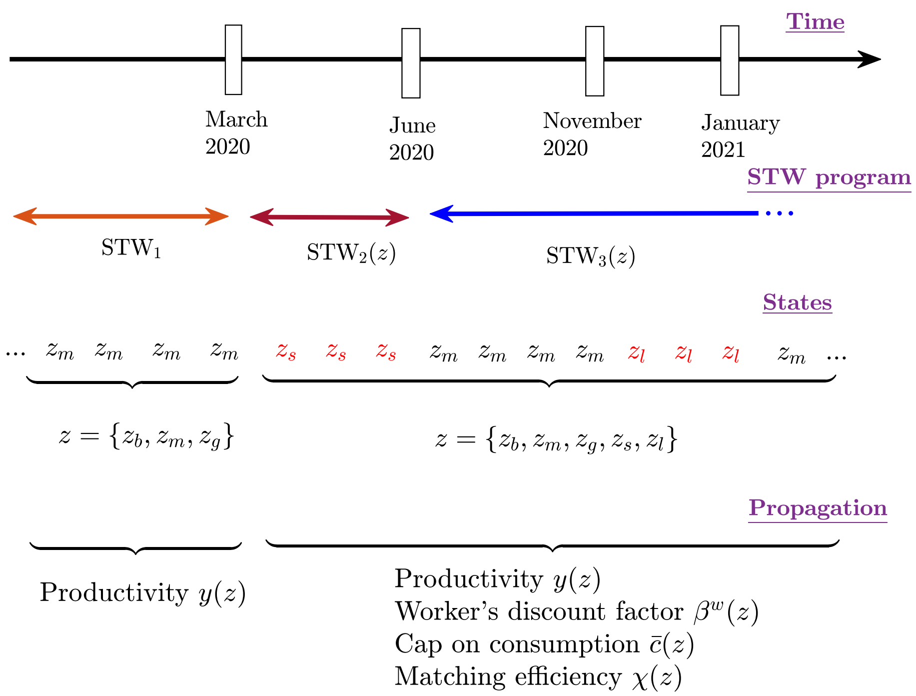
- **Before March 2020:** Initially, we consider that rare disasters do not exist, i.e., the transition matrix of the aggregate state only contains normal business cycle states $z = \{z_b, z_m, z_g\}$. We also assume that STW_2 and STW_3 do not exist, i.e. firms can only use STW_1. The regime is $\boldsymbol{s} = 1 \ \forall z$. Agents anticipate neither the possibility of rare disasters, nor potential changes in the STW scheme. We simulate the model under these assumptions to determine the initial condition of the economy in February 2020.

- **From March to May 2020:** The COVID-19 crisis starts in March 2020. The government's response is immediate and is twofold: i) A strict lockdown is implemented from March to May 2020. The aggregate state switches from z_m to z_s. Over this period, we consider that rare disasters exist and may occur with probability λ, i.e. the transition matrix of the aggregate state contains both normal business cycles states $z = \{z_b, z_m, z_g\}$ and rare disasters $z = \{z_s, z_l\}$. Agents now anticipate the possibility of rare disasters; ii) A more generous STW scheme is implemented from March to May 2020. The STW policy switches from STW_1 to STW_2. We assume that STW_3 does not exist, i.e. firms can only use STW_2. Note that agents also take into account the possibility that the STW scheme switches between STW_2 and STW_1. Indeed, z may come back to a normal situation $z = \{z_b, z_m, z_g\}$ and the regime may come back to $\boldsymbol{s} = 1$. These probabilities are taken into account by economic agents when forming their decisions. We simulate the model up to May 2020 under these assumptions and given the initial condition of the economy in February 2020.

3 **From June to September 2020:** In June 2020, the economy is affected by two changes: i) Lockdown measures are lifted. The aggregate state switches from z_s to z_m. ii) A slightly less generous STW scheme is implemented from June to December 2020. The new emergency STW policy is STW_3. We assume that STW_2 no longer exists, i.e. firms can only use STW_3, provided that $\boldsymbol{s} = 3$. The set of information used by the economic agents to form their decision rules is similar than that in the previous period except that STW_3 replaces STW_2. We simulate the model up to October 2020 under these assumptions and given the initial condition of the economy in May 2020.

4 **From November to January 2021:** In November 2020, a light lockdown is implemented. The aggregate state switches from z_m to z_l. The STW scheme as well as the set of information used to form the decision rules do not change. We simulate the model up to December 2021 under these assumptions and given the initial condition of the economy in October 2020.

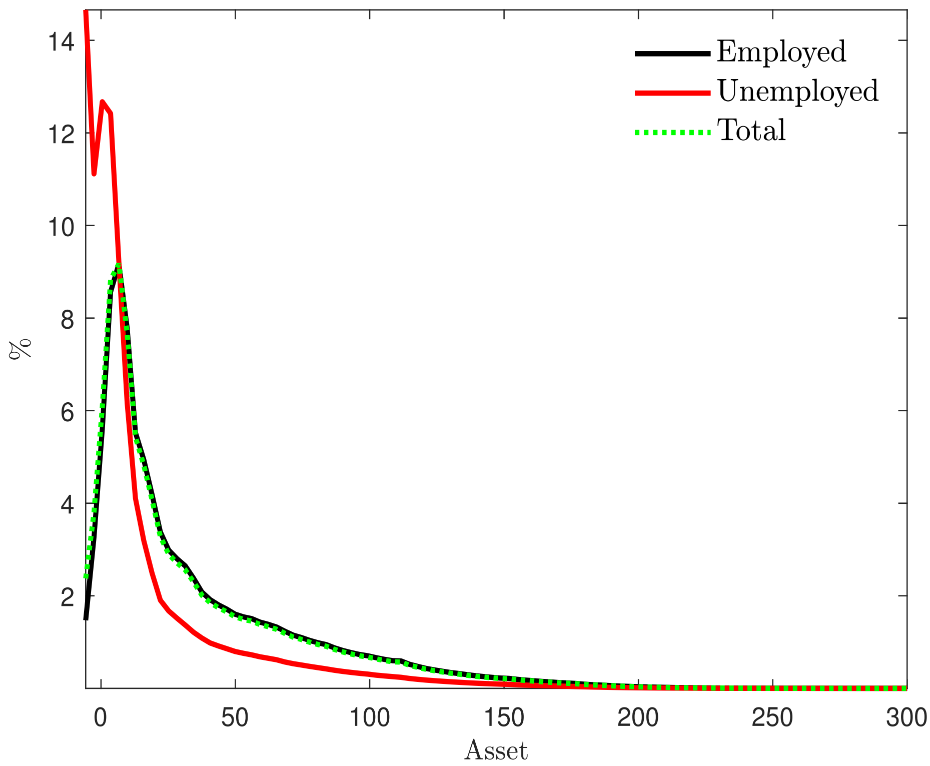
Figure 3: Timing of event



4.4.1 Steady state

Figure 4 shows the distribution of the population along the distribution of assets. The distributions are concentrated to the left, especially for the unemployed. Table 5 compares moments generated by the model to those observed in the data. Simulated moments appear broadly consistent with the data. The model tends to slightly underestimate the inter-decile wealth ratios D9/D1 and D5/D1, and the Mean/Median ratio. However, it does a good job at generating the high concentration of workers paid at the minimum wage. Overall, the model provides a good fit of the wealth distribution. It does quite well at capturing moments related to the labor market. The model slightly underestimates the unemployment rate and the job separation rate but it reproduces the job finding rate quite well.

Figure 4: Distributions over assets



Stationary distribution of employed, unemployed, and all workers. Each distribution is normalized so that it sums to one.

Table 5: LONG RUN TARGETS

Variables	Model	Data	Source
Wealth D9/D1	195.7	211.4	INSEE
Wealth D9/D5	4.9	4.7	INSEE
Wealth D5/D1	39.9	45.0	INSEE
Mean/Median	1.0	2.0	INSEE
Proportion workers at $w_{\min}$	10.2	11.5	DARES
Unemployment rate	6.9	8.0	OECD
Job finding rate	14.3	13.5	Langot and Pizzo (2019)
Job separation rate	1.1	1.7	Langot and Pizzo (2019)
Autocorrelation output	0.008	0.009	Eurostat
Persistance output	0.78	0.89	Eurostat

INSEE: Enquêtes Patrimoine, households.

4.4.2 Dynamics

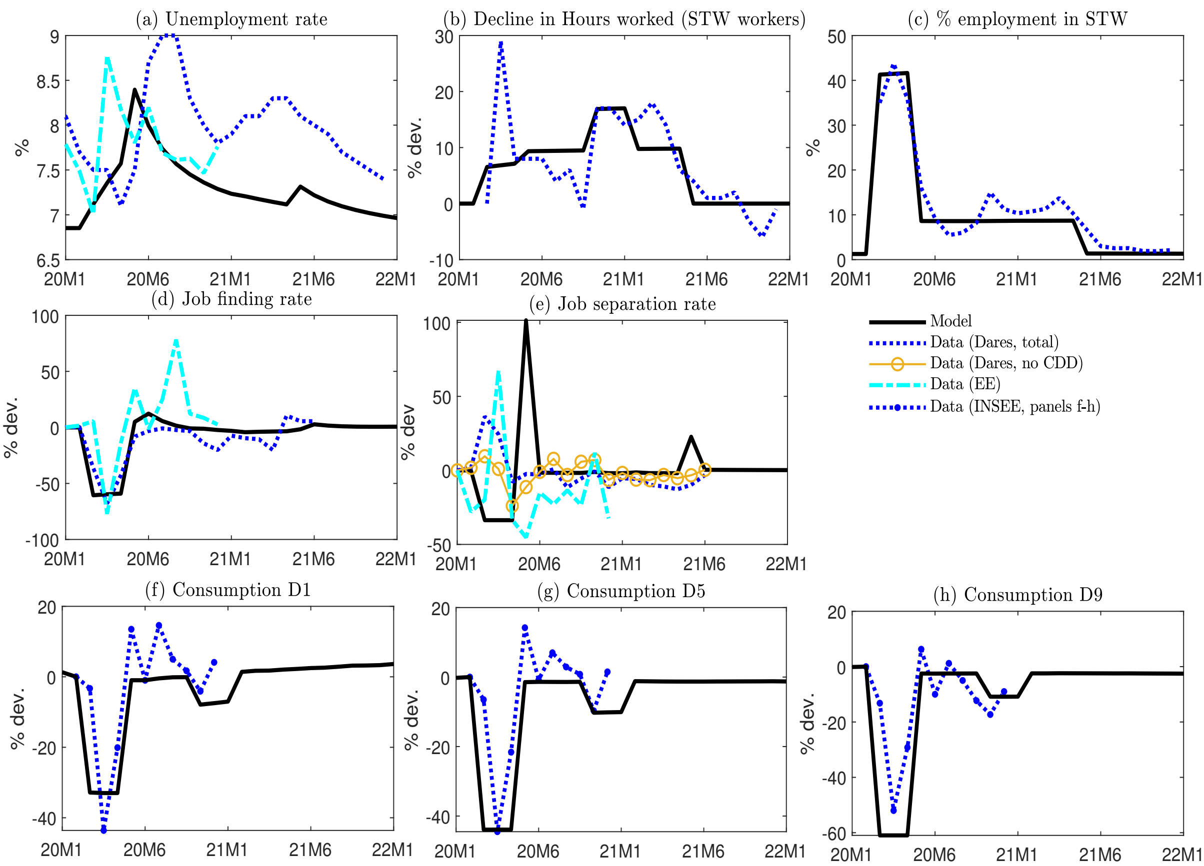
In the previous section, we assessed the ability of our model to reproduce steady-state values regarding wealth distribution or labor market stocks and flows. In this section, we test the capacity of our model to reproduce the labor market response and the household response to the COVID-19 crisis. As shown in Figure 5, our model replicates the dynamics of the labor market during the pandemic quite well. We use two different measures of the unemployment rate. The first comes from the French Ministry of Labor (DARES) and the second from the National Institute of Statistics and Economic Studies (INSEE). The first shows a peak in August 2020 while the second shows the peak in May 2020. Our model stands in-between the two. We almost replicate the increase in the unemployment rate that followed the first lockdown and its persistence. Our model also reproduces properly the changes in hours worked and the proportion of workers in STW from March to December 2020. Unlike many models that focus only on pre-COVID steady-state variables, our model also captures the labor market response to the outbreak, both at the extensive and the intensive margin. We consider this to be an important property for analyzing the effects of STW during the pandemic, as a model can be effective at reproducing steady-state values but incapable of replicating the response of the economy to the COVID-19 crisis.

