Optimal vs. simple capital control rules in a sudden stop environment

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Abstract

The aim of this paper is to contribute to the growing literature on the effectiveness of prudential capital controls in Emerging Markets Economies by examining the outcomes of three capital control policies in open economy models with pecuniary externalities due to flow collateral constraints. We provide an explanation for the pro-cyclicality of Ramsey optimal capital control policy stressed by Schmitt-Grohé and Uribe [2017]. We investigate the outcomes of two alternative policies, e.g. fixed debt-tax on foreign debt that counteract households impatience, and a simple macroprudential policy based on foreign debt-to-GDP ratio targeting. Our results, using Bianchi [2011] model calibration, show that the fixed tax policy shifts the economy far away from binding collateral constraint region and reduces macroeconomic volatility, while the implementation of the simple macroprudential leads to a welfare gain relatively higher than that of the regulated economy with fixed debt-tax and more than seven times higher than that of Ramsey economy.

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1 Introduction

Emerging market economies (EMEs) have experienced similar financial crisis events known in the existing literature as Balance-of-Payments (BoP) crisis also called Currency crisis. Many academic attempts to explain this phenomenon have focused first on the distortionary effect of public policies that jeopardise the sustainability of exchange rate regime (first generation models, Krugman [1979], Flood and Garber [1984]). Indeed, major of Latin American countries suffering from debt default crisis that occurred between 1982 – 89, pursued unsustainable macroeconomic policies mainly a loose fiscal policy financed by domestic money creation incompatible with a fixed exchange rate regime. Nevertheless, the series of speculative attacks on the European monetary system currencies in 1992 – 3 called into question the basic idea that seigniorage is the sole underlying cause of currency instability. Thus, another point of view arose explaining crisis as the product of investors’ expectation about exchange rate regime sustainability and doubt on whether a government is willing to maintain its exchange rate peg or to pursue a more expansionary monetary policy (second generation models, Obstfeld [1994]). These models highlighted the possibility of self-fulfilling crisis in which a deterioration of investor confidence could generate a currency depreciation that justified investor pessimism and could lead, in fine, to currency attacks. The third generation models has been emerging since the East Asian crisis of 1997 – 98, as the crisis did not fit perfectly the two precedent approaches, the need for a third plausible explanation inspired three main variants’ models (Krugman [2001]). The first version involves theory on moral-hazard-driven investment. In these models, special attention is given to currency mismatch and moral hazard that lead banks to engage on excessive risk taking and over-borrowing (McKinnon and Pill [1996], Corsetti et al. [1998], Krugman [1999]). The second version, largely associated to Chang and Velasco [1999]’ work, advocated the role of international illiquidity-driven channel as a main factor of currency crashes, while the third version emphasizes the role of financial amplification effect that arise from balance sheets deterioration associated with fluctuations in asset prices (Krugman [1999], Aghion et al. [2004]).
Another class of models, attributed to the third version, investigate how Sudden Stops (SS) in foreign capital inflows may bring about financial and balance of payments crises. The term Sudden Stop was first introduced by Dornbusch et al. [1995] who stressed the role of real appreciation in hurting growth and endangering financial stability. The defining characteristic of a Sudden Stop is an abrupt halt in capital inflows measured by a reversal in current account followed by a deep recession (Calvo et al. [2004]). The most common explanation of currency crisis triggered following a SS episodes, draws a parallel between Fisherian debt-deflation dynamic (Fisher [1933]) and the feedback loop between asset prices and economic activity (Mendoza [2002], Korinek [2010], Korinek and Mendoza [2014]). Fisherian mechanism describes how SS of foreign capital inflows bring the economy into a spiraling decline in asset prices and collateral assets, that deteriorate financial conditions and avoid private agents to smooth their inter-temporal expenditure. In these models, financial frictions are highlighted by an endogenous borrowing constraint that itself is a function of the endogenous aggregate states of the economy, which play an important role as the determinants of the market prices at which collateral is valued.

In the present paper we study liability dollarization credit constraint introduced in open economy models by Mendoza [2002]. This kind of collateral supposes that banks intermediate capital inflows denominated in hard currencies into domestic loans generally denominated in national currencies and value the collateral in domestic asset prices. Following a negative and persistent shock, collateral value collapses and amplifies the shock in the economy. The transmission and amplification channel of the endogenous borrowing constraint open doors for public intervention. Korinek [2011] discusses whether macroprudential policies can address this issue by adapting prudential capital policies. He presents a survey on theoretical and empirical research on ex-ante and ex-post policies. Mendoza [2016] shows that macro-prudential policy is a powerful tool for preventing such financial crises, in the sense that a constrained-efficient financial regulator can reduce significantly the severity and frequency of such crises. However, macroprudential policy is not free of implementation
challenges. \[\text{Bianchi} \, 2011\] shows that the pecuniary externality related to the endogenous credit constraint leads to overborrowing and that the optimal capital control tax is positive on average, while \[\text{Schmitt-Grohé and Uribe} \, 2017\] states that the Ramsey optimal capital tax is pro-cyclical and thus can not be considered as a macro-prudential instrument. \[\text{Jeanne and Korinek} \, 2010\] find that externalities are likely to be largest during booms when risk builds up and prudential controls should be raised during such times. \[\text{Bianchi and Mendoza} \, 2018\] show that in open economies with stock collateral constraints the unregulated economy over-borrow in comparison with regulated economy and optimal capital control policy is time inconsistent, This policy features a state-contingent macroprudential debt tax that is strictly positive at date $t$ if a crisis has positive probability at $t + 1$. \[\text{Schmitt-Grohé and Uribe} \, 2016\] demonstrate that models with collateral constraint in the style of \[\text{Mendoza} \, 2002\] displays multiple equilibria with a possible self fulfilling crisis. They show that under different calibrated parameters the economy features under borrowing instead of overborrowing stressed in the related literature.

We aim to attribute to this growing literature by studying capital control and macroprudential policies in the context of liability dollarization SS models. We adopt the same analytical framework as in \[\text{Bianchi} \, 2011\] and \[\text{Schmitt-Grohé and Uribe} \, 2017\]. We examine first Ramsey economy in which a social planner set an optimal capital control tax to make private agents internalizing the externality created by the presence of the endogenous borrowing constraint, and we suggest a solution that helps to deduce an expression for the optimal control tax. As stressed by \[\text{Schmitt-Grohé and Uribe} \, 2017\], the optimal tax is procyclical during typical boom-bust cycles. It should be positive in good times to prevent overborrowing negative effect during immediate successor states in which the collateral constraint is more likely to bind, and to be increased in bust cycle to prevent binding collateral constraint. By contrast to \[\text{Schmitt-Grohé and Uribe} \, 2017\], our results suggest that the optimal capital control tax should be lowered to reach negative values during typical crisis period. Indeed, setting a negative tax during crisis would encourage households to increase their aggregate
absorption up to the feasible maximum level, which would lead to an appreciation of the real exchange rate and relax the constraint.

In addition, we investigate the effectiveness of macroprudential policy by introducing a debt-tax that follows a simple macroprudential rule of the same spirit of Taylor rules. Our results suggest that debt-to-GDP ratio targeting is a second-best policy choice that generates high economic welfare and reduces the volatility of the economy and thus the frequency of crises, while a fixed debt-tax could generate an adverse effect depending on initial conditions.

The remainder of the paper is organized as follows: Section 2 presents the theoretical framework and equilibrium conditions under laissez-faire stance; in Section 3 we introduce three policy instruments set by a social planner that seeks to maximize households’ life-utility taking into account the externality created by the endogenous collateral constraint. Section 3 presents quantitative analysis under endowment shocks and provides conclusions.

