An Experimental Investigation of
Simultaneous Multi-battle Contests with Complementarities*

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Abstract

This paper reports the results of laboratory experiments that are designed to test theoretical predictions in a multi-battle contest with geographic complementarities. The specific setting is a game of Hex where control of each region is determined by a Tullock contest and the overall winner is determined by the combination of claimed regions. We find that in a game with only a few regions, aggregate behavior across regions is largely consistent with the theoretical predictions. However, examining individual level behavior suggests that bidders are not behaving in accordance with the model, but rather pursuing focused attacks. This intuitive behavioral approach is also found to occur in larger games where the theory is undeveloped.

Keywords: Contests, Multibattle Complementarities, Hex Game, Experiments

JEL Codes: C7, C9, D7

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Contests have long been used to model competitive situations like lobbying (Krueger, 1974; Tullock, 1980; and Synder 1989) and patent races (Fudenberg et al., 1983; Haris and Vickers, 1985, 1987). Recently, there has been renewed interest in contests as summarized in the recent survey of theoretical work by Kovenock and Roberson (2010a) and the book by Konrad (2009). A particular area of current interest involves contest that are composed of multiple battles as society worries about questions such as how to protect a pipeline or a computer network that is only as strong as its weakest link or how a state can assign its electoral college votes to influence the outcome of a US presidential election. However, concern about multi-battle contests goes back to discussions of the Colonel Blotto game (Borel, 1921; Borel and Ville, 1938; Gross, 1950; Gross and Wagner, 1950 and Freidman, 1958) in which two militaries allocate soldiers to a series of \( n \) battles and the winning side is the one that wins the most battles. In these games the outcome of a given battle depends on who has the larger army in that battle and the battlefields are linked by a budget constraint which captures the fact that the number of soldiers available is fixed.

The more recent work on multi-battle contests allows for asymmetric budgets and a positive opportunity cost of the resource in games with both continuous and discrete strategy spaces (Hart, 2008; Kvasov, 2007; Laslier, 2002; Laslier and Picard, 2002; Roberson, 2006; and Weinstein, 2005). Additionally, researchers have considered alternative forms of determining the overall winner based upon the outcomes of individual battles. Szentes and Rosenthal (2003a, b) examine a more general game in which one needs to win \( m \) battles to claim the prize (the majority rule is a special case where \( m = \frac{n+1}{2} \)). One example of such a contest would be the lobbying effort needed to achieve a supermajority. Clark and Konrad (2007) and Golman and Page (2009) consider a situation in which one party must win all of the battles while the other needs only a single victory. Such weak link games can be thought of as the defense and attack of a computer network or a pipeline. Deck and Sheremeta (2012) study a game of siege in which an attacker has repeated sequential opportunities to achieve a single victory.

In a large leap, Kovenock, et al. (2012) move beyond the situation where the overall winner is determined solely by the number of victories to a setting where success depends on the specific combination of individual victories. This structure more realistically captures many strategic problems. For example, new products from electronics to pharmaceuticals are often not based on a single patent or innovation, but rather a combination of them. Apple holds over 1300 patents related to the iPhone (Thomson Reuters, 2012). While not all of those patents may be in use, clearly a smartphone is of no use without a user interface, a power system and an antenna. Having three user interfaces and no power system or antenna would not result in a successful product. However, a portable video player requires an advanced screen and large storage capacity along with the power supply and user interface, but may not need an antenna.
In another context, a developer seeking to acquire a large number of lots from separate homeowners cares about the number of contiguous pieces that can be claimed, not the total number.

The specific problem that Kovenock, et al. (2012) solve derives from the 2 × 2 game of Hex. In this game, there are two players who are attempting to complete a path through the game board connecting their two peripheral regions (see Figure 1) using a standard Tullock contest function. While there are multiple winning combinations for each player and the set of winning combinations is not identical for each player, there will always be a single winner in the game once every region is claimed. As described by Kovenock, et al. (2012) this setup mimics a telecommunications or internet network. There are multiple ways to navigate through the network and no particular relay is critical given the redundancies. Therefore, an attack on such a network has to successfully block all possible routes. This situation is distinct from a weakest link game as no specific battle is decisive for overall victory, but some nodes are more important than others because of the number of potential winning paths of which it is a part.