We then discuss the fit of the model regarding consumption. Our data allows us to compute the evolution of consumption during the pandemic by income deciles.[26] We use our model to simulate individual trajectories and compute the dynamics of consumption by income decile during the outbreak. We then compare the consumption profiles generated by the model to their empirical counterpart (see Figure 5). Our model does a good job of matching the consumption response by income decile. During the first lockdown, consumption fell by around 40% for the poorest individuals, while it dropped by more than 50% for the richest. As noted by Bonnet et al. (2021), during the first lockdown, the consumption of basic necessities remained relatively stable, while the consumption of leisure and luxury goods collapsed. As the share of leisure and luxury goods in total consumption rises with income, the drop in con-

[26] The income deciles are calculated before the pandemic. As such we observe the consumption evolution for workers initially in income deciles 1, 5, and 9.

sumption was more pronounced among the richest households. In our model, the cap on consumption allows us to proxy this composition effect. As the cap on consumption is expressed in level, only the wealthier households are affected by the constraint. As a result, the decline in consumption is larger (smaller) for individuals belonging to the top (bottom) income decile. Our model is also able to reproduce the decline in consumption observed during the second (light) lockdown. The capacity of the model to fit these consumption profiles provides additional validation and strengthens the credibility of the counterfactual experiments presented in the next section.

Figure 5: Labor market during the COVID-19 - Model vs Data



Note. (b): shows by how much hours worked decline for employed workers in a STW program. (d) to (e): consumption by income decile (unemployed + employed), % dev. from March 2020. It shows the evolution of consumption for workers belonging to an income category defined in 2019. The consumption responses are expressed in deviation from the pre-crisis level. Data on consumption are from INSEE.

5 Counterfactual analysis

The second objective of our paper is to run counterfactual experiments. More precisely, we wonder what the labor market trajectory would have been like under different STW scenarios. The sequence of shocks is common to all scenarios, i.e., the aggregate state at a given date is the same.

5.1 Methodology

- **Scenario** STW_1**:** We consider that only STW_1 applies. STW_2 and STW_3 do not exist, i.e. firms can only use STW_1 ($\boldsymbol{s} = 1$) whatever the aggregate state. Agents do not anticipate potential changes in the STW scheme.

- **Scenario** STW_2**:** We consider that STW_1 and STW_2 coexist but only STW_2 is allowed under specific circumstances concerning the aggregate state. STW_2 applies if the aggregate state involves a lockdown (strict or light), i.e. if $z \in \{z_s, z_l\}$ or if following a lockdown the government decides to prolong its duration. Agents thus anticipate the possibility of the STW scheme switching from STW_1 to STW_2 and vice-versa. We assume that STW_3 does not exist.

- **Scenario** STW_3**:** In this scenario, STW_3 is used instead of STW_2 but the extent to which this program can be used as well as the way anticipations are formed is similar to those in the previous scenario.

Note that we have also analyzed what the labor market trajectory would have been in the absence of STW (scenario "No STW"). As the results are almost the same as in scenario STW_1, we do not report the results in the main text. Simulations related to the scenario "No STW" are provided in Appendix A.2.

5.2 Long-run impact

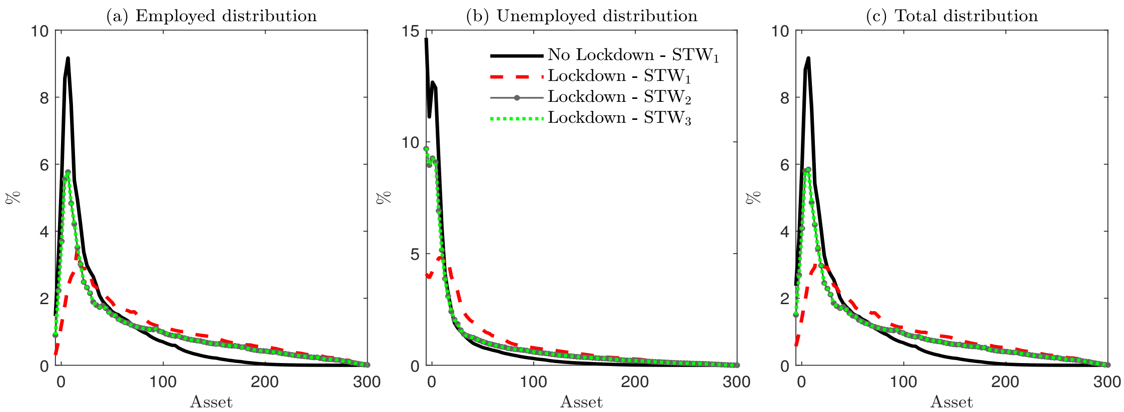
To understand the overall impact of STW, we first take a look at the long-run effects since they represent the mean levels around which the economy fluctuates. Indeed, introducing rare disaster shocks in the form of lockdown policies as well as a different design for STW programs involves structural changes that may affect the propagation of shocks. Our objective is to analyze how these two ingredients affect asset distribution and labor market outcomes.

5.2.1 Precautionary saving effects

Introducing larger shocks in the form of rare disaster shocks is likely to increase the **precautionary motive** because of consumption smoothing. One might expect the asset distribution to shift to the right. Following the timeline presented in Figure 3, STW_2 was implemented at the same time as the lockdown shock occurred. The impact of the emergency STW programs (STW_2 and STW_3) on asset accumulation is, *a priori*, ambiguous. On the one side, they reduce the precautionary motive because they alleviate the unemployment risk thanks to the flexibility to adjust employment at the intensive margin. In the presence of labor market frictions and incomplete markets, the unemployment risk generates an additional precautionary saving motive (see Albertini et al. (2021a)). By reducing this risk, the emergency STW programs reduce the incentive to save so as to smooth consumption. On the other side, these generous government spending programs act as high replacement incomes during periods where the demand for goods declines substantially. Consequently, this additional amount of liquidity may reinforce the precautionary motive outlined previously.

Our objective is to disentangle how much of the long-run changes in asset distribution are accounted for by the presence of rare disasters and the STW regimes through the **precautionary channel** and the **generosity channel**. To do so, we first compare the stationary distributions of assets under the different scenarios (see Figure 6). In the three experiments, we take into account the implementation of lockdowns ("Lockdown - STW_1", with $i = \{1, 2, 3\}$). We consider an additional scenario equivalent to scenario STW_1, except there is no lockdown ("No Lockdown - STW_1). Comparing "Lockdown - STW_1" and "No Lockdown - STW_1" allows us to highlight the impact of lockdowns on precautionary savings. As can be seen from Figure 6, lockdown policies have a huge impact on precautionary saving, and thus on wealth distribution.

Figure 6: Stationary distribution



Stationary distribution of employed, non employed, and all workers. Each distribution is normalized so that it sums to one.

5.2.2 Labor market effects

We now turn to the long-run impact on the labor market. Table 6 reveals that neither rare disaster shocks nor the particular design of STW_2 and STW_3 have a significant impact on the labor market in the long run. Comparing the no lockdown to the lockdown column under STW_1 reveals that unemployment does not change with the lockdown policy, while comparing $STW_1/STW_2/STW_3$ shows that the STW policy does not impact the unemployment rate in the long run.

Table 6: LONG-RUN IMPACT

Variables	STW_1		$STW_2(z)$	$STW_3(z)$
	No lockdown	Lockdown		
Unemployment	6.9	6.9	6.9	6.9
JFR	14.34	14.27	14.32	14.29
low skill	7.06	7.02	7.05	7.03
high skill	34.08	34.07	34.07	34.07
JSR	1.05	1.05	1.05	1.05
low skill	1.21	1.21	1.21	1.21
high skill	1.37	1.37	1.37	1.37

5.3 Short-run impact

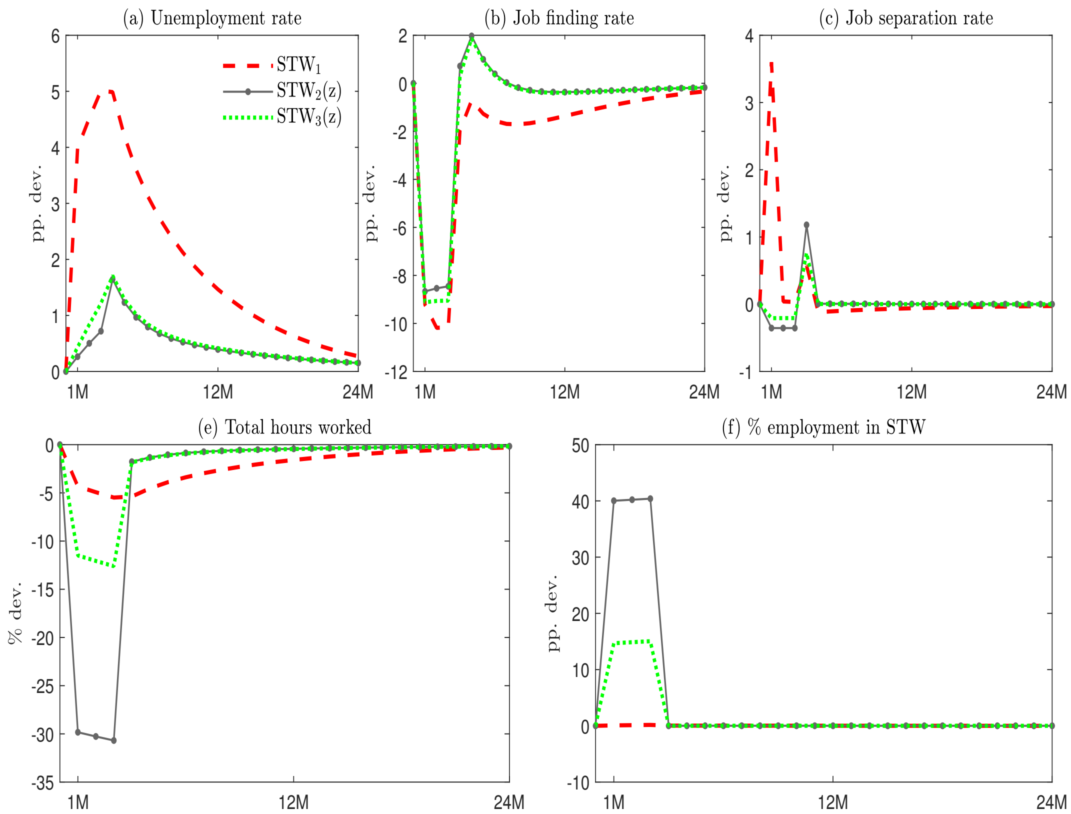
We now investigate the response of the labor market to the COVID-19 crisis. More precisely, we wonder what the labor market trajectory would have been in response to a lockdown shock under the different STW scenarios (see Figures 7 and 8). We simulate a strict lockdown policy of three months but show in Appendix A that the results remain qualitatively similar when the aggregate shock is a light lockdown.

In the absence of a government response (STW_1), the unemployment rate would have increased by 4.5 percentage points (Panel (a)). As shown by Panel (c), the job separation rate would have skyrocketed. Although this jump is short-lived, it would have caused a dramatic fall in employment. The job finding rates would have been similar across STW programs because all scenarios are subject to the same strong match efficiency shock. Panels (d) to (e) make it clear that STW_2 and STW_3 would have avoided a prolonged collapse in employment thanks to the marked use of the intensive margin. Under the STW_1 program, the number of workers affected by reduced hours would have remained at the pre-COVID-19 level while it would have increased by 40 and 15 percentage points under STW_2 and STW_3 respectively. In addition, the decline in hours worked per worker with the STW_2 and STW_3 programs is far greater. In this sense, STW_2 and STW_3 succeeded in limiting separations and preserving jobs by means of major adjustments in hours worked. Figure 8 shows that low-skilled workers benefited the most from the emergency STW programs. On the contrary, STW had little effect on skilled workers.