2 Theoretical environment

We adopt a prototypical theoretical environment of a representative agent DSGE model of a small open economy (SOE) with two sectors tradable and nontradable goods, similar to those studied by Mendoza [2002] and Bianchi [2011]. The economy is populated by a continuum of identical, infinitely-lived households of measure unity with preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

With $E$ is the expectation operator conditional on information available in period $t$, $c_t$ denotes consumption in period $t$, $U(.)$ denotes an increasing and concave period utility function and $\beta$ is the discount factor. The period utility function $U(.)$ has the constant-relative-risk-aversion (CRRA) form:

$$U(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$
With $\sigma > 0$, we assume that consumption basket $c_t$ is an Armington-type CES aggregator with elasticity of substitution $\zeta$ between tradable $c_T^t$ and nontradable goods $c_N^t$ given by:

$$
c_t = A \left( c_T^t, c_N^t \right) = \left[ a c_T^{1-\frac{1}{\zeta}} + (1-a) c_N^{1-\frac{1}{\zeta}} \right]^{\frac{1}{1-\frac{1}{\zeta}}}
$$

With $a \in (0, 1)$ is the weight of tradables in CES aggregator.

In each period, households receive an exogenous endowments of tradable goods $y_T^t$ and non-tradable goods $y_N^t$. The vector of endowments is supposed to follow a first-order Markov process. As in Bianchi [2011], endowment shocks are the only source of uncertainty in the model.

Household have access to a single internationally-traded bond denominated in terms of tradable goods that pays an interest rate $R_t$ when it is held from period $t$ to period $t+1$. Normalizing the price of tradables to 1 and denoting the price of non tradable goods by $p^n_t$, the households’ sequential budget constraint is:

$$
c_T^t + p^n_t c_N^t + d_t = y_T^t + p^n_t y_N^t + \frac{d_{t+1}}{R_t}
$$

$d_{t+1}$ denotes the amount of debt assumed in the beginning of the period $t$ and maturing in $t+1$. $p^n_t$ denotes the relative price of nontradables in terms of tradables. In the same way as in standard SS models e.g. Mendoza [2002, 2006, 2016], Bianchi [2011], the model counts for a second feature of the international credit market incompleteness as borrowing requires collateral in the form:

$$
d_{t+1} \leq \kappa \left( y_T^t + p^n_t y_N^t \right)
$$

Only a fraction $\kappa$ of the agent’s income is pledgeable as collateral, and as a result, the agent cannot borrow more than this fraction of total income in units of tradables. The pecuniary externality characterizing this model is highlighted by the endogenous character of the borrowing constraint because each individual household takes the real exchange rate
as exogenously determined, nevertheless in equilibrium their collective absorption is a key
determinant of this relative price. The trade balance \((tb_t)\) is given by:

\[
tb_t = y^T_t - c^T_t
\]

The balance-of-payment implies that the real value of current account balance (in terms of
traded goods’ price) equals the net foreign capital outflows. Thus, we have:

\[
ca_t = d_t - \frac{d_{t+1}}{R_t}
\]

### 2.1 Equilibrium dynamics under laissez-faire stance

Households choose a set of processes \(\{c^T_t, c^N_t, c_t, d_{t+1}\}\) to maximize Eq.(1) subject to Eq.(2)
and Eq.(3), given the exogenous processes \(\{y^T_t, y^N_t, R_t\}\) and the initial debt position \(d_0\). The
first-order conditions are:

\[
c_t = A (c^T_t, c^N_t) = \left[ ac^1 - \frac{1}{\xi} + (1 - a) c^1 - \frac{1}{\xi} \right]^{ \frac{1}{1 - \xi}}
\]

\[
c^T_t + p^N_t c^N_t + d_t = y^T_t + p^N_t y^N_t + \frac{d_{t+1}}{R_t}
\]

\[
\lambda_t = U' \left( A \left( c^T_t, c^N_t \right) \right) A' \left( c^T_t, c^N_t \right)
\]

\[
p^N_t = \left( \frac{1 - a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{\frac{1}{\xi}}
\]

\[
\left( \frac{1}{R_t} - \mu_t \right) \lambda_t = \beta E_t \lambda_{t+1}
\]

\[
d_{t+1} \leq \kappa(y^T_t + p^N_t y^N_t), \left[ \kappa \left( y^T_t + p^N_t y^N_t \right) - d_{t+1} \right] \mu_t = 0, \mu_t \geq 0
\]
In equilibrium, the market for nontradenables must clear. That is, \( c_t^N = y_t^N \). Then, a competitive equilibrium is a set of processes \( \{c_t, d_{t+1}, \mu_t\} \) satisfying:

\[
c_t^T + d_t = y_t^T + \frac{d_{t+1}}{R_t} \tag{9}
\]

\[
\lambda_t = a \left( \frac{c_t^T}{c_t} \right)^\frac{1}{\zeta - \sigma} \tag{10}
\]

\[
p_t^n = \left( \frac{1 - a}{a} \right) \left( \frac{c_t^T}{c_t^N} \right)^\frac{1}{\zeta} \tag{11}
\]

\[
(1 - R_t \mu_t) = \beta R_t E_t \frac{\lambda_{t+1}}{\lambda_t} \tag{12}
\]

\[
d_{t+1} \leq \kappa \left( y_t^T + p_t^n y_t^N \right), \left[ \kappa \left( y_t^T + p_t^n y_t^N \right) - d_{t+1} \right] \mu_t = 0, \mu_t \geq 0 \tag{13}
\]

Eq.9 equates total household expenditure to income measured in units of tradable good. Eq.10 equates the current shadow value of wealth to the marginal utility of tradable consumption. Eq.11 equalizes the marginal rate of substitution between tradable and nontradable consumption with the relative price or the real exchange rate. Eq.12 is Euler equation that equalizes the marginal benefit with the marginal cost of increasing consumption of tradables in current period. As households face a borrowing limit given by the market value of the collateral, the marginal utility of increasing debt falls to \( \left( \frac{1}{R_t} - \mu_t \right) \), reflecting a shadow penalty for trying to increase debt when the collateral constraint is binding. Eq.13 is the complementary slackness condition. It highlights an important feature that characterizes the economy during financial crisis (e.g. binding collateral constraint periods) which is reflected by the presence of the relative price \( p_t^n \) in the collateral constraint. This feature describes how financial amplification effect arises from the endogenous credit constraint. Indeed, when the economy is hit by a negative shock, the decrease of households income reduces their credit access, which tightens the budget constraint and forces them to cut back on consumption leading to a depreciation in the real exchange rate and thus a decline of the market value of the collateral. As a result, households’ borrowing ability reduces further and the economy could be involved in a financial crisis generated by the feedback loop between the endogenous
borrowing constraint and the economic activity. The feedback loop dynamic is described by the equilibrium conditions given in Eq.12 and by replacing $p_t^n$ using Eq.10:

$$d_{t+1} \leq \kappa \left( y_t^T + \left( \frac{1-a}{a} \right) (c_t^T)^{\frac{1}{1-\alpha}} y_t^{N(1-\alpha)} \right)$$

(14)

It follows that foreign debt is a strictly increasing function $c_t^T$, then a decrease of consumption of tradables tightens households’ financial conditions leading to a more reduction of $d_{t+1}$ and then of $c_t^T$ via budget constraint 9. In turn as it is described in Figure.1, the new decrease of $c_t^T$ accentuates the depreciation of the real exchange rate which tightens more the credit constraint and decreases more $c_t^T$.