In this paper we report the results of controlled laboratory experiments designed to test the theoretical predictions of Kovenock, et al. (2012). We also report the results of experiments involving a larger 4 × 4 game board. While the size of this larger game is still relatively small, the computations involved make developing theoretical predictions for it overly cumbersome. In fact, such scalability is a clear advantage of a behavioral approach to studying contests with complicated complementarities.

As a prelude to the results, we find that aggregate behavior in the 2 × 2 game is generally consistent with the theoretical predictions. However, we do observe substantial overbidding, a robust phenomenon in contest experiments (see Dechenaux, et al. 2012 for a review of contest experiments). Despite the overbidding, the relative bids between different regions are in line with the theoretical predictions. However, instead of bidding on each region as the theory predicts, individuals tend to focus their bids on a path through the game. This same behavioral pattern is observed in the larger game as well, with the result being that on average a larger amount is bid on those cells that have a greater number of winning paths running through them.

The remainder of the paper is organized as follows. The next section presents the specific hypotheses to be tested based upon the theoretical model. The third section details the experimental design and the fourth section analyses the results. Concluding remarks are given in the final section.

**Model and Hypotheses**
The model presented here is from Kovenock, et al. (2012). Consider the $2 \times 2$ game of Hex shown in Figure 1. There are two players, X and Y. Player X controls the top left and bottom right corners of the board, while Y controls the other two corners. The objective for each player is to form a contiguous path connecting their pair of corner regions. Thus player X wins a prize valued at $V$ if he captures any of the following sets of regions \{North, South, East, West\}, \{North, South, East\}, \{North, South, West\}, \{North, East, West\}, \{South, East, West\}, \{North, South\}, \{North, East\}, \{South, West\}. There are also 8 winning combinations for Player Y, some of which are the same as those for player X and some that are different. For example, either player wins by capturing the set \{North, South\}, while only player X wins with the set \{North, East\} and only player Y wins with the set \{South, East\}. Let $X^*$ and $Y^*$ denote the set of winning sets for players X and Y respectively.

![Diagram of the 2x2 Hex Game](image)

Figure 1. The 2x2 Hex Game of Kovenock, et al. (2012)

The players are assumed to be risk neutral, have a common value for winning the game, and do not face a budget constraint. Further, the winner of each region is determined by a standard Tullock contest success function. Letting $X_r$ ($Y_r$) denote the investment by player X (Y) in region $r \in R = \{\text{North, South, East, West}\}$. The probability that player X claims region $r$ is $\frac{X_r}{X_r + Y_r}$. Hence the probability that X wins the overall prize is given by

$$P = \sum_{a \in X^*} \left[ \prod_{r \in R \cap a} \left( \frac{X_r}{X_r + Y_r} \right) \prod_{r \notin R \cap a} \left( \frac{Y_r}{X_r + Y_r} \right) \right]$$

with a similar calculation for the probability that Y wins the prize. As any investment is forgone regardless of the outcome, player X’s profit function is given by $\pi_X = VP - \sum_{r \in R} X_r$ and similarly for player Y.

The unique Nash equilibrium of this game is $X_{\text{North}} = X_{\text{South}} = Y_{\text{North}} = Y_{\text{South}} = V/8$, and $X_{\text{East}} = X_{\text{West}} = Y_{\text{East}} = Y_{\text{West}} = V/16$. Notice, that each payer should invest a positive amount in each region.
The equilibrium calculation is straightforward, but tedious involving the simultaneous solution to four first order conditions of the profit maximization problem for each player. In equilibrium, each player has a 50% chance of winning and an expected payoff of $V/8$. Notice also that aggregate investment by the two players together is $4 \times V/8 + 4 \times V/16 = 3V/4$.

For comparison, if there were no complementarities and each of the regions was valued at $V_r$, then the standard result would hold for each region. Specifically, each bidder should bid $V_r/4$ for each region, resulting in a 50% chance of claiming any region and an expected profit of $V_r/4$ in each region. The total investment in $R$ would be $\sum_r V_r/2$. If $\sum_r V_r = V$ so that the total prize was the same in the two games then without complementarities each player would invest a total of $V/4$, the expected profit per player would be $V/4$, and the total investment would be $V/2$. Therefore, for the same total prize the complementarity increases total investment and the expected profits of the players are lower.