When comparing STW_2 to STW_3, it is worth noting that the simulation predicts almost the same impact on unemployment, the job finding rate, and the separation rate. This result is particularly interesting because cuts in hours worked are significantly lower under STW_3 than under STW_2, both in terms of workers concerned and hours per worker. Firms cut hours worked more sharply under STW_2 (due to the generosity of the program) compared to STW_3, without any additional effects on employment. Under the most generous program (STW_2), the reduction in hours worked seems unjustified in terms of saving jobs. This result is of great importance for the design of future STW policies. While the implementation of more generous and flexible programs like those triggered during the COVID-19 crisis has succeeded in stabilizing employment, over-generous programs create an incentive for firms to abuse reductions in hours without saving more jobs. This may have damaging consequences not only on the government budget but also on employed workers who receive a replacement income of 70%.

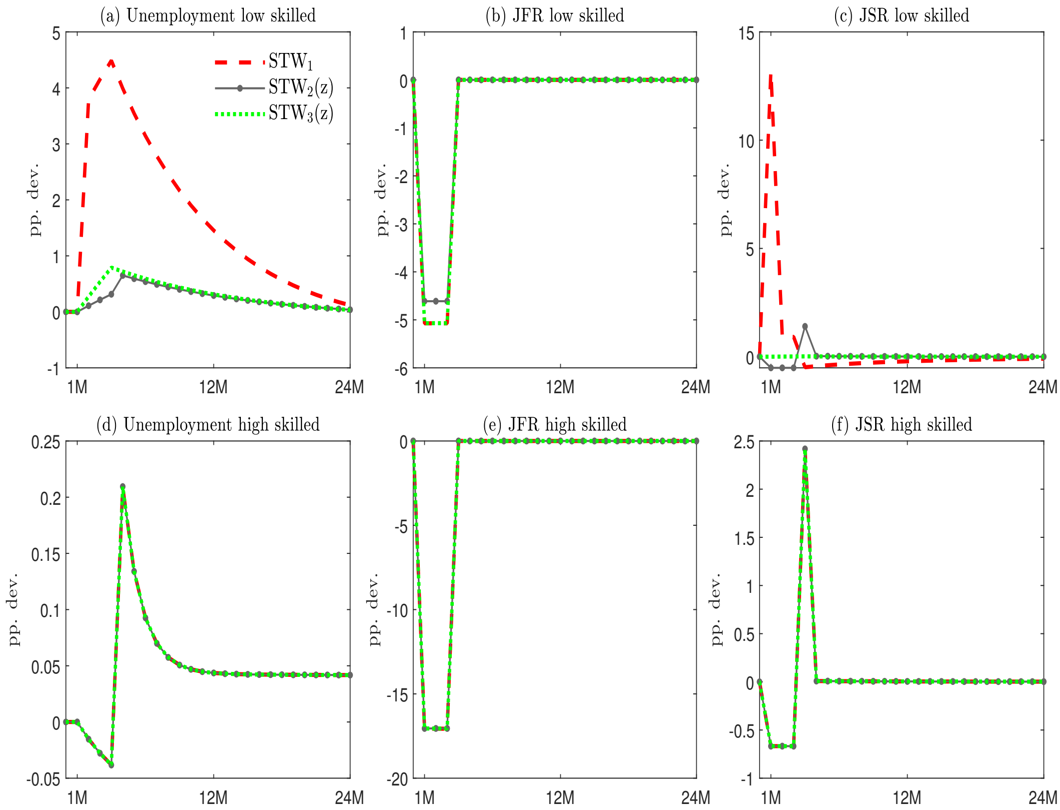
We fully acknowledge that the unprecedented situation of the COVID-19 did not leave much time for the government to find the optimal trade-off between saving jobs and flexibility in hours adjustments. Our study does not intend to criticize the design of STW during the pandemic. At the onset of the pandemic, little was known about the duration of the lockdown, how many firms would be impacted and how much flexibility in hours worked was needed to avoid a major labor market collapse. Our results aim at highlighting the interaction between the design of the STW policy and the number of jobs saved. The objective is to provide grounds for the understanding of the optimal design of STW policies as a function of the economic context.

Figure 7: Labor market response - strict lockdown



All impulse response functions are expressed in percentage deviation from initial value in month 0.

Figure 8: Labor market response - strict lockdown - by skill



All impulse response functions are expressed in percentage deviation from initial value in month 0. Low skill is the lowest skill category, $h = 1$. High skill is the highest skill category, $h = 5$

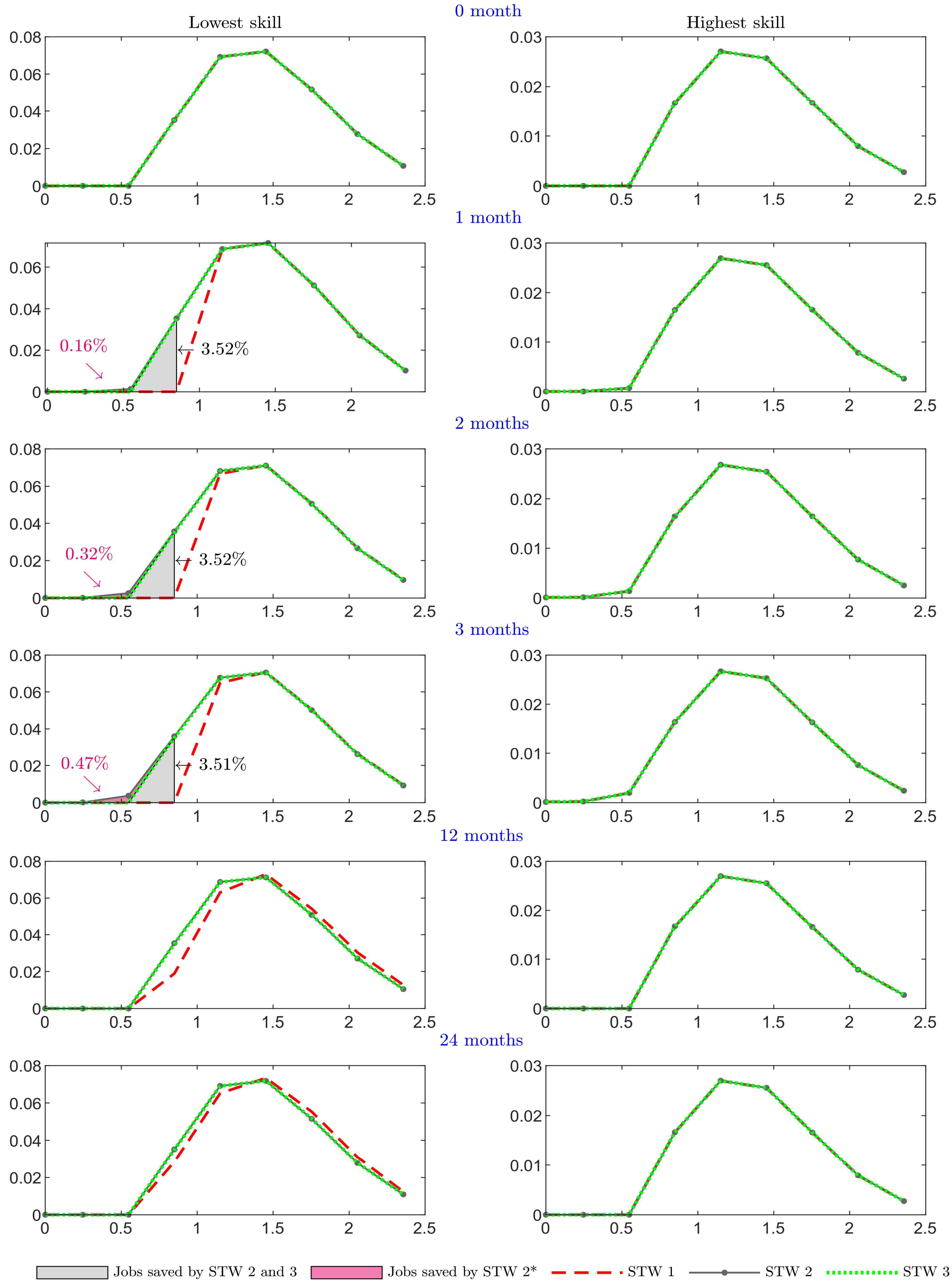
5.4 Disentangling the windfall effect

As discussed in the previous sections, STW_2 achieved its objective by preventing job destruction. However, we showed that a less generous policy, STW_3, would also have avoided a rise in unemployment. Our next exercises will help us to appreciate the proportion of jobs saved and to disentangle the potential windfall effect of STW_2.

Figure 9 shows the impact of a strict lockdown on the evolution of the employment distribution over time, under different STW scenarios. The economy is initially in normal times (i.e. $z = z_m$ at "0 month"). "). A strict lockdown hits the economy in month 1 and lasts three months. How many jobs have been saved by STW programs during the lockdown? According to our simulations, 3 to 4% of low-skilled employment is saved by the implementation of STW_2 at the beginning of the lockdown, while the effect tends toward zero for high-skilled employment. In this sense, the increase in the generosity of STW programs at the beginning of the pandemic succeeds in saving jobs. The figure reveals another interesting result. If the government had implemented STW_3 (a slightly less generous program) instead of STW_2 at the beginning of the lockdown, almost as many jobs would have been saved, at a lower cost. Indeed, relatively to STW_3, STW_2 only saved a modest number of jobs (between 0.16 and 0.47% of low-skilled employment). These findings confirm the intuitions of Figures 7 and 8: under

STW_2, some firms without any economic difficulties used STW to adjust hours. Put differently, the generosity of STW2 generated substantial windfall effects. Note that our conclusions are the same if we consider a light lockdown instead of a strict lockdown (See Appendix A.4). All these simulations suggest that the decision to (slightly) reduce the generosity of the STW program in June 2020, to stabilize employment while avoiding windfall effects, was well-founded.

Figure 9: Employment distribution



*Note. The Figure shows the distribution of employment across individual productivity by skill group (h_1 and h_H) aggregated over asset a. The shock hit the economy in period 1, so that "0 month" is the steady state where $z = z_m$. The economy remains in a strict lockdown $z = z_s$ for three periods before returning to the steady state $z = z_m$. Percentages in the figure indicate the amount of employment saved if any. * additional jobs saved by STW2 w.r.t. STW3.*

6 Conclusion

STW is intended to assist firms experiencing a temporary shock, by allowing employers to reduce workers' hours instead of laying them off. On the workers' side, STW limits lost earnings in the short term, while mitigating the impact of the shock on human capital and labor market trajectories in the long run. On the firms' side, STW allows skilled workers to be kept on and avoids the costs associated with dismissal, hiring, or training. STW can also promote faster economic recovery.

Compared to other labor-market institutions, relatively little research has been conducted on STW. The "German jobs miracle" observed during the Great Recession, partially attributed to work-sharing (*Kurzarbeit*), sparked a renewed interest for STW among economists and policymakers. While some studies investigate the role played by STW in stabilizing employment during the Great Recession, little is known about its effects during the pandemic, and in particular during lockdowns.

In this paper, we develop a heterogeneous agents model with search frictions, human capital, and aggregate and idiosyncratic productivity shocks. Firms have two options to respond to a shock. First, they can lay off employees. In this case, firms face dismissal costs and have to pay hiring costs again when the economy recovers, while the laid-off workers face income risks as well as risks of human capital depreciation. Second, firms can respond by reducing hours worked through STW. In this case, firms have to pay compensation to employees but they receive a subsidy that covers a large part of the employees' remuneration.

We calibrate the model using French data. We simulate the model to quantify the extent to which STW has contributed to saving jobs and limiting lost earnings during the pandemic by exploring what the labor market response would have been under alternative STW schemes. Our simulations support the conclusion that STW was successful in preserving jobs and limiting lost earnings during the outbreak. However, the policy also generated substantial windfall effects: some jobs that would have been maintained in the absence of STW benefited from the program. We then use the model to explore what the labor market response would have been under alternative STW schemes.

The study of STW policies during the pandemic is certainly in its infancy. When microeconomic data on firms' and workers' adjustments along the intensive margin are released, it will be possible to conduct a retrospective analysis to provide additional assessments on the impact of the STW program on the labor market. From the theoretical side, our model can be expanded in various directions. First, we do not analyze the welfare impact of short-time work. Second, in our model, human capital is considered as general and is the result of a learning-by-doing process. The implications of STW could be different in the presence of firm–or occupation–specific human capital, or by assuming that skills are the results of an investment in vocational training (Terriau, 2018). Third, as noted by Barrero et al. (2020), COVID-19 also produced a reallocation shock. Policies that subsidize employee retention impede reallocation of workers from contracting sectors to expanding sectors (Cahuc, 2019). Although our model includes firm heterogeneity, we ignore differences between sectors. Although STW saves jobs in firms in structural decline, like the so-called *zombie companies,* it may lead to excessive labor hoarding (Giupponi and Landais, 2018), delay reallocation, and put a damper on economic recovery.[27] It is probably too early to assess the full labor market response

[27] This negative effect on reallocation is temporary, as STW programs cannot last more than 12 months.

to the COVID-19 crisis. More research is needed in the future to understand how STW may inhibit the reallocation response and affect the labor market in the long run.