3 Social planner

3.1 Ramsey economy

We presented previously the equilibrium conditions of the unregulated economy specially when households take the relative price as exogenous. We consider now, as in [Bianchi] [2011], a social planner that seeks to maximize the well-being of the representative household. We suppose that it has the ability to choose directly the level of debt and to allow goods market to clear competitively. The social planner is also subject to the collateral constraint and internalize the financial amplification effect arising from the endogenous credit constraint.
Social planner problem is equivalent to the following maximization problem\textsuperscript{1}:

$$\text{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t \ U \left( A \left( c_t^T, y_t^N \right) \right)$$

subject to

$$c_{t+1}^T + d_t = y_t^T + \frac{d_{t+1}}{R_t}$$

$$d_{t+1} \leq \kappa \left( y_t^T + \left( \frac{1-a}{a} \right) \left( c_t^T \right)^{\frac{1}{\zeta}} y_t^{N1-\frac{1}{\zeta}} \right)$$

The optimal conditions of Ramsey problem are summarized below:

$$\lambda_t = a \left( \frac{c_t^T}{c_t} \right)^{\frac{1}{\zeta}} c_t^{-\sigma} \quad (15)$$

$$\lambda_t = \lambda_t^{SP} \left( 1 - \mu_t^{SP} \psi_t \right) \quad (16)$$

$$\left( \frac{1}{R_t} - \mu_t^{SP} \right) \lambda_t^{SP} = \beta E_t \lambda_{t+1}^{SP} \quad (17)$$

$$\left[ \kappa \left( y_t^T + \left( \frac{1-a}{a} \right) \left( c_t^T \right)^{\frac{1}{\zeta}} y_t^{N1-\frac{1}{\zeta}} \right) - d_{t+1} \right] \mu_t^{SP} = 0; \mu_t^{SP} \geq 0 \quad (18)$$

With $\lambda_t$ corresponds to the Lagrangian multiplier of the regulated economy with an appropriate policy instrument and SP denotes Lagrangian multiplier related to the social planner optimization and $\psi_t$ represents the externality term given by:

$$\psi_t = \kappa \left( \frac{1-a}{a} \right) \frac{1}{\zeta} \left( \frac{c_t^T}{y_t^N} \right)^{\frac{1}{\zeta} - 1} \quad (19)$$

The key difference between the unregulated economy and Ramsey equilibrium conditions is described by Eq.\textsuperscript{17}. Indeed, the current shadow value of wealth of the social planner is higher than that of the decentralized economy when the collateral constraint is binding $\mu_t^{SP} \neq 0$. In such state, an increase in tradable consumption increases the price of nontradables and

\cite{Schmitt-Grohé2017} show that the set of equations can be reduced to only two equations 9 and 14.
relaxes the credit constraint of all households by $\psi_t^\prime$, which has a shadow value of $\mu_t^{SP}$. When the collateral constraint is not binding in current and all immediate following states, both unregulated economy and Ramsey allocations coincide. We introduce a policy instrument that helps to achieve Ramsey allocation. As in the existing literature, we study optimal capital control tax on external borrowing, which is the variable most directly affected by the pecuniary externality. The aim of this policy is to induce the representative household to internalize the effect of the aggregate absorption on the relative price of nontradables and therefore on the value of the collateral. Following Bianchi [2011], let $\tau_t$ the proportional tax on debt imposed in period $t$. If $\tau_t$ is positive, it represents a proper capital control tax, whereas if it is negative it has the interpretation of a borrowing subsidy. The revenue from capital control taxes is given by $\frac{\tau_t}{R_t} d_t + 1$. We assume that government consumes no goods and that it rebates all revenues from capital controls to the public in the form of lump-sum transfers (lump-sum taxes if $\tau_t < 0$), denoted $\ell_t$. The budget constraint of the representative household becomes:

$$c_t^T + d_t = y_t^T + (1 - \tau_t) \frac{d_{t+1}}{R_t} + \ell_t$$  

(20)

The new competitive equilibrium conditions of private agents is given by:

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{R_t}$$  

(21)

$$\lambda_t = a \left( \frac{c_t}{c_t^T} \right) \left( \frac{c_t}{\frac{c_t^T}{y_t^N}} \right)^{\sigma - 1}$$  

(22)

$$p^n_t = \left( \frac{1 - a}{a} \right) \left( \frac{c_t^T}{y_t^N} \right)^{\frac{1}{\xi}}$$  

(23)

$$\left( \frac{1 - \tau_t}{R_t} - \mu_t \right) \lambda_t = \beta E_t \lambda_{t+1}$$  

(24)

$$d_{t+1} \leq \kappa \left( y_t^T + p^n_t y_t^N \right), \left[ \kappa \left( y_t^T + p^n_t y_t^N \right) - d_{t+1} \right] \mu_t = 0, \mu_t \geq 0$$  

(25)

Given a policy process $\tau_t$, exogenous endowments $y_t^T$ and $y_t^N$, and the initial condition $d_0$. The social planner sets capital control taxes to maximize households lifetime utility subject
to the restriction that the optimal allocation be supportable as a competitive equilibrium.

When $\tau_t > 0$, the interest rate perceived by households becomes $\frac{R_t}{1-\tau_t}$ which is higher than $R_t$. The aim of capital controls in this case is to discourage households from borrowing, as one unit of debt payable in $t+1$ increases consumption of tradables by $\frac{1-\tau_t}{R_t}$ units in period $t$ and utility by $\frac{1-\tau_t}{R_t} \lambda_t$ instead of $\frac{1}{R_t}$ and $\frac{\lambda_t}{R_t}$ respectively. When $\tau_t < 0$, households receive more incentive to borrow as they perceive a lower interest rate $\frac{R_t}{1-\tau_t} < R_t$.

3.1.1 The cyclical behavior of the optimal policy tax

When the borrowing constraint is binding, the optimal capital tax is undetermined as stressed by Schmitt-Grohé and Uribe [2017]. Any linear combination of $(\mu_t, \tau_t)$ that makes Euler equation 22 hold is a solution. It means that during financial crisis the implementation of the optimal tax is ambiguous and we can not conclude how the social planner should set capital control tax to restore the constrained efficient allocation.

In what follows we derive a solution of the optimal tax that helps to explain the pro-cyclicality behavior stressed by Schmitt-Grohé and Uribe [2017] in boom-bust cycles and accordingly we emphasize tax behavior during crisis periods.

Indeed, we can show that $\left(\mu_t = \mu_t^{SP}, \tau_t = \tau_t^{\mu=\mu_t^{SP}}\right)$ is a solution of $(\mu_t, \tau_t)$ that delivers the same allocation in the regulated economy (described by Eq.21 to Eq.25) as in Ramsey economy (described by Eq.15 to Eq.18).

First we can show that when $\mu_t = 0$ (or $\mu_t \neq 0$) it implies that $\mu_t^{SP} = 0$ (or $\mu_t^{SP} \neq 0$) and vice-versa. In fact, Ramsey planner allows goods market to clear competitively and sets the tax rate $\tau_t$ to restore the constrained efficient allocation by imposing a debt choice, then debt levels in the regulated economy and Ramsey economy should be the same. Consequently, according to households budget constraint, consumptions of tradable goods in the regulated and Ramsey economies are the same too.

We conclude that when the collateral constraint is not binding we then have $\left[\kappa (y_t^T + p_t^ny_t^N) - d_{t+1}\right] > 0$ in regulated and Ramsey economies which implies $\mu_t = \mu_t^{SP} = 0$. When the collateral con-
straint is binding, \( d_{t+1} \) is given by the credit limit e.g. \( \kappa \left( y_t^T + p_t^N y_t^N \right) \) in both regulated economy and Ramsey economy so it implies that \( \mu_t \neq 0, \mu_t^{SP} \neq 0 \). In both cases, taking \( \mu_t = \mu_t^{SP} \) keeps the slackness condition satisfied in regulated economy (Eq. 25). We then deduce tax rate as follows:

\[
\tau_t = \tau_t^{\mu_t = \mu_t^{SP}} = \beta R_t \left[ \frac{E_t \left( \lambda_{t+1}^{SP} \right)}{\lambda_t^{SP}} - \frac{E_t \left( \lambda_{t+1} \right)}{\lambda_t} \right]
\]  

(26)

Setting \( \tau_t = \tau_t^{\mu_t = \mu_t^{SP}} \) keeps Euler equation (Eq. 22) satisfied while all other equilibrium conditions in regulated economy are still satisfied also.