The equilibrium investments serve as the basic hypotheses to be tested in the lab. However, given the robust finding from previous experiments that people tend to overbid in simple contests we also investigate the following relative hypotheses that focus on the role of complementarities.

1. *Without complementarities, bids in a region are proportional to the equilibrium bid for the region and thus also proportional to the value of that region.*
2. *With complementarities, bids in a region are proportional to the equilibrium bid for the region.*

Before continuing to the experimental design, we briefly point out a few additional items. First, the theoretical problem described above can be extended to an $n \times n$ size game of hex. The equilibrium condition is determined by the simultaneous solution of $2n^2$ first order conditions. Further, the profit function itself depends on the elements of $X^*$ and $Y^*$ which each contain $2^{n^2-1}$ entries. So for a $4 \times 4$ game there are 32768 winning combinations for player $X$ and 32768 winning combinations for player $Y$, and the equilibrium level of investment depends on 32 simultaneous equations based on those winning combinations. Clearly, this problem quickly becomes intractable, but despite the large number of winning combinations, it is trivial for a person to look at a $4 \times 4$ game and see who has won.

Finally, notice that in the $2 \times 2$ game every winning path for $X$ is either {North, South}, {North, East}, {South, West}, or some superset of one of these. With this in mind we define the notion of a minimal winning set. *Minimal winning sets* are the sets of cells which are sufficient for victory, but no proper subset of which is a winning set. There are also three minimal winning paths for player $Y$ and again each involves 2 regions. For both players, the regions North and South are in two of their minimal winning sets and East and West are only in one of the minimal
winning sets. Therefore, if a player were to invest uniformly along a minimal winning path and not bid off of that path, then on average twice as much would be invested in North and South than in East and West, the same aggregate pattern predicted by the equilibrium despite the difference in individual behavior. As the game becomes larger \((n > 2)\), it is no longer the case that every minimal winning path is of length \(n\), although each player does always have minimal winning paths of length \(n\). Therefore, in addition to the optimal strategy that involves investing in all \(n^2\) regions, players could pursue a strategy of investing only on some minimal winning path or further restrict investment only to winning paths of length \(n\). Still, as noted earlier both the equilibrium strategy and either path investment strategy would lead to greater average investment in regions that are in more winning sets.

**Experimental Design**

To evaluate the theoretical predictions of the model, we conducted a series of contest experiments using a between subjects design. In one treatment subjects participated in a series of contests that did not involve complementarities and in the other treatment the contests involved complementarities.

Throughout the experiment, monetary amounts are denoted in Experimental Currency (EC), which would be converted to $US at the rate of $EC 25 = $US 1. This exchange rate is explained to the subjects when the experiment begins. Unless otherwise noted, all monetary amounts below are in EC. Subjects also received a $US 5 payment for showing up to the lab on time for the one hour session, as is standard policy in the Behavioral Business Research Laboratory at the University of Arkansas where the experiments were conducted. Participants were 72 undergraduate students from the lab’s database of approximately 2000 volunteers. While some subjects had previously participated in other economics experiments, none had participated in any related studies.

Both treatments involved three phases as described below. Subjects read phase specific instructions just prior to the start of each phase. Subjects did not know how many periods were in any phase nor did they know of the existence of future phases.

**Phase 1: Sequential Play in a 2 \(\times\) 2 Game with No Investment**

The first phase consisted of 10 games on a 2 \(\times\) 2 board. For technical ease, the regions were actually squares instead of hexagons, but the arrangement was such that the winning combinations were the same as discussed in the previous section for the game with complementarities (see Figure 2). In each game one player moved first and was able to claim a region (at no cost) by simply clicking it on their screen. The second player could observe the
choice of the first player and then select a region to claim. This choice was revealed to the first mover who could then claim a second region leaving the final region for the second mover.