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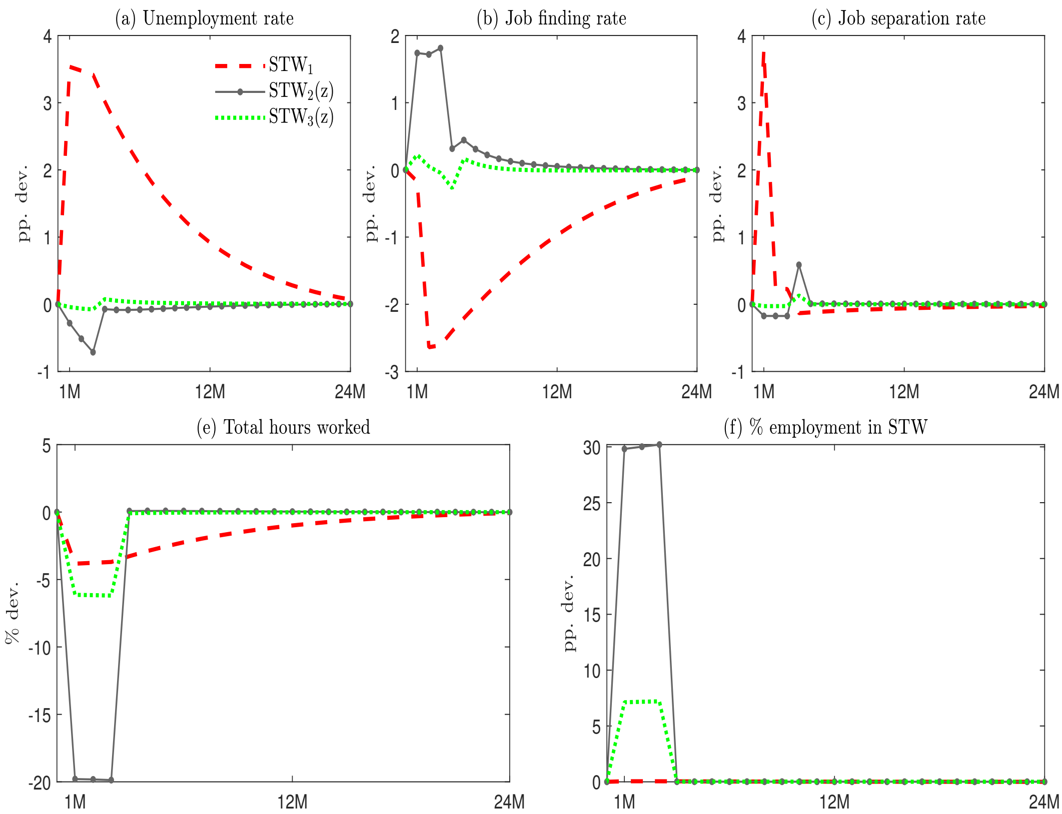
Online appendix, not for paper publication

A Additional simulations

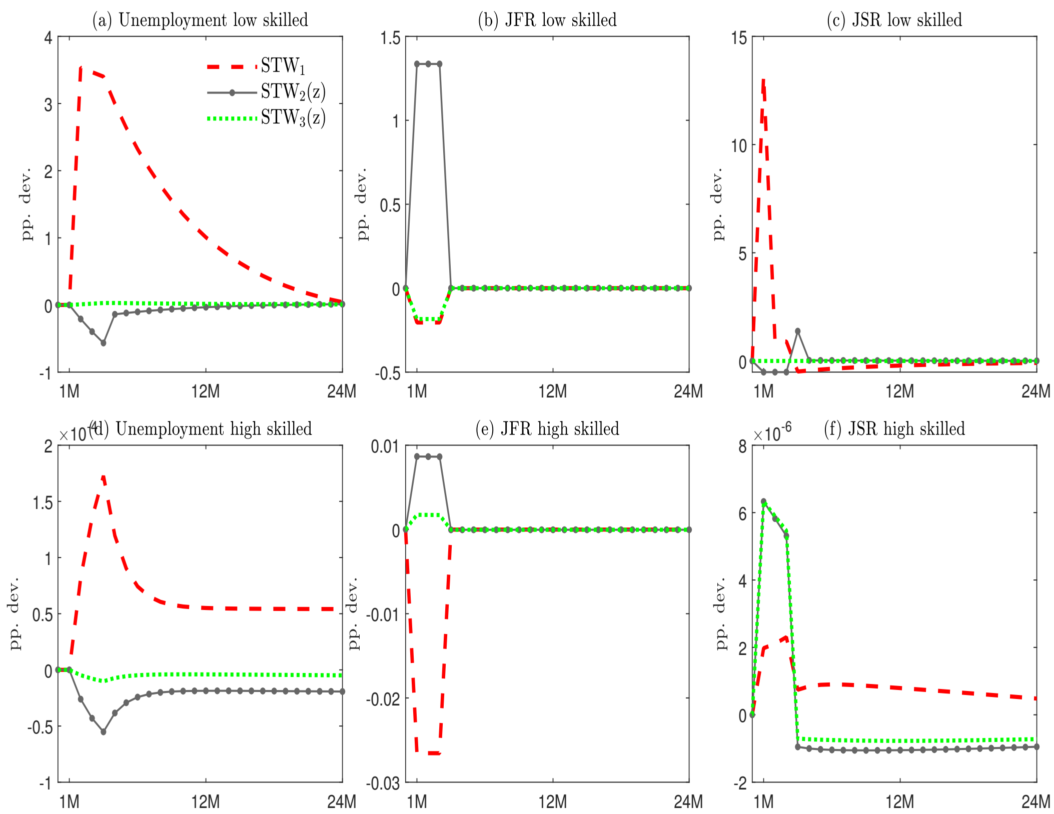
A.1 Light lockdown

Figure 10: Labor market response - light lockdown



All impulse response functions are expressed in percentage deviation from initial value in month 0.

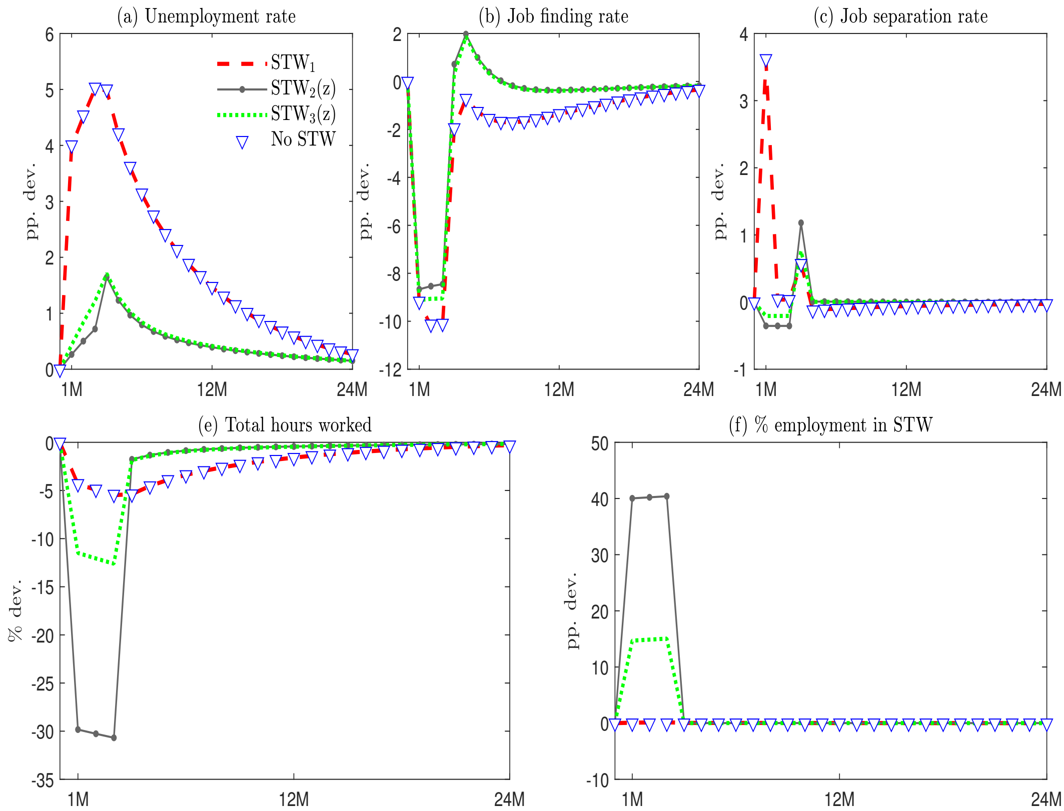
Figure 11: Labor market response - light lockdown - by skill



All impulse response functions are expressed in percentage deviation from initial value in month 0. Low skill is the lowest skill category, $h = 1$. High skill is the highest skill category, $h = 5$

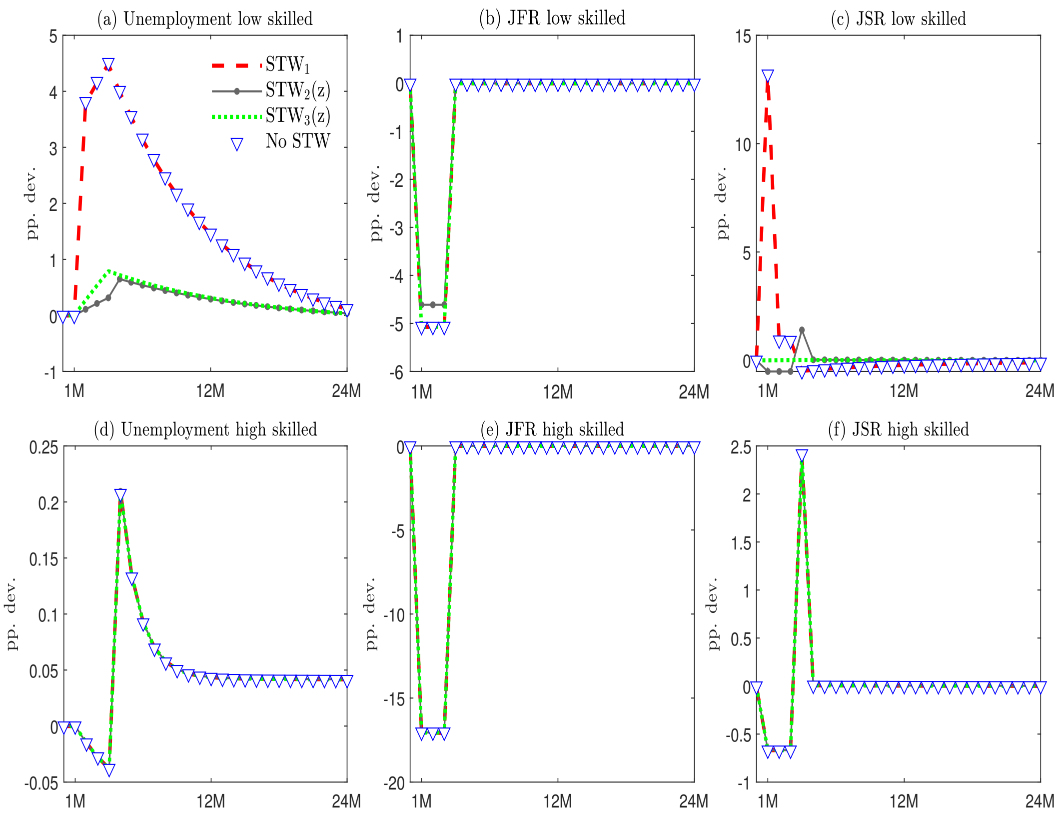
A.2 No STW

Figure 12: Labor market response - strict lockdown



All impulse response functions are expressed in percentage deviation from initial value in month 0.