Suppose that it exists another solution \( \mu'_t \) and \( \tau'_t \) that satisfies all equilibrium conditions of regulated economy. Using Euler equation (Eq. 22) under the two solutions \( \{ (\mu_t', \tau_t'), (\mu_t^{SP}, \tau_t^{\mu_t = \mu_t^{SP}}) \} \) we deduce the following relationship:

\[
\tau'_t - \tau_t^{\mu_t = \mu_t^{SP}} = R_t \left( \mu_t^{SP} - \mu'_t \right)
\]

When the collateral constraint is not binding \( \mu_t^{SP} = \mu'_t = 0 \) which implies that \( \tau_t' = \tau_t^{\mu_t = \mu_t^{SP}} \). We then find a unique solution for \( (\mu_t, \tau_t) \) given by \( (\mu_t^{SP}, \tau_t^{\mu_t = \mu_t^{SP}}) \) in states in which the collateral constraint is not binding. When the collateral constraint is binding, setting \( \tau_t' = \tau_t^{\mu_t = \mu_t^{SP}} \) ensures that Ramsey allocation and regulated economy allocation are the same as it implies that households make the same inter-temporal choice as the social planner.

The optimal tax rate expression given in Eq. (26) provides a relevant explanation of the procyclicalit y behavior of the optimal tax. In fact, it describes the logic behind the intervention of the social planner who acts only when the borrowing constraint is binding or/and the probability that it will bind in immediate successor state becomes positive. Eq. (26) can be expressed as follows:

\[
\tau_t^{\mu_t = \mu_t^{SP}} = \beta R_t \left[ \frac{1}{1 - \frac{E_t \left( \mu_t^{SP} \psi_{t+1} \right)}{1 - \mu_t^{SP} \psi_t}} - \frac{1}{1 - \mu_t^{SP} \psi_t} \right]
\]
As emphasized by Schmitt-Grohé and Uribe [2017], when the collateral constraint is slack in current state and all immediate following states \( \mu^{SP}_{t+1} = \mu^{SP}_t = 0 \), the optimal capital control tax is zero. In this case, households marginal cost/benefit ratio of holding an extra unit of debt coincides with that of Ramsey planner, and no intervention is needed. If the current state the collateral constraint is slack and there are immediate successor states in which it will bind, Eq. (26) becomes:

\[
\tau^\mu=\mu^{SP}_t = \beta R_t \frac{E_t (\lambda^{SP}_{t+1})}{\lambda^{SP}_t} E_t (\mu^{SP}_{t+1}\psi_{t+1})
\]

In such state, the tax rate is positive as the marginal cost of holding an extra unit of debt for the social planner is higher than that of the representative household \( \lambda^{SP}_{t+1} > \lambda_{t+1} \). In this case, Ramsey planner set positive tax rate to avoid large amount of debt holding in current period that would induce a large decline of \( c^T_{t+1} \) in the following state when the collateral constraint will bind. This suggests that during a boom cycle, the probability that the economy hits the credit limit in current state becomes weak and then the optimal tax rate is set to zero. During bust cycle, when the credit limit is more likely to bind, the social planner increases optimal tax rate to avoid a large decline in consumption during the following state as the probability that \( \mu^{SP}_{t+1} \neq 0 \) becomes positive.

During crisis periods, specially, when the collateral constraint is binding in current state and it is not in the immediate following state, Eq. (26) suggests a negative tax rate:

\[
\tau^\mu=\mu^{SP}_t = -\beta R_t \frac{E_t (\lambda^{SP}_{t+1})}{\lambda^{SP}_t} \mu^{SP}_t\psi_t
\]

In such state, private agents undervalue wealth \( \lambda_t < \lambda^{SP}_t \). Indeed, households’ marginal benefit from an increase of tradable consumption includes only the direct increase in utility \( \lambda_t \) and not the indirect increase in utility \( \mu^{SP}_t\psi_t \) that arises from relaxing the collateral constraint (via the increase of the relative price and that of the collateral value). Ramsey planner set negative tax to encourage households to increase their debt holding in current
period by internalizing the collateral constraint effect on the current shadow value of wealth \( \lambda_t \). However, during crisis periods, when private agents borrow up to the credit limit, it seems that the only way for the social planner to affect current consumption of tradables depends on ex-ante policy that has an effect on past foreign debt, given traded and non traded endowment that are exogenously determined in the economy, as it is indicated by household’s budget constraint:

\[
c^T_t = y^T_t + \frac{\kappa (1 - \tau_t)}{R_t} \left( y^T_t + \left( \frac{1 - a}{a} \right) \left( \frac{c^T_t}{y^N_t} \right)^{1/\zeta} y^N_t \right) - d_t + \ell_t
\]  

(27)

This relationship shows prima-facie that the larger is past foreign debt the lower would be the current consumption which calls for a positive optimal capital tax in period \( t - 1 \) in line with the result derived from Eq. (21). It suggests that a high amount of external debt in periods when the borrowing constraint is not binding would increase the risk of financial crisis specifically when the economy is hit by an unexpected negative shock. It shows also that the higher is current consumption of tradables, the higher would be household borrowing ability and the relative price, and the lower would be the effect of the externality. In such state, the elasticity of current consumption of tradables to past external debt highlights the relevance of optimal tax even when the borrowing constraint is binding. Consider the derivative of \( c^T_t \) with respect to \( d_t \) taking \( \ell_t \) as exogenous, we can obtain the following relationship:

\[
\frac{\partial c^T_t}{\partial d_t} = \left( \frac{(1 - \tau_t)}{R_t} \psi_t - 1 \right)^{-1}
\]  

(28)

As Eq. (26) suggests a negative value of \( \tau_t \) we then have \( (1 - \tau_t) > 1 \). Accordingly, we can show that:

\[
\left( \frac{(1 - \tau_t)}{R_t} \psi_t - 1 \right)^{-1} < \left( \frac{\psi_t}{R_t} - 1 \right)^{-1}
\]

In fact, setting a negative tax rate during crisis period would encourage household to increase their debt holding to coincide with the maximum feasible level of that of the social planner.
and thus to increase their consumption of tradable as much as possible. Consequently, the impact of past foreign debt on current consumption would be reduced.

In order to compare our results with those of Schmitt-Grohé and Uribe [2017] we define $\tau^{SU}_{t}$ as defined by authors:

$$\tau^{SU}_{t} = 1 - E_{t} \left( \frac{\lambda_{t+1}^{SP}}{\lambda_{t+1}^{SP}} \right)$$  \hspace{1cm} (29)

### 3.2 Simple macro-prudential policy rule

In practice, the implementation of the optimal policy such as $\tau_{t}$ is not free from difficulties and challenges. We investigate whether a simple macro-prudential policy rule of the same esprit of Taylor rule could help to achieve a better-off welfare. Let $\tau^{MP}_{t}$ a debt-tax on foreign borrowing $d_{t+1}$ that varies with the aggregate debt level according to the following rule:

$$\tau^{MP}_{t} = \left( \frac{d_{t+1}}{R y_{t}} \right) \psi - 1$$ \hspace{1cm} (30)

With $\frac{d}{R R_{y}}$ the debt-to-GDP ratio target and $\psi$ is the elasticity of the debt-tax with respect to debt-to-GDP ratio variations. The choice of the form of this target, rather than targeting debt gap, is two-fold. First, the change in the credit-to-GDP ratio is a better indicator than the credit gap as the latter misses too many crises as emphasized by Mitra et al. [2011]. Second, when the economy is hit by a positive productivity shock leading to an increase of current income which would relax the credit constraint and ease households’ financial conditions, foreign debt holding would increase naturally without endangering financial stability. Increasing debt-tax in this case will be welfare reducing.