For the no complementarities treatment the North and South Regions were valued at 8 while the other two regions was valued at 4 each. Thus, the first mover should take either North or South and the second mover should take the other of these relatively high valued regions. For the complementarities treatment, the value of completing a winning path was 48. In this case, the first mover should pick North or South initially. The second mover should then select whichever remains unclaimed of North or South in the hope that the first mover makes a subsequent mistake. The first mover should then make a winning move and hence the first player should always win. This first mover advantage holds in the sequential game of Hex regardless of the board size (including the 11 × 11 board offered by Parker Brothers).

Each player alternated between playing the first and second mover in these games. Further, there were 6 people in each session and players were randomly and anonymously matched each game to eliminate any issues with repeated play or reputation.

The purpose of phase 1 is twofold. First, it allows players to discover the strategic value of each region to both players in the game of Hex without explicitly being instructed about the importance of the North and South regions, which might bias bidding behavior. Second, it provides an opportunity for the subjects to earn money which can be used as an endowment during the later phases of the experiment where one risks losing money. In fact, since both players forfeit their investment and only one player can claim the prize, there must be a financial loser in each contest. Therefore, to maintain control over subject incentives the standard procedure of providing an endowment from which losses can be deducted is used. Because the values in the two games are different, for reasons explained below, the subjects were also given a treatment specific initial endowment. Those in the complements treatment received an endowment of 160. Those in the no complements treatment received an endowment of 280. The end result is that after phase 1, each subject should have earned a cumulative profit of 400 and thus start phase 2 with the same amount of money.

**Phase 2: 2 × 2 Game with Regions Decided by Contests**

The second phase of the experiment consisted of 20 games on a 2 × 2 board. Again, players were randomly and anonymously matched each period. In this phase the players privately and simultaneously submitted bids for each region as shown in Figure 2 with each region being awarded probabilistically according to the Tullock success function. For the games with complementarities, the prize for winning was again 48. Without complementarities, North and South were valued at 8 and East and West were valued at 4. Table 1 summarizes the equilibrium bids and profits for both treatments. As described in the model section, for the
same total value available the expected profits differ between treatments. Therefore, the values in the no complements treatment were adjusted so that (i) the expected profit per bidder is held constant across treatments, and (ii) the relative value of each region is held constant across treatments. The first point is important for ensuring that subjects have the same incentives in both treatments. The second point enables a clearer test to determine how the complementarity impacts investment as well as allowing for a test of the standard contest model as the value changes.

Table 1. Summary of Experiment Parameters and Predictions for 2 × 2 Games

<table>
<thead>
<tr>
<th>Value of Winning</th>
<th>No Complementarities</th>
<th>With Complementarities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North/South worth 8</td>
<td>Completed Path worth 48</td>
</tr>
<tr>
<td></td>
<td>East/West worth 4</td>
<td></td>
</tr>
<tr>
<td>Equilibrium Bid on</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>North/South</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equilibrium Bid on</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>East/West</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected Profit per Player</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 2. Screen Shots of 2 × 2 game for

a) Contests with Complementarities (Hex Game)
b) Contests without Complementarities

Phase 3: 4 × 4 Game with Regions Decided by Contests

The procedures for this phase were identical to those in phase 2, except that the game board was increased to 4 × 4 and that subjects only played the game 5 times (and thus placed the same total number of bids as in phase 2 since there are four times as many regions in phase 3). Figure 3 shows a sample outcome of the 4 × 4 game with complementarities. The value of a winning path remained 48 for the treatment with complementarities. For the game without complements, each of the 16 regions was valued at 3 so that the total value was the same between the two treatments allowing for an evaluation of the impact of incentives and additional variation in prize values for standard stand-alone contests. Given the exploratory nature of this larger game and the inherent increase in complexity when there are complementarities, this phase was always conducted last so as to (i) not influence behavior in the 2 × 2 game which is the main focus of the project, and (ii) provide subjects an opportunity to learn in the simpler game so that observed behavior in this game is meaningful. While these two concerns are not as important when there are no complementarities, the two treatments are kept parallel to make direct between treatment comparisons.