Figure 13: Labor market response - strict lockdown - by skill



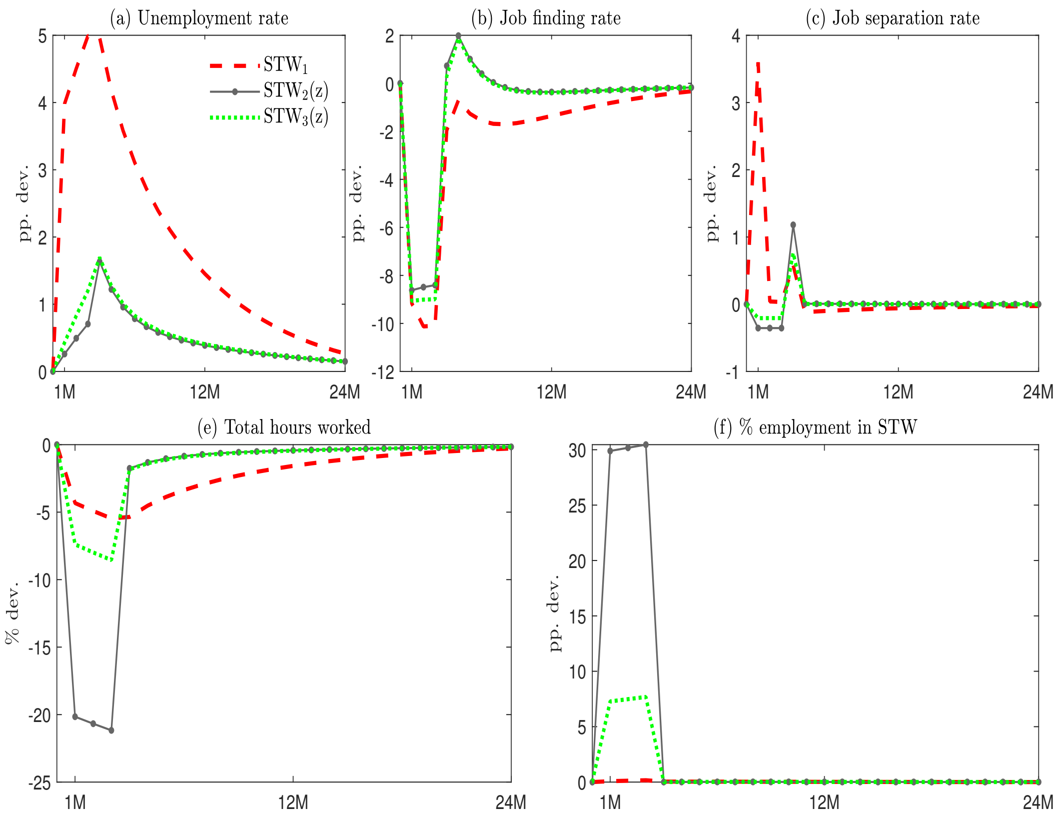
All impulse response functions are expressed in percentage deviation from initial value in month 0. Low skill is the lowest skill category, $h = 1$. High skill is the highest skill category, $h = 5$

A.3 State-dependent hourly wage

Consider now the following hourly wage equation:

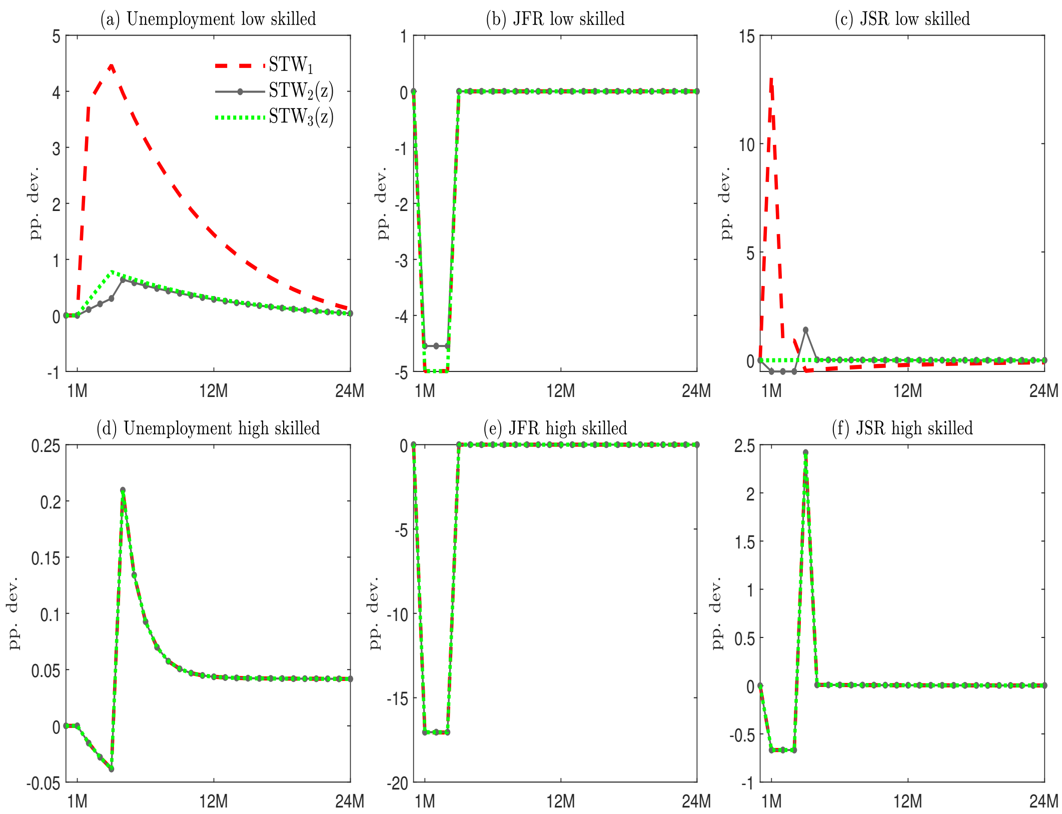
$$w(\varepsilon, h, z) \quad = \quad \max(w_{\min}, (z\varepsilon h)^{\gamma})$$

Figure 14: Labor market response - strict lockdown



All impulse response functions are expressed in percentage deviation from initial value in month 0.

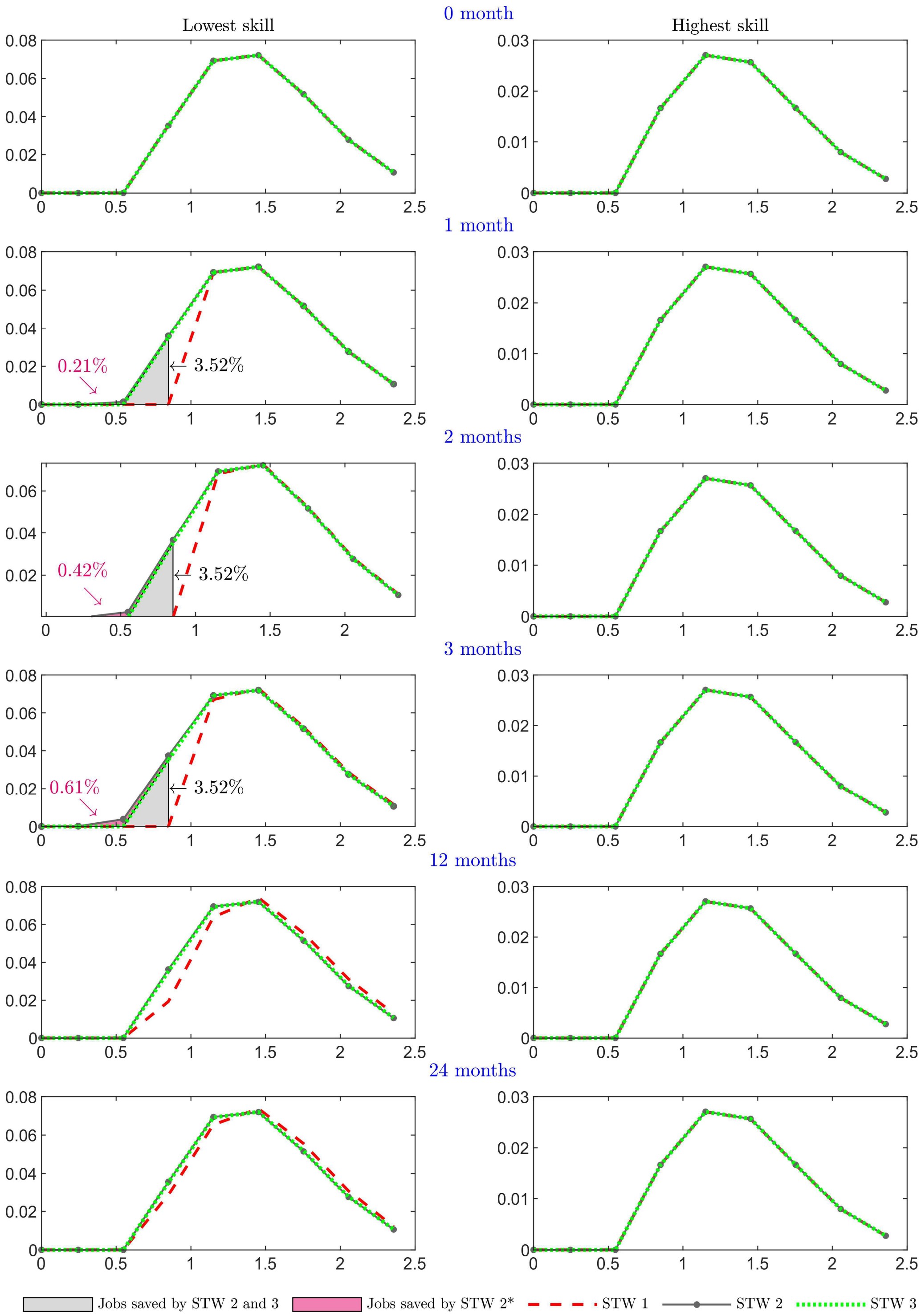
Figure 15: Labor market response - strict lockdown - by skill



All impulse response functions are expressed in percentage deviation from initial value in month 0. Low skill is the lowest skill category, $h = 1$. High skill is the highest skill category, $h = 5$

A.4 Employment distribution - light lockdown

Figure 16: Employment distribution - light lockdown



Note. The Figure shows the distribution of employment across individual productivity by skill group (h_1 and h_H) aggregated over asset a. The shock hit the economy in period 1, so that "0 month" is the steady state where $z = z_m$. The economy remains 3 periods in a strict lockdown $z = z_s$ and returns to the steady state $z = z_m$. Percentages in the figure indicate the amount of employment saved if any. additional jobs saved by STW2 w.r.t. STW3.*

B Aggregate states

B.1 Aggregate shock

Let z be the aggregate state of the economy. We consider that $z = \{z_b, z_m, z_g, z_s, z_l\}$ with the following states respectively:

$$\begin{array}{ll} z_b & \text{bad state} \\ z_m & \text{middle state} \\ z_g & \text{good state} \\ z_s & \text{strict lockdown} \\ z_l & \text{light lockdown} \end{array} \tag{21}$$

z follows a Markov process with transition probability from state z to z', $P(z'|z)$ as:

$$P(z'|z) = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} \\ p_{41} & p_{42} & p_{33} & p_{44} & p_{45} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} \end{bmatrix} \tag{22}$$

The definition of p_{ij} is particular due to the presence of rare disasters. We describe here the steps for defining each p_{ij}.

- Following Barro (2006), the occurrence of rare disaster is 1.7% on a annual basis. We denote by λ the probability of entering lockdown. The lockdown can be strict, with rate ϖ, or light, with rate $(1-\varpi)$. $\forall\, i = \{1,2,3\}$, $p_{i4} = \lambda\varpi = (1-(1-0.017)^{1/12})\varpi = \varpi 0.14\%$ on a monthly frequency. Similarly, $\forall\, i = \{1,2,3\}$, $p_{i5} = \lambda(1-\varpi) = (1-(1-0.017)^{1/12})(1-\varpi) = (1-\varpi)0.14\%$

- The switch between z_1, z_2 and z_3 is governed by the discretization of a first-order autoregressive process with persistence ρ_z, and standard deviation σ_z. Using the Rouwenhorst discretization technique, we obtain the following transition matrix for a three-states process:

  $$Q_0 = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}$$

  It follows that:

  $$\begin{aligned} p_{ij} &= q_{ij} \times \left(1 - \sum_{k=4}^{5} p_{ik}\right) \quad i = \{1,2,3\}, \quad j = \{1,2,3\} \quad j \neq i \\ p_{ij} &= q_{ij} \times (1-\lambda) \end{aligned}$$

- Barro (2006) estimates that rare disasters last between one and five years. While COVID-19 fits the definition of a rare disaster, governments impose lockdowns of shorter duration. Lockdowns range from one month to up to 9 months. We take an agnostic view and consider a value of 6 months. Consequently, the probability of remaining in a lockdown each month is $p_{44} = p_{55} = \varphi = \frac{5}{6}$.