In addition, the implementation of such policy is equivalent to the activation of Counter-Cyclical-Buffers which is based on the deviation of credit-to-GDP ratio from its trend, e.i. the equilibrium credit-to-GDP ratio. As recommended by IMF [2013], the CCB should be imposed if the credit-to-GDP ratio exceeds its trend value and consists on increasing regulatory capital requirements. Since capital requirements are linked to the amount of credit, banks
may cut lending to satisfy the requirements. Consequently, the cost of borrowing increases which is translated in our model by a higher debt-tax.

The cyclicality behavior of $\tau_t^{MP}$ depends on the cyclicality behavior of debt-to-GDP ratio. Intuitively, during boom cycle debt-to-GDP ratio decreases as GDP increases and thus the debt-tax decreases. By contrast, during bust cycle the debt-tax would be pushed up due to the decline of GDP. We explore numerically cyclical behaviors of policy instruments in section 5.

3.3 Debt-tax to address households’ impatience

The theoretical environment of such a Sudden Stop model features two important characteristics. The presence of the endogenous collateral constraint creates a feedback loop between the economic activity and households’ financial conditions as described previously. Second, a lower discount factor that refers to impatient private agents generates a permanent desire of households to front-load consumption by accumulating large external debt amount. The first one is addressed by the social planner presented in section 3 as in Bianchi [2011], Schmitt-Grohé and Uribe [2017], who activates optimal policy tax to make households internalizing the effect of the collateral constraint when the probability of hitting the credit limit becomes positive in current and/or immediate successor states, while the second characteristic is largely ignored. Indeed, as private agents are impatient, the presence of collateral constraint describes a self insurance of both households and lenders against adverse shocks under which the economy would be unable to reimburse its debt. Accordingly, addressing households’ impatience could reduce the frequency of crisis and affect life-time households’ utility. In order to investigate this fact, we introduce a debt-tax that makes private agents as much patient as the rest of the world by setting:

$$\tau_t^{IM} = 1 - \beta R_t \quad (31)$$
Consequently, Euler equation under this debt-tax becomes:

$$E_t \frac{\lambda_{t+1}}{\lambda_t} = \left(1 - \frac{\mu_t}{\beta}\right)$$

When the collateral constraint is not binding, the presence of $\tau_t^{IM}$ equates the cost and benefit of an extra unit of debt and enforces households to carry a lower amount of debt than that in the unregulated economy. We can show that the new Euler equation implies:

$$\ln \left[ \frac{c_{t+1}^T}{c_t^T} \right] = (1 - \zeta \sigma) \ln \left[ \frac{c_{t+1}}{c_t} \right]$$

Accordingly, the correlation sign between when $c_{t+1}^T$ and $c_t$ is given by parameters values $\zeta$ and $\sigma$. When the elasticity of substitution between tradables and nontradables equals the inverse of risk aversion; $\zeta = \frac{1}{\sigma}$, consumption of tradable goods becomes constant $c_t^T = c_{t+1}^T$ and depends strongly on the initial value of foreign debt. When $\zeta > \frac{1}{\sigma}$, $c_t^T$ and $c_t$ become negatively correlated and thus $c_t^T$ and $y_t^N$ are too, while a value of $\zeta$ under $\frac{1}{\sigma}$ implies a positive correlation $c_t^T$ and $y_t^N$. However, implementing a such tax policy could have an adverse effect on welfare and resource allocations depending on parameter values and initial debt position. In fact, avoiding households from accumulating large amount of debt could involve the economy into a deep recession via reducing aggregate demand and thus aggregate production, which would lead to decrease the collateral value and then households borrowing abilities.

When the collateral constraint is binding, the shadow value $\frac{\mu_t}{\beta}$ of increasing debt holdings is higher than that of Ramsey economy and regulated economy with debt-to-GDP ratio tax. Note that $\tau_t^{IM}$ is negatively correlated to $R_t$ and it is unvaried when the later is too.

Table 1 recapitulates the policy instruments taken into consideration in our analysis.
Table 1: Policies to correct the pecuniary externality of the endogenous collateral constraint

<table>
<thead>
<tr>
<th>Policy rule $\tau_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ramsey economy</strong></td>
</tr>
<tr>
<td>Baseline optimal capital control tax $\beta R_t \left[ \frac{E_t(\lambda_{t+1}^{SP})}{\lambda_{t+1}^{SP}} - \frac{E_t(\lambda_{t+1})}{\lambda_t} \right]$</td>
</tr>
<tr>
<td>Schmitt-Grohé and Uribe [2017] optimal capital tax $1 - E_t \left( \frac{\lambda_{t+1}^{SP}}{\lambda_{t+1}^{SP}} \right)$</td>
</tr>
<tr>
<td>Simple macroprudential policy $(\frac{d_{t+1}}{R_{t+1}} \frac{\ddot{a}}{R_{t+1}})^\psi - 1$</td>
</tr>
<tr>
<td>Capital control tax to address households’ impatience $1 - \beta R_t$</td>
</tr>
</tbody>
</table>

4 Welfare analysis

Following Lucas and Lucas [1987], the welfare cost of correcting the externality in the present model can be measured by a compensation parameter $\gamma$ that equals the expected lifetime utility of the representative household in the unregulated economy and that in the regulated economy under different policy instruments as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left((1 + \gamma (b, y)) c_{t}^{de}\right) = E_0 \sum_{t=0}^{\infty} \beta^t U \left(c_{t}^{re}\right)$$

With $c_{t}^{de}$ and $c_{t}^{re}$ represent the historical optimal choices of consumption in the unregulated and in regulated economies respectively given the initial state of the economy $(b, y)$. $\gamma$ represents the quantitative measure of the welfare gain in moving from the unregulated economy to the regulated economy, or equivalently, the welfare cost of not correcting the externality. Alternatively, equation 32 can also be interpreted as showing the compensation, in units of percentage of consumption, given by the parameter $\gamma$, that would make the representative household indifferent between remaining in the unregulated economy and correcting the externality. Because of the homotheticity of the utility function, the welfare
gain \( \gamma \) a state \((b,y)\) is given by:

\[
(1 + \gamma (b, y))^{(1-\sigma)} V^{de} (b, y) = V^{re} (b, y)
\]

With \( V^{de} \) and \( V^{re} \) value functions related to the unregulated and regulated economies respectively.

5 Quantitative analysis

In order to evaluate quantitatively policy implications on resource allocations and welfare, we solve the competitive equilibrium and the social planner’s problems numerically using global non-linear methods described in Appendix A. A period in the model represents a year. We adopt exactly the same calibration as in Bianchi [2011] to evaluate the quantitative implications of the cyclicality behavior of policy instruments. The baseline calibration in Bianchi [2011] exploits data from Argentina to model endowment shocks which are assumed to follow a bi-variate auto-regressive \( AR(1) \) process:

\[
\begin{bmatrix}
    \ln y^T_t \\
    \ln y^N_t
\end{bmatrix} =
\begin{bmatrix}
    0.901 & -0.453 \\
    0.495 & 0.225
\end{bmatrix}
\begin{bmatrix}
    \ln y^T_{t-1} \\
    \ln y^N_{t-1}
\end{bmatrix} + \epsilon_t; \epsilon_t \sim N \left( 0, \begin{bmatrix}
    0.00219 & 0.00162 \\
    0.00162 & 0.00167
\end{bmatrix} \right)
\]

\( \epsilon_t \) is assumed to be i.i.d. This process implies unconditional standard deviations of 6% and serial correlations of about 0.53 and 0.61 for traded and non-traded endowments respectively, and a contemporaneous correlation of 0.8. The three remaining parameters \( \{\beta, a, \kappa\} \) are calibrated to match Argentina historical data. The parameter \( a \) represents the tradable share in the CES aggregator and it is set at 0.31 which corresponds to a share of tradable sector of about 32%. The discount factor is set at 0.91 so that the average net foreign asset position-to-GDP ratio in the model equals its historical average in Argentina; −29 percent of GDP, which makes the domestic agents impatient regarding world market and discount
the future at a higher rate than world interest rate 0.04 assumed to be constant. We set $\kappa$ at 0.32 $R_t$ the same value used in Bianchi [2011], Schmitt-Grohé and Uribe [2017] and that delivers a frequency of crisis of 5.5%. Other parameters are calibrated following DSGE-SOE.