Figure 3. A Sample Screen of the Outcome from a 4 × 4 Contest with Complementarities
Behavioral Results

We begin our analysis with behavior in Phase 1 of the experiment. Overall, players made an optimal choice 98% of the time, where optimal is defined as claiming the higher valued region if it is available. For the more complicated sequential hex game, optimal behavior, defined as following the subgame perfect equilibrium strategy conditional on the decision point, is observed 95% of the time. Further, no suboptimal behavior was observed in the last three rounds for either treatment. This finding clearly indicates that subjects understood the strategic value of the different regions before beginning phase 2.

We now turn to analyzing behavior in phase 2. Given the symmetry in the game, regions are standardized around a vertical line drawn through the center of the game and reported with respect to the position of a region from the perspective of the Yellow player who is trying to complete a path from the top left to the bottom right. For the $2 \times 2$ game this means that for reporting purposes bids Green placed nominally for the West region are combined with Yellow’s bids for the East and vice versa. The average bid for each standardized region is shown in Table 2.

We begin by considering the no complementarities treatment, the results of which are shown in the left column of Table 2. Several interesting features of the data are readily apparent. First, for all four regions players are bidding almost twice the equilibrium level on average. This
is clear evidence that the subjects are not bidding according to the equilibrium predictions. Such overbidding is actually typical in simple contests experiments. The second main feature is that players are bidding basically the same amount for North and South, consistent with Hypothesis 1. The players are also bidding nearly identically on average for East and West, also consistent Hypothesis 1. Finally, players are bidding twice as much for North and South as for East and West, again consistent with Hypothesis 1.

Table 2. Average Bids by Region in the $2 \times 2$ Games

<table>
<thead>
<tr>
<th>Region</th>
<th>No Complementarities</th>
<th>With Complementarities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Percent Overbid</td>
</tr>
<tr>
<td>North</td>
<td>3.81</td>
<td>91%</td>
</tr>
<tr>
<td>South</td>
<td>3.75</td>
<td>88%</td>
</tr>
<tr>
<td>East</td>
<td>1.89</td>
<td>89%</td>
</tr>
<tr>
<td>West</td>
<td>1.85</td>
<td>85%</td>
</tr>
<tr>
<td>North/South</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>East/West</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>$(\text{North} + \text{South}) / (\text{East} + \text{West})$</td>
<td>2.03</td>
<td>1.92</td>
</tr>
</tbody>
</table>

Table 3 reports the estimation results from regression analysis where the dependent variable is the amount bid on a region relative to the equilibrium bid for that region and South and West are dummy variables for those respective regions and Side is a dummy variable that takes the value of 1 for the East and West regions and is 0 otherwise. Thus the omitted case is the North region. Side captures the effect of being in the East region while West captures any differential effect between the East and West regions. Standard errors are clustered at the session level while each individual bidder is treated as having a random effect. The joint lack of significance for the dummy variables in Table 3 provides statistical support in favor of Hypothesis 1 (p-value for F - test = 0.5989). To test whether or not players are bidding according to the equilibrium predictions, involves comparing the constant term in Table 3 to the predicted value of 1. This hypothesis can be rejected in favor of systematic overbidding (p-value < 0.00005).
Table 3. Regression of actual bids / theoretical bids in the absence of complementarities

<table>
<thead>
<tr>
<th>Region</th>
<th>Coefficient</th>
<th>Robust Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.9070***</td>
<td>0.0447</td>
</tr>
<tr>
<td>South</td>
<td>-0.0310</td>
<td>0.0385</td>
</tr>
<tr>
<td>Side</td>
<td>-0.0535</td>
<td>0.0736</td>
</tr>
<tr>
<td>West</td>
<td>0.0238</td>
<td>0.0278</td>
</tr>
</tbody>
</table>

*** indicates significance at the 1% level.

Turning to the treatment with complementarities, the data presented in the right column of Table 2 provide strong evidence in support of Hypothesis 2. The average bids are very similar in North and South as predicted. Further, average bids are similar in the East and West, as predicted. Finally, bids are predicted to be twice as high in the North and South as in the east and West and this is what is observed. Statistical evidence is provided in Table 4, which reports a similar regression to that done for the no complements treatment. Hypothesis 2 is supported by the joint lack of significance on the three dummy variables in Table 4 (p-value for F-test = 0.6526). Table 2 also reveals that subjects are at least nominally overbidding in each region, but this behavior is not significant as the constant term in Table 4 is not statistically different from 1 (p-value = 0.1868). In part the reduction in overbidding that occurs with complementarities is being driven by the increase in the equilibrium bid levels rather than observed bids falling.