- The probability of exiting the lockdown is $1 - p_{jj} = 1 - \varphi$, $j = \{4, 5\}$. It is distributed across the other events: (i) a different lockdown and (ii) the return to a normal business cycle.
  - (i) $p_{45} = (1 - p_{44})\lambda(1 - \varpi) = (1 - \varphi)\lambda(1 - \varpi)$ and $p_{54} = (1 - p_{55})\lambda\varpi = (1 - \varphi)\lambda\varpi$
  - (ii) The switch from any lockdown to a normal business cycle p_{ji}, $i = \{1, 2, 3\}$, $j = \{4, 5\}$. It is determined using the unconditional distribution of Q_0. Let π_i, $i = \{1, 2, 3\}$ be the unconditional distribution of the Markov process. One has:

$$\pi = Q_0' \pi$$

with Q_0' being the transpose of Q_0 and,

$$\begin{aligned} p_{4i} &= \pi_i(1 - \varphi)(1 - \lambda(1 - \varpi)) \quad i = \{1, 2, 3\} \\ p_{5i} &= \pi_i(1 - \varphi)(1 - \lambda\varpi) \quad i = \{1, 2, 3\} \end{aligned}$$

The transition matrix can be rewritten as:

$$P(z'|z) = \begin{bmatrix} q_{11}(1-\lambda) & q_{12}(1-\lambda) & q_{13}(1-\lambda) & \lambda\varpi & \lambda(1-\varpi) \\ q_{21}(1-\lambda) & q_{22}(1-\lambda) & q_{23}(1-\lambda) & \lambda\varpi & \lambda(1-\varpi) \\ q_{31}(1-\lambda) & q_{32}(1-\lambda) & q_{33}(1-\lambda) & \lambda\varpi & \lambda(1-\varpi) \\ \pi_1(1-\varphi)(1-\lambda(1-\varpi)) & \pi_2(1-\varphi)(1-\lambda(1-\varpi)) & \pi_3(1-\varphi)(1-\lambda(1-\varpi)) & \varphi & (1-\varphi)\lambda(1-\varpi) \\ \pi_1(1-\varphi)(1-\lambda\varpi) & \pi_2(1-\varphi)(1-\lambda\varpi) & \pi_3(1-\varphi)(1-\lambda\varpi) & (1-\varphi)\lambda\varpi & \varphi \end{bmatrix}$$

Considering the following values:

Table 7: CALIBRATION OF STOCHASTIC PROCESSES

Variables	Symbol	Value
Persistence aggregate shock	ρ_z	0.96
Standard deviation aggregate shock	σ_z	0.003
Standard deviation aggregate shock	σ_z	0.007
Probability of enter in a lockdown	λ	0.14%
Rate of strict lockdown	ϖ	0.5
Probability of stay in a lockdown	φ	83.33%

The transition matrix is:

$$P(z'|z) = \begin{bmatrix} 0.9590 & 0.0391 & 0.0004 & 0.0007 & 0.0007 \\ 0.0196 & 0.9594 & 0.0196 & 0.0007 & 0.0007 \\ 0.0004 & 0.0391 & 0.9590 & 0.0007 & 0.0007 \\ 0.0416 & 0.0833 & 0.0416 & 0.8333 & 0.0001 \\ 0.0416 & 0.0833 & 0.0416 & 0.0001 & 0.8333 \end{bmatrix}$$

Remark When solving the model before the pandemic, states $z = 4$ and $z = 5$ did not exist and are thus ignored when solving for the optimal decision rules and stationary distribution. In that case, $z = \{1, 2, 3\}$ and the transition matrix is $P(z'|z) = Q_0$.

B.2 Regime-switching STW policy

Let $\boldsymbol{s} = \{1, 2, 3\}$ be the STW regime with

$$\begin{array}{ll} \boldsymbol{s} = 1 & \text{Pre-COVID STW} \\ \boldsymbol{s} = 2 & \text{March-June STW} \\ \boldsymbol{s} = 3 & \text{Post-June STW} \end{array}$$

We make several assumptions regarding the process followed by $\boldsymbol{s}$.

- Before the pandemic: there is no emergency STW program. $\boldsymbol{s}$ is always equal to 1 (pre-COVID situation). In addition, there is no anticipation about a potential switch from $\boldsymbol{s} = 1$ to $\boldsymbol{s} > 1$. The use of the pre-COVID STW program is costly. Furthermore, with $z = \{1, 2, 3\}$, the transition matrix is $P(z'|z) = Q_0$.

- The economy moves from $\boldsymbol{s} = 1$ to $\boldsymbol{s} = 2$ in March 2020:

  - Agents update their decision rules and anticipations.
  - We assume that they know the stochastic process for the aggregate state defined by Equation (22) and they also take into account the process followed by the regime switching policy. We consider the following transition matrix:

$$S(\boldsymbol{s}'|\boldsymbol{s}; z) = \begin{bmatrix} \eta_{11}(z) & \eta_{12}(z) \\ \eta_{21}(z) & \eta_{22}(z) \end{bmatrix} \tag{23}$$

  In that case there are two potential regime $\boldsymbol{s} = \{1, 2\}$. Agents take into account the probability that the authorities may switch between the two types of STW programs, depending on the aggregate state. One has,

$$S(\boldsymbol{s}'|\boldsymbol{s}; z) = \begin{cases} \begin{bmatrix} 1 & 0 \\ \nu & 1-\nu \end{bmatrix} & \forall z \in \{z_b, z_m, z_g\} \\[2ex] \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} & \forall z \in \{z_s, z_l\} \end{cases}$$

  Remark. The probability that STW_2 is triggered if the economy is in regime $\boldsymbol{s} = 1$ and outside a lockdown is zero. Outside the lockdown ($z \in \{z_b, z_m, z_g\}$), if the initial regime is $\boldsymbol{s} = 2$, it means that there was previously a lockdown. In this situation, the probability that the emergency programs ends is ν. ν represents the lag between the decision to end the emergency STW program since the end of the lockdown. If $z \in \{z_s, z_l\}$, the use of STW_2 is automatically triggered, whatever the initial regime $\boldsymbol{s}$. Said differently, the government decides to maintain the emergency program as long as the economy is in a lockdown.

- The economy moves from $\boldsymbol{s} = 2$ to $\boldsymbol{s} = 3$ in June 2020:

  - Agents update their decision rules and anticipations again.
  - We assume that they know the new stochastic process that governs the regime-switching policy. Equation (23) applies but with $\boldsymbol{s} = \{1, 3\}$. We assume that there is no possibility of returning to regime $\boldsymbol{s} = 2$. The transition probability $S(\boldsymbol{s}'|\boldsymbol{s}; z)$ is strictly identical to the previous case.

C Solution method

The model is solved recursively and we use its block recursive structure, which allows to solve the model numerically with aggregate shocks without having recourse to Krusell Smith methodology. A first set of equations, including in particular the value functions, is solved. These equations are independent of the distribution of workers. Finally, a second set of equations provides the distribution of workers.

C.1 Discretization of the state-space and of the shocks

To solve the model, we discretize the state-space and the shocks.

- The wealth interval $[a_{\min}, a_{\max}]$ is discretized and a takes its values in the grid $\mathcal{A} = \{a_1, ..., a_{N_a}\}$.
- We consider a discrete approximation of the the conditional distribution of the idiosyncratic productivity and ε takes its values in the grid $\mathcal{E} = \{\varepsilon_1, ..., \varepsilon_{N_\varepsilon}\}$. The transition probability is given by $G(\varepsilon_{j'}|\varepsilon_j) = G(j, j')$. A new productivity is drawn with probability $G_0(\varepsilon_j) = G_0(j)$.
- Human capital h takes its values in the grid $h \in \mathcal{H} = \{h_1, ..., h_H\}$. The transition probability is given by $\mu_n(h_k, h_{k'})$.
- We consider a discrete approximation of the conditional distribution of the aggregate productivity. z takes its values in the grid $\mathcal{Z} = \{z_1, ..., z_{N_z}\}$. The transition probability is given by $P(\varepsilon_{l'}|\varepsilon_l) = P(l, l')$.
- Finally, there are $N_{\boldsymbol{s}}$ regimes $\boldsymbol{s} \in \mathcal{S} = \{1, 2, 3, ..., N_{\boldsymbol{s}}\}$. The transition probability is $S(\boldsymbol{s}_{m'}|\boldsymbol{s}_m, z_{l'}) = S(m, l', m')$.

C.2 Algorithm

The model has a block recursive structure. In a first time, we solve a set of equations providing the value functions and the decision rules. In a second time, we determine the distribution of the agent.

C.2.1 Definition of equilibrium

DEFINITION 1. *Given exogenous processes for human capital h, aggregate productivity z, idiosyncratic productivity ε, and STW regime $\boldsymbol{s}$; the equilibrium is a list of (i) quantities $m(h, z, \boldsymbol{s})$, and $v(h, z, \boldsymbol{s})$; (ii) probabilities $f(h, z, \boldsymbol{s})$, $q(h, z, \boldsymbol{s})$ (iii) the price $w(\varepsilon, h)$ and the productivity $y(\varepsilon, h, \ell, z, \boldsymbol{s})$; (iv) value functions $J(\varepsilon, h, z, \boldsymbol{s})$, $V(h, z, \boldsymbol{s})$, $W(a, \varepsilon, h, z, \boldsymbol{s})$, and $U(a, h, z, \boldsymbol{s})$; (v) optimal hours decision $\ell^\star(\varepsilon, h, z, \boldsymbol{s})$; (vi) optimal separation decision $\mathbb{1}(a, \varepsilon, h, z, \boldsymbol{s})$; (vii) stationary distributions of employment $n(a, \varepsilon, h, z, \boldsymbol{s})$ and unemployment $u(a, h, z, \boldsymbol{s})$; satisfying the following conditions:*

(i) *$m(h, z, \boldsymbol{s})$, $f(h, z, \boldsymbol{s})$, $q(h, z, \boldsymbol{s})$, and $v(h, z, \boldsymbol{s})$ are the solutions of the matching function* (3)*, the job finding rate* (4)*,the vacancy filling rate* (5)*, and the job creation condition* (16)*, respectively;*

(ii) *Prices $y(\varepsilon, h, \ell, z)$ and $w(\varepsilon, h)$ satisfy equations* (6) *and* (7)*;*

(iii) *Value functions $J(\varepsilon, h, z, \boldsymbol{s})$, $V(h, z, \boldsymbol{s})$, $W(a, \varepsilon, h, z, \boldsymbol{s})$, and $U(a, h, z, \boldsymbol{s})$ are solutions of the system that combines* (11), (13), (14) *and*(15)

(iv) *The optimal hours worked decision $\ell^{\star}(\varepsilon, h, z, \boldsymbol{s})$ solves* (12);

(iv) *The optimal separation decision $\mathbb{1}(a, \varepsilon, h, z, \boldsymbol{s})$ is derived from* (18);

(vii) *The distributions $n(a, \varepsilon, h, z, \boldsymbol{s})$ and $u(a, h, z, \boldsymbol{s})$ solve the law of motion described by* (19) *and* (20).

C.2.2 Value function iteration

Step 1 Guess an initial value for the probability $\{q\}$.

Step 2 Compute the value functions $\{J, V, W, U\}$.

Step 3 Given the values obtained at step 2, using the job creation condition, compute the new value of the probability q'.

Step 4 Calculate the norm:

$$\mathcal{N} = \frac{|q' - q|}{|q|}$$

Step 5 If $\mathcal{N} < \epsilon_q$, with ϵ_q the convergence criteria, stop. Otherwise, set $q = q'$ and return to step 2.

C.2.3 Distribution of workers

Step 1 Using the previous results, compute the elements of the transition probability matrix.