Table 2: Calibration of the Economy following Bianchi [2011]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.3328</td>
<td>Parameter of collateral constraint</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.91</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$R_t$</td>
<td>1.04</td>
<td>World interest rate</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Inverse of intertemporal elast. of subst.</td>
</tr>
<tr>
<td>$a$</td>
<td>0.31</td>
<td>Weight on tradables in CES aggregator</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1</td>
<td>The baseline calibration of the elasticity of the debt-tax with respect to debt-to-GDP ratio change</td>
</tr>
</tbody>
</table>

$\frac{R}{R^*}$ 23.5% Debt-to-GDP ratio equilibrium
$y^N$ 1 Steady-state nontradable output
$y^T$ 1 Steady-state tradable output
$n_{y^T}$ 4 Number of grid points for $y^T_t$
$n_{y^N}$ 4 Number of grid points for $y^N_t$
$n_d$ 800 Number of grid points for $d_t$, equally spaced

$[lny^T_t, y^T_t]$ $[-0.1093, 0.1093]$ Range for traded output
$[lny^N_t, y^N_t]$ $[-0.1328, 0.1328]$ Range for nontraded output
$[d_t, \frac{a}{R_t}]$ $[0.4, 1.02]$ Range for debt

literature; the inverse of intertemporal elasticity of substitution $\sigma$ is set at 2, a standard value in the existing related literature. The intratemporal elasticity of substitution $\zeta$ is set at 0.83 as in Bianchi [2011]. We set $\psi$ at 1 as a baseline calibration and we conduct sensitivity analysis to evaluate the economic allocation outcomes under different values. We derive an endogenous estimation for debt-to-GDP ratio equilibrium.
5.0.1 Policy outcomes

We solve equilibrium conditions for both unregulated, Ramsey and regulated economies. We use the optimal set of processes derived from social planner solution to solve the regulated economies with $\tau^\mu = \mu^{SP}$ and $\tau^{SU}$. We solve also using the same methods the regulated economies with $\tau^{MP}$ and $\tau^{IM}$. We then simulate the resulting set of processes for one million years.

Figure 2 displays the ergodic distribution of bond holdings for the unregulated, Ramsey and regulated economies using different debt-taxes e.i. $\tau^\mu = \mu^{SP}$, $\tau^{SU}$, $\tau^{MP}$ and $\tau^{IM}$. It shows obviously a different density distributions of bond holdings in the regulated economy with $\tau^{MP}$ and $\tau^{IM}$, that are centered around 80% of traded output with high probabilities associated to a moderate amount of foreign debt levels. Indeed, under $\tau^{MP}$, the highest probability of 13.4% is associated with a debt level of about 81% in terms of traded output, while the unregulated economy has a probability of 12.6% to accumulate a foreign debt level of about 95, 4%, and the higher probability of the social planner (17.7%) is associated with a debt level of about 93.3%. Under $\tau^{IM}$, the presence of the permanent fixed tax enforces households to moderate their debt holdings which is translated into a moderate aggregate debt holding; the larger amount of debt does not exceed 96% and the highest probability of 5.3% is associated with 83%.

The assessment of median debt levels in the resulting simulated data highlights that the amount of debt carried in the economy with $\tau^{IM}$ is 0.793 relatively similar to that of the regulated economy with $\tau^{MP}$ (0.81) significantly small than 0.94 of Ramsey economy and 0.95 of the unregulated economy.
Figure 2: The ergodic distribution of foreign debt holdings of unregulated economy and of the regulated economy with different policy instruments.

Figure 2 shows also that the two capital control taxes, $\tau^\mu_t = \mu^{SP}_t$ and $\tau^SU_t$, deliver the same foreign debt density distribution as Ramsey economy and thus both help to achieve the
social planner constrained efficient allocation. In addition, the average of debt-to-output ratios (Table 3) are similar, 28.9% while this ratio is slightly higher for unregulated economy 29.2%. By contrast, the average of debt-to-output ratio in the regulated economy with $\tau_t^{MP}$ and $\tau_t^{IM}$ are only about 24.8% and 24.5% respectively, largely smaller than that of the unregulated economy and that of the social planner. The reduced exposition to higher debt holdings of the regulated economy with $\tau_t^{MP}$ and $\tau_t^{IM}$ helps to shift the economy away from binding constraint region and thus avoiding crisis, while the social planner crisis frequency is once every 16 years and every 12 years in the unregulated economy.

Table 3: Debt-to-output ratios, Frequency of Crises and Welfare

<table>
<thead>
<tr>
<th></th>
<th>Debt-to-Output Ratio</th>
<th>Frequency of Crises</th>
<th>Welfare gain in percent of permanent consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unregulated economy</td>
<td>29.2%</td>
<td>12 years</td>
<td>-</td>
</tr>
<tr>
<td>Ramsey economy</td>
<td>28.9%</td>
<td>16 years</td>
<td>0.046</td>
</tr>
<tr>
<td>Regulated economy</td>
<td>28.9%</td>
<td>16 years</td>
<td>0.046</td>
</tr>
<tr>
<td>with $\tau_t^{MP}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regulated economy</td>
<td>28.9%</td>
<td>16 years</td>
<td>0.046</td>
</tr>
<tr>
<td>with $\tau_t^{SU}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regulated economy</td>
<td>24.8%</td>
<td>No crisis</td>
<td>0.33</td>
</tr>
<tr>
<td>with $\tau_t^{MP}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regulated economy</td>
<td>24.5%</td>
<td>No crisis</td>
<td>0.196</td>
</tr>
<tr>
<td>with $\tau_t^{IM}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The absence of crises in the regulated economy with $\tau_t^{MP}$ and $\tau_t^{IM}$ is translated into higher welfare gain of about 0.33 and 0.19 percent of permanent consumption respectively. However, the welfare gain of implementing optimal capital tax is only around 0.046 per-
In terms of macroeconomic volatility, the regulated economy with $\tau_t^{IM}$ displays a very low macroeconomic volatility. As $\tau_t^{IM}$ shifts households’ consumption preferences toward nontraded goods and thus it creates a negative correlation between consumption of tradables $c_t^T$ and nontraded endowment $y_t^N$ of about $-0.19$. As a result, the opposite movement in nontraded endowments and households’ consumption of tradables stabilize relatively aggregate consumption that is characterized by a volatility of about 3.74 against 5.8 in the unregulated economy, 5.8 in Ramsey economy and 5.06 in the regulated economy with simple macroprudential policy. In addition, the resulting correlation between $c_t^T$ and traded endowment $y_t^T$ is only about 0.32 instead of 0.68 in the unregulated economy, 0.57 and 0.80 in Ramsey economy and the regulated economy with $\tau_t^{MP}$ respectively. Consequently, a significant change of traded and nontraded endowments is not accompanied by significant consumption of tradables and foreign debt variations which reduce the amplification effect of the borrowing collateral constraint and lead to less aggregate output volatility (Table 4).