Table 3. Regression of actual bids / theoretical bids with complementarities

<table>
<thead>
<tr>
<th>Region</th>
<th>Coefficient</th>
<th>Robust Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.2297***</td>
<td>0.1503</td>
</tr>
<tr>
<td>South</td>
<td>0.1626</td>
<td>0.1318</td>
</tr>
<tr>
<td>Side</td>
<td>0.2409</td>
<td>0.2457</td>
</tr>
<tr>
<td>West</td>
<td>-0.2066</td>
<td>0.3342</td>
</tr>
</tbody>
</table>

*** indicates significance at the 1% level.

The above analysis focuses on aggregate behavior and the results seem to support the relative theoretical predictions for both cases. However, these results are masking a distinct behavioral pattern. The theoretical prediction is that a player should bid on every region regardless of the
complementarity. This is done over 80% of the time in the absence of complementarities, but in the presence of the complementarity this pattern is observed in only 35% of the realizations. Further, only 1% of the time does the set of regions on which a subject bids constitute a non-winning set in the game with complementarities. The missing majority of observations, 64% of the behavior in the game with complementarities, are such that subjects bid on a proper subset of regions that contained a winning path and the vast majority of these were for minimal winning sets.\(^1\) This is shown visually in Figure 4 which gives a histogram for the number of regions upon which a subject bid.

**Figure 4.** Frequency of 2 × 2 behavior based upon number of regions with positive bids

As discussed previously, if one randomly selects a minimum winning path and bids uniformly on it, the result would also be average bids that are twice as high for North and South as for East and West. Further, if the total bid equaled half of the value of winning, as occurred on average in the independent contests of the no complementarities treatment, then one would bid 12 on North two-thirds of the time, 12 on South two-thirds of the time, and 12 on East and West each one-third of the time. The result would be an average bid of 8 for North and South and an

\(^1\) 307 out of the 309 bids for two regions in the 2 × 2 game with complementarities constitute a minimal winning set or a path.
average bid of 4 for East and West, the pattern revealed in Table 2. Hence it appears that theoretical model of Kovenock, et al. (2013) works in aggregate, but not for the right reason.

We now turn to behavior in the exploratory third phase of the experiment. Figure 5 shows the average bid in each region for the no complementarities treatment. In this case, each region had a value of 3 and thus the equilibrium bid is 0.75. The average observed bid across all 16 independent regions was 1.09 and there are no major differences between regions. The average rate of overbidding was 45%, which is smaller than in phase 2. One reason for this drop in overbidding is that subjects only bid on every region 64% of the time with the other observations typically involving a few ignored regions. This pattern is shown in Figure 6. Once non-bids are accounted for the rate of overbidding is similar to that in phase 2, suggesting that the level of incentives are not driving overbidding directly. Of course, failure to bid on every region could be due to fatigue or the large number of choices that had to be made, the effects of which may in turn be influenced by the level of the stakes.

In the 4 × 4 game with complementarities, the optimal bid for each region is related to the number of winning sets that contain the region. Figure 7 shows both the number of winning sets and the average observed bid by region for this case. The correlation between the two measures is quite high at ρ = 0.898. However, this aggregate success again masks individual heterogeneity. Rather that bidding on every region, 36% of bids are for a 4 region path and only 31% of bids involve more than half of the regions, see Figure 6.
Figure 6. Frequency of 4 × 4 behavior based upon number of regions with positive bids

![Chart showing frequency of 4 × 4 behavior](chart)

Figure 5. Exploration of the 4 × 4 Game with Complementarities

![Game representation with numbers and labels](game)

Number of Winning Sets Containing Region

Average Bid by Region

### Concluding Remarks

Contests have long been used to model a wide array of activity. Recently, researchers have begun to look at ever more complicated strategic situations where outcomes are based on a
series of interconnected battles. One such example is the recent model by Kovenock, et al. (2013), which focuses on the game of Hex, but is representative of a wider class of games that involve complementarities in outcomes. This model is particularly relevant for the protection of networks that have built in redundancies, such as telecommunications or computer networks. Unfortunately, the tradeoff of additional complexity is often tractability. For example, Kovenock, et al. (2013) restrict attention to a $2 \times 2$ game board.