Step 2 Guess an initial value for the distribution probability $\mathcal{D} = \{n, u\}$.

Step 3 Using the transition probability matrix, compute the new distribution $\mathcal{D}'$.

Step 4 Calculate the norm:

$$\mathcal{N} = \frac{|\mathcal{D}' - \mathcal{D}|}{|\mathcal{D}|}$$

Step 5 If $\mathcal{N} < \epsilon_d$, with ϵ_d the convergence criteria, stop. Otherwise, set $\mathcal{D} = \mathcal{D}'$ and return to step 2.

C.3 Solving the Bellman equations and distribution

In this section, we provide some details about the numerical determination of the value functions and of the distribution of workers.

C.3.1 Solving the Bellman equation of the workers

The problem of the worker is summarized by equations (14) and (15). Eliminating consumption by means of the budget constraint in the worker's problems provides:

$$
\begin{aligned}
W(a,\varepsilon,h,z,\mathbf{s}) &= \max_{a'\geq \underline{a}} \Big\{ \Upsilon_n(a,\varepsilon,h,z,\mathbf{s},a') \\
&+ \beta_w(z)\sum_{h'}\mu_n(h,h')\int\int\int \begin{bmatrix} (1-\delta)\,\Omega(a',\varepsilon',h',z',\mathbf{s}') \\ +\delta\; U(a',h',\mathbf{s}') \end{bmatrix} dG(\varepsilon'|\varepsilon)dS(\mathbf{s}'|\mathbf{s};z')dP(z'|z). \Big\}
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
U(a,h,z,\mathbf{s}) &= \max_{a'\geq \underline{a}} \Big\{ \Upsilon_u(a,h,z,\mathbf{s},a') \\
&+ \beta_w(z)\sum_{h'}\mu_u(h,h')\int\int\int \begin{bmatrix} (1-f(h,z,\mathbf{s}))U(a',h',z',\mathbf{s}') \\ +f(h,z,\mathbf{s})\Omega(a',\varepsilon',h',z',\mathbf{s}') \end{bmatrix} dG_0(\varepsilon')dS(\mathbf{s}'|\mathbf{s};z')dP(z'|z) \Big\}
\end{aligned} \tag{25}
$$

with

$$\Omega(a,\varepsilon,h,z,\mathbf{s}) = \max(W(a,\varepsilon,h,z,\mathbf{s}),U(a,h,z,\mathbf{s}))$$

$\Upsilon_n(a,\varepsilon,h,z,\mathbf{s},a')$ and $\Upsilon_u(a,h,z,\mathbf{s},a')$ represent the instantaneous utility (obtained after elimination of consumption by mean of the budget constraint) of an employed and unemployed worker respectively.

The worker's problem is solved recursively using a discretized version of the problem. To implement the model resolution, we build a multidimensional grid constructed using the Cartesian product, that is:

$$\mathcal{G} = \mathcal{A}\times\mathcal{E}\times\mathcal{H}\times\mathcal{Z}\times\mathcal{S}$$

Let $(a_i,\varepsilon_j,h_k,z_l,\mathbf{s}_m)$ be an element of $\mathcal{G}$. For each point of the grid $\mathcal{G}$, the agents choose a' maximizing its value. We suppose a' is selected in a moving grid depending on the asset level a_i. The moving grid is constructed in such a way that the optimal a' belongs to it. Let $\mathcal{A}'_i$ the moving grid associated to an asset level a_i. This moving grid is defined over a small interval and has a large number of points. One has $\mathcal{A}'_i = \{a^i_1,...,a^i_{i'},...,a^i_{N_{a'}}\}$.

Before writing the distretized problem and solving it, we have to describe the linear interpolation used in our numerical algorithm. Suppose that the values taken by a function $M(a)$ are only known for values of a chosen in the grid $\mathcal{A}$. We evaluate $M(a)$ with $a\in[a_i,a_{i+1}[$ by:

$$\mathcal{M}(a) = \frac{a_{i+1}-a}{a_{i+1}-a_i}\mathcal{M}(a_i) + \frac{a-a_i}{a_{i+1}-a_i}\mathcal{M}(a_{i+1})$$

This expression will be used to evaluate the expected values in the Bellman equations (25) and (26) and to compute the distribution of workers. In that case, the coefficients $\frac{a_{i+1}-a}{a_{i+1}-a_i}$ and $\frac{a-a_i}{a_{i+1}-a_i}$ will represent the probability of drawing a_i and a_{i+1} respectively. We define:

$$\Delta_i(a) = \frac{a_{i+1}-a}{a_{i+1}-a_i}$$

$$\Delta_{i+1}(a) = \frac{a-a_i}{a_{i+1}-a_i}$$

To solve equations (25) and (26), we start by making an initial guess about the values W and U, that is $W^{(0)}$ and $U^{(0)}$ defined on the grid $\mathcal{G}$.

Suppose we are at step j with $W^{(j)}$ and $U^{(j)}$. The updated value functions are $W^{(j+1)}$ and $U^{(j+1)}$. Consider a point $(a_i, \varepsilon_j, h_k, z_l, \boldsymbol{s}_m)$ of the grid $\mathcal{G}$. For each point $a^i_{i'}$ of the moving grid $\mathcal{A}'_i$ determines the interval of $\mathcal{A}$ to which it belongs (there exist ι such that $a^i_{i'} \in [a_\iota, a_{\iota+1}[$) and then evaluate the right-hand side of equations (25) and (26), that is:

$$\begin{aligned}
& \Phi_n(a^i_{i'}; a_i, \varepsilon_j, h_k, z_l, \boldsymbol{s}_m) \equiv g(a_i, \varepsilon_j, h_k, z_l, \boldsymbol{s}_m, a^i_{i'}) \\
+ \quad & \beta_w(z) \sum_{k'=1}^{H} \mu_n(h_k, h_{k'}) \sum_{l'=1}^{N_z} \sum_{m'=1}^{N_{\boldsymbol{s}}} \sum_{j'=1}^{N_\varepsilon} \Big[\Delta_\iota(a^i_{i'}) \{ (1-\delta) \Omega^{(j)}(a_\iota, \varepsilon_{j'}, h_{k'}, z_{l'}, \boldsymbol{s}_{m'}) + \delta U^{(j)}(a_\iota, h_{j'}, z_{l'}, \boldsymbol{s}_{m'}) \} \\
+ \quad & \Delta_{\iota+1}(a^i_{i'}) \{ (1-\delta) \Omega^{(j)}(a_{\iota+1}, \varepsilon_{j'}, h_{k'}, z_{l'}, \boldsymbol{s}_{m'}) + \delta U^{(j)}(a_{\iota+1}, h_{j'}, z_{l'}, \boldsymbol{s}_{m'}) \} \Big] G(j, j') S(m, l', m') P(l, l')
\end{aligned}$$

$$\begin{aligned}
& \Phi_u(a^i_{i'}; a_i, h_k, z_l, \boldsymbol{s}_m) \equiv h(a_i, \varepsilon_j, h_k, z_l, \boldsymbol{s}_m, a^i_{i'}) \\
+ \quad & \beta_w(z) \sum_{k'=1}^{H} \mu_n(h_k, h_{k'}) \sum_{l'=1}^{N_z} \sum_{m'=1}^{N_{\boldsymbol{s}}} \Big[\Delta_\iota(a^i_{i'}) \{ f(h_k, z_l, \boldsymbol{s}) \sum_{j'=1}^{N_\varepsilon} \Omega^{(j)}(a_\iota, \varepsilon_{j'}, h_{k'}, z_{l'}, \boldsymbol{s}_{m'}) G_0(j') \\
+ \quad & (1 - f(h_k, z_l, \boldsymbol{s}')) U^{(j)}(a_\iota, h_{k'}, z_{l'}, \boldsymbol{s}_{m'}) \} \\
+ \quad & \Delta_{\iota+1}(a^i_{i'}) \{ f(h_k, z_l, \boldsymbol{s}_{m'}) \sum_{j'=1}^{N_\varepsilon} \Omega^{(j)}(a_{\iota+1}, \varepsilon_{j'}, h_{k'}, z_{l'}, \boldsymbol{s}') G_0(j') \\
+ \quad & (1 - f(h_k, z_l, \boldsymbol{s}')) U^{(j)}(a_{\iota+1}, h_{k'}, z_{l'}, \boldsymbol{s}_{m'}) \} \Big] S(m, l', m') P(l, l')
\end{aligned}$$

The updated value function are given by:

$$\begin{aligned}
W^{(j+1)}(a_i, \varepsilon_j, h_k, z_l, \boldsymbol{s}_m) &= \max_{a^i_{i'} \in \mathcal{A}'_i} \Phi_n(a^i_{i'}; a_i, \varepsilon_j, h_k, z_l, \boldsymbol{s}_m) \\
U^{(j+1)}(a_i, h_k, z_l, \boldsymbol{s}_m) &= \max_{a^i_{i'} \in \mathcal{A}'_i} \Phi_u(a^i_{i'}; a_i, h_k, z_l, \boldsymbol{s}_m)
\end{aligned}$$

The value functions are updated until convergence.

C.3.2 Solving the Bellman equation of firms

The free entry condition implies that $V(h, z, \boldsymbol{s}) = 0$. Equation (16) providing the value of a job can be written synthetically as follows:

$$\begin{aligned}
J(\varepsilon, h, z, \boldsymbol{s}) &= \pi(\varepsilon, h, z, \boldsymbol{s}) \\
&+ \beta_f (1-\delta) \sum_{h'} \mu_n(h, h') \int \int \int \Lambda^o(\varepsilon', h', z', \boldsymbol{s}') dG(\varepsilon'|\varepsilon) dS(\boldsymbol{s}'|\boldsymbol{s}; z') dP(z'|z), \quad (26)
\end{aligned}$$

where

$$\Lambda^o(\varepsilon, h, z, \boldsymbol{s}) = \max(J(\varepsilon, h, z, \boldsymbol{s}), V(h, z, \boldsymbol{s})),$$

and $\pi(\varepsilon, h, z, \boldsymbol{s})$ is instantaneous profit evaluated for at the optimal hours worked level $\ell^\star(\varepsilon, h, z, \boldsymbol{s})$. β_f is the discount factor of the firm. The firm value is computed recursively using a discrete version of equation (26). To begin, we consider an initial guess

$J^{(0)}$ defined over the grid $\mathcal{J} = \mathcal{E} \times \mathcal{H} \times \mathcal{Z} \times \mathcal{S}$. The value J is updated using the following equation:

$$\begin{aligned} J^{(j+1)}(\varepsilon_j, h_k, z_l, \boldsymbol{s}_m) &= \pi(\varepsilon_j, h_k, z_l, \boldsymbol{s}_m) \\ + \quad \beta_f(1-\delta) \sum_{k'=1}^{H} \mu_n(h_k, h_{k'}) & \sum_{l'=1}^{N_z} \sum_{m'=1}^{N_s} \sum_{j'=1}^{N_\varepsilon} \Lambda^{o(j)}(\varepsilon_{j'}, h_{k'}, z_{l'}, \boldsymbol{s}_{m'}) G(j,j') S(m,l',m') P(l,l') \end{aligned}$$

The value function is updated until convergence.
Finally, the filling rate is deduced from:

$$\frac{c}{q(h_k, z_l, \boldsymbol{s}_m)} = \beta_f \sum_{l'=1}^{N_z} \sum_{m'=1}^{N_s} \sum_{j'=1}^{N_\varepsilon} \Lambda^n(\varepsilon_{j'}, h_k, z_{l'}, \boldsymbol{s}_{m'}) G_0(j') S(m,l',m') P(l,l')$$

with

$$\Lambda^n(\varepsilon, h, z, \boldsymbol{s}) = \max(J(\varepsilon, h, z, \boldsymbol{s}), 0))$$

C.3.3 The distribution of the workers

Stationary distribution An employed worker and an unemployed worker are characterized by the individual states (a, ε, h) and (a, h) respectively. The aggregate state is $(z, \boldsymbol{s})$. $n(a, \varepsilon, h; z, \boldsymbol{s})$ is the number of employed workers in state $(a, \varepsilon, h; z, \boldsymbol{s})$. Likewise, $u(a, h; z, \boldsymbol{s})$ is the number of unemployed workers in state $(a, h; z, \boldsymbol{s})$. We previously described the resolution of the worker's problem. If the optimal a' is selected in the grid $\mathcal{A}$, the agent decision rules are of the form $a' = \phi_n(a, \varepsilon, h; z, \boldsymbol{s})$ and $a' = \phi_u(a, h; z, \boldsymbol{s})$.