Table 4: Macroeconomic volatility

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation (%)</th>
<th>Correlation with GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unregulated economy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate output</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>Aggregate consumption</td>
<td>5.8</td>
<td>0.8331</td>
</tr>
<tr>
<td>Consumption of tradables</td>
<td>8.8</td>
<td>0.99</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>9.96</td>
<td>0.8</td>
</tr>
<tr>
<td>Current account</td>
<td>6.3</td>
<td>-0.35</td>
</tr>
<tr>
<td>Trade balance</td>
<td>6.3</td>
<td>-0.67</td>
</tr>
<tr>
<td><strong>Ramsey economy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate output</td>
<td>6.75</td>
<td>-</td>
</tr>
<tr>
<td>Aggregate consumption</td>
<td>5.4</td>
<td>0.828</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Consumption of tradables</td>
<td>7.03</td>
<td>0.99</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>7.5</td>
<td>0.63</td>
</tr>
<tr>
<td>Current account</td>
<td>4.2</td>
<td>-0.37</td>
</tr>
<tr>
<td>Trade balance</td>
<td>4.2</td>
<td>-0.46</td>
</tr>
<tr>
<td><strong>Regulated economy with</strong> $\tau_t^{\mu t=\mu^S_P}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate output</td>
<td>6.75</td>
<td>-</td>
</tr>
<tr>
<td>Aggregate consumption</td>
<td>5.4</td>
<td>0.828</td>
</tr>
<tr>
<td>Consumption of tradables</td>
<td>7.03</td>
<td>0.99</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>7.5</td>
<td>0.63</td>
</tr>
<tr>
<td>Current account</td>
<td>4.2</td>
<td>-0.37</td>
</tr>
<tr>
<td>Trade balance</td>
<td>4.2</td>
<td>-0.46</td>
</tr>
<tr>
<td><strong>Regulated economy with</strong> $\tau_t^{SU}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate output</td>
<td>6.75</td>
<td>-</td>
</tr>
<tr>
<td>Aggregate consumption</td>
<td>5.4</td>
<td>0.828</td>
</tr>
<tr>
<td>Consumption of tradables</td>
<td>7.03</td>
<td>0.99</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>7.5</td>
<td>0.63</td>
</tr>
<tr>
<td>Current account</td>
<td>4.2</td>
<td>-0.37</td>
</tr>
<tr>
<td>Trade balance</td>
<td>4.2</td>
<td>-0.46</td>
</tr>
<tr>
<td><strong>Regulated economy with</strong> $\tau_t^{MP}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate output</td>
<td>4.9</td>
<td>-</td>
</tr>
<tr>
<td>Aggregate consumption</td>
<td>5.06</td>
<td>0.87</td>
</tr>
<tr>
<td>Consumption of tradables</td>
<td>4.65</td>
<td>0.9886</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>3.9</td>
<td>0.08</td>
</tr>
<tr>
<td>Current account</td>
<td>2.6</td>
<td>0.21</td>
</tr>
<tr>
<td>Trade balance</td>
<td>2.6</td>
<td>0.36</td>
</tr>
<tr>
<td><strong>Regulated economy with</strong> $\tau_t^{IM}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate output</td>
<td>2.2</td>
<td>-</td>
</tr>
</tbody>
</table>
### 5.0.2 Boom-bust cycles under endogenous endowments shocks

Following Schmitt-Grohé and Uribe [2017], we define a boom-bust episode as a situation in which tradable output starts above trend and is below trend three years later. To characterize the typical boom-bust cycle, we extract all windows containing a boom-bust cycle from the simulated data. This yields 12 non-overlapping boom-bust episodes every century equivalent to 254,127 windows. We refer to the average dynamics of an economy over all boom-bust episodes as the typical boom-bust cycle. We take the median rather than the average for debt-tax rates $\tau_t^{\mu_t=\mu_t^{SP}}$, $\tau_t^{SU}$ and $\tau_t^{MP}$ as their distributions are negatively skewed.

Figure 3 displays with solid line the cyclical behavior of the unregulated economy, with dashed lines the Ramsey economy (SP) and the regulated economy using policy instruments. The exogenous boom-bust cycle in traded and non traded outputs produces endogenous boom-bust cycles in total output ($y_t = y^T_t + p_t y^N_t$), consumption, the relative price of nontradables, and the foreign debt holdings.

In one hand, Figure 3 points prominently the same cyclical dynamics for Ramsey economy and the regulated economy using optimal capital taxes; $\tau_t^{\mu_t=\mu_t^{SP}}$ and $\tau_t^{SU}$. The social planner restores the constrained efficient allocation by avoiding households to accumulate relatively high foreign debt levels in comparison with that of the unregulated economy.
Figure 3: The Typical Boom-Bust Cycle in the Endowment-Shock Economy
However, the constrained efficient allocation achieved by the social planner is characterized by an economic contraction as much large as the contraction in the unregulated economy. Indeed, the social planner set positive capital tax only when the constraint is binding or about to bind. When the collateral constraint is not binding in current state and all immediate successor states, unregulated economy allocation and that of Ramsey economy coincide, thus, the social planner intervention aims only to avoid binding collateral constraint and not to remedy the desire of households to front-load consumption. Consequently, foreign debt in both Ramsey economy and unregulated economy do not respond significantly to exogenous dynamics.

In other hand, Figure 3 highlights different cyclical dynamics of the regulated economy with $\tau^M$ and $\tau^I$. Indeed, the average of debt-tax during the typical boom-bust cycle is 5.33% of $\tau^M$ which prevent households to carry a large amount of debt, the resulting debt-to-GDP ratio is 24.7% largely smaller than 28.8% of Ramsey economy. This fact leads to higher consumption of tradable and thus higher relative price level and collateral value. As a result, households have the ability to smooth their consumption during bust cycle without involving the economy in a financial crisis. In addition, as the mean of consumption of tradable is enhanced, trade balance contraction is more large in comparison with the unregulated and Ramsey economies. The cyclical dynamics of the regulated economy with $\tau^I$ shows less amplification effect of the collateral constraint, the presence of the fixed debt-tax enforces households to carry a moderate debt level and to stabilize consumption of tradable around 96% in terms of traded output. Consequently, the exogenous boom-bust cycle in traded and non traded outputs drives endogenous boom-bust cycle only in the relative price and trade balance that follows the same dynamics behavior as nontraded and traded outputs. In addition, foreign debt becomes counter-cyclical, it decreases during a boom cycle and increases during bust cycle which helps households to smooth their consumption of tradables during bad times. Figure 3 shows also the pro-cyclicality of the three time-varying policy instruments during the typical boom-bust cycle. In fact, policy instruments display
Table 5: Cyclicality of capital control policy instruments

<table>
<thead>
<tr>
<th>Regulated economy with</th>
<th>unconditional median</th>
<th>corr(.,yt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1%</td>
<td>0.27\textsuperscript{a} and -0.85\textsuperscript{b}</td>
<td></td>
</tr>
<tr>
<td>1.2%</td>
<td>-0.85</td>
<td></td>
</tr>
<tr>
<td>5.1%</td>
<td>-0.6</td>
<td></td>
</tr>
<tr>
<td>5.4%</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a} including all simulated data
\textsuperscript{b} excluding crisis periods

a negative correlation with output (table 5) except for the policy instrument \(\tau_t^{\mu_t=\mu_t^{SP}}\) which is positively correlated to output due to its negative values during crisis periods. Excluding crisis periods, \(\tau_t^{\mu_t=\mu_t^{SP}}\) displays the same correlation to GDP as \(\tau_t^{SP}\).

5.0.3 Typical crisis under exogenous endowments shocks

The cyclical behaviors of policy instruments indicates that they are pro-cyclical during a typical boom-bust cycle. In this section we explore the behavior of policy instruments during a typical crisis period. To this end, we characterize the behavior of the unregulated, Ramsey and regulated economies around episodes in which the collateral constraint binds in the unregulated economy. As previously, we extract all eleven-year windows centered around a period in which the collateral constraint binds. This yields 84,221 windows. Thus, the unregulated economy suffers on average one financial crisis every 12 years (table 3) while the Ramsey economy experiences crisis once every 16 years. The regulated economy using the two versions of optimal capital tax experiences the same crisis frequency, while the regulated economies using \(\tau_t^{MP}\) and \(\tau_t^{IM}\) do not experience at all any binding constraint during the typical crisis period.