In this paper we report the results of controlled laboratory experiment designed to test the predictions of the Kovenock, et al. (2013) model specifically and also explore how players actually approach more complicated contests. What we find from the experiment is that in aggregate the model’s predictions accurately reflect the relative bids in the different regional battles. However, the model’s relative predictions are correct, but for the wrong reason. Rather than fighting in every region in accordance with the model, people actually concentrate on specific winning combinations and largely ignore the other battles. As in the case of independent values, people bid just less than half of the prize value, but in the Hex game they spread that amount along a winning path.

An advantage of a behavioral approach to investigating contest behavior is that one can directly investigate situations beyond what is computationally attractive. Here we also considered a more complex $4 \times 4$ that involves 32768 winning combinations for each player. Again aggregate behavior was such that players bid more for regions that are in more winning combinations as the model would predict. However, this aggregate pattern is again explained by individuals focusing on winning combinations. This finding indicates that concentrated attacks are robust behavior. The implications of these patterns are potentially quite broad as researchers attempt to identify optimal strategies for attack and defense of networks in many naturally occurring applications such as cyber-security or supply chains.

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Appendix: Subject Instructions

*On the following pages there are two sets of instructions. The first set is for the treatment with no complementarities and the second set is for the treatment with complementarities. Text in [brackets] was not observed by subjects.*

[No Complementarities Phase 1]

This is an experiment in the economics of decision making. You will be paid in cash at the end of the experiment based upon your decisions, so it is important that you understand the directions completely. Therefore, if you have a question at any point, please raise your hand and someone will assist you. Otherwise we ask that you do not talk or communicate in any other way with anyone else. If you do, you may be asked to leave the experiment and will forfeit any payment.

The experiment will proceed in three parts. You will receive the directions for part 2 after part 1 is completed and for part 3 after part 2 is completed. What happens in part 2 does not depend on what happens in part 1, and what happens in part 3 does not depend on what happens in part 1 or 2. But whatever money you earn in part 1 will carry over to part 2 and whatever you earn in part 2 will carry over to part 3.

Each part contains a series of decision tasks that require you to make choices. For each decision task, you will be randomly matched with another participant in the lab. None of the participants will ever learn the identity of the person they are matched with on any particular round.

In each task you have the opportunity to earn Lab Dollars. At the end of the experiment your lab dollars will be converted into US dollars at the rate $25 Lab Dollars = $US 1.

You have been randomly assigned to be either a “Green” or a “Yellow” participant. You will keep this color throughout the experiment. For every task you will be randomly matched with a person who has been assigned the other color. Your color is indicated in a box on the right side of your screen. This portion of your screen also shows you your earnings on a task once it is completed and your cumulative earnings in the experiment. You will start off with an earning balance of $280.

In the center of your screen you can see a map with four regions: North, South, East, and West. The North is worth $8. The South is worth $8. The East is worth $4. The West is worth $4. This information is displayed at the top of your screen above the map. In part 1 of the experiment, when it is your turn you can claim any one unclaimed region on the map by clicking on it. When you claim a region, that part of the map will be colored in with your color and your earnings will be increases by the value of the region you claimed. When the person you are matched with claims a region, it will be colored with that person’s color and that person’s earnings will be increased by the value of the claimed region.

On each task Green and Yellow alternate turns until all of the regions on a map are claimed. Once all of the regions are claimed the task is complete. At that point, you will be randomly rematched with another participant for the next task. Which color gets to go first alternates between tasks. You will go through this process several times.
[No Complementarities Phase 2]

Part 2 of this experiment is very similar to part 1. The map is the same, as are the values of each region. Your color will be the same and you will be randomly and anonymously rematched with someone in the opposite role for each task.

What is different is how the regions are claimed. Now you and the person you are matched up with have to bid for each of the four regions at the same time. However, any amount you bid on a region is deducted from your earnings regardless of whether or not you get to claim the region. Since you have to pay what you bid, the sum of your bids for the four regions cannot exceed the amount of earnings you have when placing your bids.