The stationary distribution of the workers (consider the aggregate state $(z, \boldsymbol{s})$ as given) is given by equation (19) and (20), that is:

$$\begin{aligned} n(a', \varepsilon', h'; z, \boldsymbol{s}) &= (1-\delta) \sum_{a \in \mathcal{A}} \sum_{\varepsilon \in \mathcal{E}} \sum_{h \in \mathcal{H}} \mathbb{1}(a' = \phi_n(a, \varepsilon, h, z, \boldsymbol{s})) G(\varepsilon'|\varepsilon) \mu_n(h, h') I^o(a', \varepsilon', h', z, \boldsymbol{s}) n(a, \varepsilon, h; z, \boldsymbol{s}) \\ &+ \sum_{a \in \mathcal{A}} \sum_{h \in \mathcal{H}} f(h, z, \boldsymbol{s}) \mathbb{1}(a' = \phi_u(a, h, z, \boldsymbol{s})) G_0(\varepsilon') \mu_u(h, h') I^n(a', \varepsilon', h', z, \boldsymbol{s}) u(a, h; z, \boldsymbol{s}) \\ u(a', h'; z, \boldsymbol{s}) &= \sum_{a \in \mathcal{A}} \sum_{\varepsilon \in \mathcal{E}} \sum_{h \in \mathcal{H}} \mathbb{1}(a' = \phi_n(a, \varepsilon, h, z, \boldsymbol{s})) G(\varepsilon'|\varepsilon) \mu_n(h, h')(\delta + (1-\delta)(1 - I^o(a', \varepsilon', h', z, \boldsymbol{s})) n(a, \varepsilon, h; z, \boldsymbol{s}) \\ &+ \sum_{a \in \mathcal{A}} \sum_{h \in \mathcal{H}} \mathbb{1}(a' = \phi_u(a, h, z, \boldsymbol{s})) G_0(\varepsilon') \mu_u(h, h')(1 - f(h, z, \boldsymbol{s}) + f(h, z, \boldsymbol{s})(1 - I^n(a', \varepsilon', h', z, \boldsymbol{s})) u(a, h; z, \boldsymbol{s}) \end{aligned}$$

with

$$\begin{aligned} I^o(a', \varepsilon', h', z, \boldsymbol{s}) &= \mathbb{1}(W(a', \varepsilon', h', z, \boldsymbol{s}) \geq U(a', h', z, \boldsymbol{s})) \times \mathbb{1}(J(\varepsilon', h', z, \boldsymbol{s}) \geq -F(\varepsilon', h')) \\ I^n(a', \varepsilon', h', z, \boldsymbol{s}) &= \mathbb{1}(W(a', \varepsilon', h', z, \boldsymbol{s}) \geq U(a', h', z, \boldsymbol{s})) \times \mathbb{1}(J(\varepsilon', h', z, \boldsymbol{s}) \geq 0) \end{aligned}$$

If the optimal a' is selected in the moving grid $\mathcal{A}'$, linear interpolations are necessary. The distribution of workers is then written:

$$
\begin{aligned}
n(a_{i'}, \varepsilon_{j'}, h_{k'}; z, \boldsymbol{s}) \quad &= \quad (1-\delta) \sum_{i=1}^{N_a} \sum_{j=1}^{N_\varepsilon} \sum_{k=1}^{H} \Big\{ \mathbb{1}(\phi_n(a_i, \varepsilon_j, h_k, z, \boldsymbol{s}) \in [a_{i'-1}, a_{i'}[) \Delta_{i'}(\phi_n(a_i, \varepsilon_j, h_k, z, \boldsymbol{s})) \\
&+ \quad \mathbb{1}(\phi_n(a_i, \varepsilon_j, h_k, z, \boldsymbol{s}) \in [a_{i'}, a_{i'+1}[) \Delta_{i'}(\phi_n(a_i, \varepsilon_j, h_k, z, \boldsymbol{s})) \Big\} \\
&\times \quad G(j, j') \mu_n(h_k, h_{k'}) I^o(a_{i'}, \varepsilon_{j'}, h_{k'}, z, \boldsymbol{s}) n(a_i, \varepsilon_j, h_k; z, \boldsymbol{s}) \\
&+ \quad \sum_{i=1}^{N_a} \sum_{j=1}^{N_\varepsilon} \sum_{k=1}^{H} f(h_k, z, \boldsymbol{s}) \Big\{ \mathbb{1}(\phi_u(a_i, \varepsilon_j, h_k, z, \boldsymbol{s}) \in [a_{i'-1}, a_{i'}[) \Delta_{i'}(\phi_u(a_i, \varepsilon_j, h_k, z, \boldsymbol{s})) \\
&+ \quad \mathbb{1}(\phi_u(a_i, \varepsilon_j, h_k, z, \boldsymbol{s}) \in [a_{i'}, a_{i'+1}[) \Delta_{i'}(\phi_u(a_i, \varepsilon_j, h_k, z, \boldsymbol{s})) \Big\} \\
&\times \quad G_0(j') \mu_u(h_k, h_{k'}) I^n(a_{j'}, \varepsilon_{j'}, h_{k'}, z, \boldsymbol{s}) u(a_i, h_k; z, \boldsymbol{s})
\end{aligned}
$$

$$
\begin{aligned}
u(a_{i'}, h_{k'}; z, \boldsymbol{s}) \quad &= \quad \sum_{i=1}^{N_a} \sum_{j=1}^{N_\varepsilon} \sum_{k=1}^{H} \Big\{ \mathbb{1}(\phi_n(a_i, \varepsilon_j, h_k, z, \boldsymbol{s}) \in [a_{i'-1}, a_{i'}[) \Delta_{i'}(\phi_n(a_i, \varepsilon_j, h_k, z, \boldsymbol{s})) \\
&+ \quad \mathbb{1}(\phi_n(a_i, \varepsilon_j, h_k, z, \boldsymbol{s}) \in [a_{i'}, a_{i'+1}[) \Delta_{i'}(\phi_n(a_i, \varepsilon_j, h_k, z, \boldsymbol{s})) \Big\} \\
&\times \quad G(j, j') \mu_n(h_k, h_{k'}) (\delta + (1-\delta)(1 - I^o(a_{j'}, \varepsilon_{i'}, h_{k'}, z, \boldsymbol{s})) n(a_i, \varepsilon_j, h_k; z, \boldsymbol{s}) \\
&+ \quad \sum_{i=1}^{N_a} \sum_{j=1}^{N_\varepsilon} \sum_{k=1}^{H} \Big\{ \mathbb{1}(\phi_u(a_i, \varepsilon_j, h_k, z, \boldsymbol{s}) \in [a_{i'-1}, a_{i'}[) \Delta_{i'}(\phi_u(a_i, \varepsilon_j, h_k, z, \boldsymbol{s})) \\
&+ \quad \mathbb{1}(\phi_u(a_i, \varepsilon_j, h_k, z, \boldsymbol{s}) \in [a_{i'}, a_{i'+1}[) \Delta_{i'}(\phi_u(a_i, \varepsilon_j, h_k, z, \boldsymbol{s})) \Big\} \\
&\times \quad G_0(j') \mu_u(h_k, h_{k'}) (1 - f(h_k, z, \boldsymbol{s}) + f(h_k, z, \boldsymbol{s})(1 - I^n(a_{i'}, \varepsilon_{j'}, h_{k'}, z, \boldsymbol{s})) u(a_i, h_k; z, \boldsymbol{s})
\end{aligned}
$$

Evolution of the distribution between two aggregate states Suppose the aggregate state moves from $(z, \boldsymbol{s})$ to $(z', \boldsymbol{s}')$. The distribution of workers is now written:

$$
\begin{aligned}
n(a_{i'},\varepsilon_{j'},h_{k'};z',\boldsymbol{s}') &= (1-\delta)\sum_{i=1}^{N_a}\sum_{j=1}^{N_\varepsilon}\sum_{k=1}^{H}\Big\{\mathbb{1}(\phi_n(a_i,\varepsilon_j,h_k,z,\boldsymbol{s})\in[a_{i'-1},a_{i'}[)\Delta_{i'}(\phi_n(a_i,\varepsilon_j,h_k,z,\boldsymbol{s}))\\
&+ \mathbb{1}(\phi_n(a_i,\varepsilon_j,h_k,z,\boldsymbol{s})\in[a_{i'},a_{i'+1}[)\Delta_{i'}(\phi_n(a_i,\varepsilon_j,h_k,z,\boldsymbol{s}))\Big\}\\
&\times G(j,j')\mu_n(h_k,h_{k'})I^o(a_{i'},\varepsilon_{j'},h_{k'},z',\boldsymbol{s}')n(a_i,\varepsilon_j,h_k;z,\boldsymbol{s})\\
&+ \sum_{i=1}^{N_a}\sum_{j=1}^{N_\varepsilon}\sum_{k=1}^{H}f(h_k,z',\boldsymbol{s}')\Big\{\mathbb{1}(\phi_u(a_i,\varepsilon_j,h_k,z,\boldsymbol{s})\in[a_{i'-1},a_{i'}[)\Delta_{i'}(\phi_u(a_i,\varepsilon_j,h_k,z,\boldsymbol{s}))\\
&+ \mathbb{1}(\phi_u(a_i,\varepsilon_j,h_k,z,\boldsymbol{s})\in[a_{i'},a_{i'+1}[)\Delta_{i'}(\phi_u(a_i,\varepsilon_j,h_k,z,\boldsymbol{s}))\Big\}\\
&\times G_0(j')\mu_u(h_k,h_{k'})I^n(a_{j'},\varepsilon_{j'},h_{k'},z',\boldsymbol{s}')u(a_i,h_k;z,\boldsymbol{s})
\end{aligned}
$$

$$
\begin{aligned}
u(a_{i'},h_{k'};z',\boldsymbol{s}') &= \sum_{i=1}^{N_a}\sum_{j=1}^{N_\varepsilon}\sum_{k=1}^{H}\Big\{\mathbb{1}(\phi_n(a_i,\varepsilon_j,h_k,z,\boldsymbol{s})\in[a_{i'-1},a_{i'}[)\Delta_{i'}(\phi_n(a_i,\varepsilon_j,h_k,z,\boldsymbol{s}))\\
&+ \mathbb{1}(\phi_n(a_i,\varepsilon_j,h_k,z,\boldsymbol{s})\in[a_{i'},a_{i'+1}[)\Delta_{i'}(\phi_n(a_i,\varepsilon_j,h_k,z,\boldsymbol{s}))\Big\}\\
&\times G(j,j')\mu_n(h_k,h_{k'})(\delta+(1-\delta)(1-I^o(a_{j'},\varepsilon_{i'},h_{k'},z',\boldsymbol{s}'))n(a_i,\varepsilon_j,h_k;z,\boldsymbol{s})\\
&+ \sum_{i=1}^{N_a}\sum_{j=1}^{N_\varepsilon}\sum_{k=1}^{H}\Big\{\mathbb{1}(\phi_u(a_i,\varepsilon_j,h_k,z,\boldsymbol{s})\in[a_{i'-1},a_{i'}[)\Delta_{i'}(\phi_u(a_i,\varepsilon_j,h_k,z,\boldsymbol{s}))\\
&+ \mathbb{1}(\phi_u(a_i,\varepsilon_j,h_k,z,\boldsymbol{s})\in[a_{i'},a_{i'+1}[)\Delta_{i'}(\phi_u(a_i,\varepsilon_j,h_k,z,\boldsymbol{s}))\Big\}\\
&\times G_0(j')\mu_u(h_k,h_{k'})(1-f(h_k,z',\boldsymbol{s}')+f(h_k,z',\boldsymbol{s}')(1-I^n(a_{i'},\varepsilon_{j'},h_{k'},z',\boldsymbol{s}'))u(a_i,h_k;z,\boldsymbol{s})
\end{aligned}
$$

The above equations may take the following matrix form:

$$\Gamma' = \boldsymbol{\Theta}(z,\boldsymbol{s},z',\boldsymbol{s}')\Gamma$$

with

$$\Gamma = \begin{pmatrix} n \\ u \end{pmatrix}$$