Figure 4 displays with solid lines the dynamics of the unregulated economy during crisis period. In the figure, the time of the crisis is normalized to period 0. The crisis occurs after
a string of increasingly negative endowment shocks. In the period of the crisis, both endowments are about 8.8% for $y_t^T$ and 7.3% for $y_t^N$ below their respective averages (1). Regarding the unregulated economy, the decline of endowments triggers a Fisherian debt deflation, first, by lowering the collateral value and deteriorating financial conditions of households, whom respond by cutting consumption of tradable. As a result, $c_t^T$ decline by 23.7% more than the contraction of traded output leading to a large improvement in the trade balance of about 16% of tradable output. Second, the decline of $c_t^T$ combined with the fall of nontraded output leads to a depreciation of the real exchange rate of about 22.7% and then a decline of aggregate output of about 20.1%. Consequently, the Fisherian debt-deflation aggravates the fall in collateral, which is already quite depressed by the fall in both endowments. At the time of the crisis, as the economy is forced to deleverage, foreign debt fall by about 14.7%. Figure 4 displays with broken lines the typical crisis in Ramsey and regulated economies. The implementation of the optimal capital taxes, $\tau_t^{\mu_t=\mu_t^{SP}}$ and $\tau_t^{SP}$, generates an output contraction of about 13%, that is driven mainly by a depreciation of the relative price of 11.7% due to a decline of consumption of tradables of about 14.6%. As a result, foreign debt decreases by 7%. The fall of aggregate absorption lead to a surplus of trade balance of about 9.8% in terms of traded output. However, under $\tau_t^{MP}$ and $\tau_t^{IM}$, the regulated economies display stable dynamic behavior during the typical crisis period. As households are enforced to accumulate a moderate debt levels, their borrowing abilities are unaffected by the decline of both traded and non-traded endowments. As a result, households smooth their consumption by increasing their debt holding during the crisis by about 3% in the regulated economy under $\tau_t^{MP}$ and about 6.7% in the regulated economy under $\tau_t^{IM}$.

The resulting crisis dynamics of policy instruments point out their pro-cyclicality except for $\tau_t^{\mu_t=\mu_t^{SP}}$ that displays negative value during the typical crisis period as explained previously. It follows that the magnitudes of financial crisis are substantially more severe because of the externality. Figure 5 shows the distribution of the response of the aggregate consumption during financial crises for the unregulated economy, Ramsey economy, regulated economy
with $\tau^M_t$ and $\tau^M_t$ expressed as a percentage deviation from the average long-run value of consumption. This figure shows that the unregulated economy assigns non-trivial probabilities to consumption drops of more than 20 percent, while such a fall in consumption is a zero probability in the regulated economies with $\tau^M_t$ and $\tau^M_t$. In the later economies, aggregate consumption drops do not exceed 10.7% and 13% under $\tau^M_t$ and $\tau^M_t$ respectively.

Regarding Ramsey economy, the implementation of the optimal capital tax helps to assign high distribution’ probability to a consumption drop of about 10.3% and low probabilities to a drop that exceeds 20%.

\footnote{The distribution of percent change in consumption is calculated during unregulated economy financial crises for the regulated economies with $\tau^M_t$ and $\tau^M_t$.}
Figure 4: The Typical Crisis in the Endowment-Shock Economy
Figure 5: Conditional distribution of the externality effect on consumption during financial crises
6 Conclusion

In the present paper, we provide an explanation for the pro-cyclicality of capital control policy tax in open economy models with pecuniary externalities due to flow collateral constraints as stressed by [Schmitt-Grohé and Uribe 2017]. We show that Ramsey planner sets non-nil optimal tax only when individual agents’ trade-off between actual benefit of borrowing and the expected future cost of repayment and that of social planner do not coincide, and sets the optimal tax according to change on the difference between cost-to-benefit ratios of borrowing in Ramsey and unregulated economies. As this difference is pro-cyclical, the optimal tax is lowered during boom cycle and set to zero when the borrowing constraint is not binding in current and all immediate future states, it is increased during bust cycle when the probability of hitting the collateral constraint in future immediate successor states becomes positive. However, by contrast to [Schmitt-Grohé and Uribe 2017], the optimal capital control tax is counter-cyclical during financial crisis and displays negatives values.

In addition, the expression derived for the optimal capital tax in this paper explains the logic behind the intervention of Ramsey planner that seeks to make households internalizing the externality of the endogenous collateral constraint only when the economy is hitting or about to hit the credit limit. Thus, Ramsey planner does not seek to affect households’ desire to front-load consumption by carrying large amount of debt and to avoid them to overborrow. Consequently, implementing a capital control policy that aims to affect households impatience and consumption preferences, e.i. a fixed debt-tax that makes households as much patient as the rest of world, shifts the economy far away from binding collateral constraint region, and thus delivers high welfare gain by stabilizing the economy. Nevertheless, the effectiveness of this debt-tax policy depends strongly on model’s parameter calibration. For instance, when the elasticity of substitution between tradable and nontradable goods equals the inverse the households’ risk aversion, adopting a such tax could involve the economy into a permanent financial crisis mainly when the collateral constraint is binding in the initial state.
Furthermore, we investigate the effectiveness of a simple macropudential policy based on foreign debt-to-GDP ratio targeting. The main advantage of such policy is its uncomplicated implementation based on simple policy rule that links debt-tax variation to changes in debt-to-GDP gap. The resulting debt-tax cyclical behavior remains pro-cyclical as output contraction during a boom-bust cycle lead to a pro-cyclical debt-to-GDP gap. Our results highlight that the implementation of a such policy leads to a welfare gain relatively higher than that of the regulated economy with fixed debt-tax and more than seven times higher than that of Ramsey economy.
Appendix: Numerical Solution Method for unregulated, Ramsey and regulated economies

The computation of the competitive equilibrium requires recursive solving for functions $d_{t+1}$, $p^n_t$, $c^T_t$. We adopt the same algorithm employed by Bianchi [2011] which is based on time iteration algorithm modified to address the occasionally binding endogenous constraint. Next period variables are presented with the prime superscript, the algorithm contains the following steps:

1. We make use of the Markov chain processes publicly published by Bianchi [2011] (available at [https://www.aeaweb.org]). Then, we use the discrete processes grid for the economy’s bond position $G_d = \{d_1, d_2, ..., d_{800}\}$ and the shock state space $G_y^i = \{y^i_1, y^i_2, ..., y^i_4\}$ with $i=T,N$;

2. Start with guess values for $c^T_t(d,y)$, $d'_t(d,y)$ and $p^n_t(d,y)$ at time $t$ for $\forall d \in G_d$ and $y \in G_y^i$;

3. Adopt an interpolation scheme to generate $\lambda'$ as a function of $(d', y^i)$;

4. Set $j = 1$

5. Solve for $d'_{t-j}$, $p^n_{t-j}$ and $c^T_{t-j}$:

(a) Set $d'_{t-j} = \kappa (p^n_{t-j} y^N + y^T)$ and compute $c^T_{t-j}(d,y)$

(b) Compute

$$U = \lambda(c^T_{t-j}(d,y), y^N) - \beta R \lambda' (d', y^i)$$

(c) If $U \succ 0$; the credit constraint binds; keep $c^T_{t-j}$ and calculate $p^n_{t-j} = \frac{1-a}{a} c^T_{t-j}(d,y)$

and $d'_t(d,y) = \kappa (p^n_{t-j} y^N + y^T)$;

(d) If else, solve the following equation for $c^T_{t-j}$ with a root finding algorithm:

$$\lambda(c^T_{t-j}(d,y), y^N) - \beta R \lambda' (d', y^i) = 0$$
Then, deduce $p^n_{t-j}$ and $d'_t$.

6. Evaluate convergence. If $\|x_{t-j} - x_{t-j+1}\| < \epsilon$ for $x = c^T, p^n$ and $d'$ we have found the competitive equilibrium. Otherwise, set $x_{t-j} = \alpha x_{t-j} + (1 - \alpha) x_{t-j+1}$ and $j = j + 1$ and go to step (5). We set the convergence criterion $\epsilon = 10^{-8}$ and $\alpha = 0.2$. 
References


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