Bidding for a region works in the following way. The chance that you claim a region is proportional to how much you bid relative to the total amount bid for that region. For example, suppose that Yellow bid $6 for the North and Green bid $2 for the North then the chance that Yellow would claim North is $6/(6+2) = 6/8 = 75\%$. The chance that Green would claim North is $2/(6+2) = 2/8 = 25\%$.

As another example, suppose that Yellow bid $0$ for the North and Green bid $0.25$ for the North then the chance that Yellow would claim North is $0/(0+0.25) = 0\%$. The chance that Green would claim North is $0.25/(0+0.25) = 100\%$.

If both bidders bid $0$ for a region then each would claim the region with a 50% chance.

You and the person you are matched with will both privately and simultaneously place your bids for all four regions at one time. The computer will then determine who claims each region based upon the probabilities associated with the bids. Each region will turn Yellow or Green to indicate who claimed that region.

As before, whoever claims a region will receive the value for that region and have that value added to their earnings. Suppose Yellow bid $5$ for the North and Green bid $2$ for the North. If Yellow claims the North then Yellow will earn $8 - $5 = $3$ and Green will earn -$2$. Thus $3$ will be added to Yellow’s earnings and Green will have $2$ subtracted from their earnings. However, if Green claims the North then Yellow will earn -$5$ and Green will earn $8 - $2 = $6$. Earnings for the other three regions will be determined in the same fashion.

[No Complementarities Phase 3]

Part 3 of this experiment is just like part 2, except that there are now 16 regions on the map and the value of each region is $3$. Regions will be claimed in the same way, your color will be the same and you will be randomly and anonymously rematched with someone in the opposite role for each task.
This is an experiment in the economics of decision making. You will be paid in cash at the end of the experiment based upon your decisions, so it is important that you understand the directions completely. Therefore, if you have a question at any point, please raise your hand and someone will assist you. Otherwise we ask that you do not talk or communicate in any other way with anyone else. If you do, you may be asked to leave the experiment and will forfeit any payment.

The experiment will proceed in three parts. You will receive the directions for part 2 after part 1 is completed and for part 3 after part 2 is completed. What happens in part 2 does not depend on what happens in part 1, and what happens in part 3 does not depend on what happens in part 1 or 2. But whatever money you earn in part 1 will carry over to part 2 and whatever you earn in part 2 will carry over to part 3.

Each part contains a series of decision tasks that require you to make choices. For each decision task, you will be randomly matched with another participant in the lab. None of the participants will ever learn the identity of the person they are matched with on any particular round.

In each task you have the opportunity to earn Lab Dollars. At the end of the experiment your lab dollars will be converted into US dollars at the rate $25 Lab Dollars = $US 1.

You have been randomly assigned to be either a “Green” or a “Yellow” participant. You will keep this color throughout the experiment. For every task you will be randomly matched with a person who has been assigned the other color. Your color is indicated in a box on the right side of your screen. This portion of your screen also shows you your earnings on a task once it is completed and your cumulative earnings in the experiment. You will start off with an earning balance of $160.

In the center of your screen you can see a map with four regions: North, South, East, and West. These regions are surrounded by large colored areas. The top left and bottom right areas are colored “Yellow”. The top right and bottom left areas are colored “Green”. In part 1 of the experiment, when it is your turn you can claim any one unclaimed region on the map by clicking on it. When you claim a region, that part of the map will be colored in with your color. If you can complete a continuous path of regions in your color connecting your two large colored areas you will earn $48. When the person you are matched with claims a region, that part of the map will be colored in with that person’s color. If the person you are matched with completes a continuous path, that person will earn $48. Exactly one person can complete a path each time and the person that does not complete a path will earn $0.

On each task Green and Yellow alternate claiming regions until all four regions are claimed. At that point, you will be randomly rematched with another participant for the next task. Which color gets to go first alternates between tasks. You will go through this process several times.
[Complementarities Phase 2]

Part 2 of this experiment is very similar to part 1. The map is the same, as is the value of completing a path. Your color will be the same and you will be randomly and anonymously rematched with someone in the opposite role for each task.

What is different is how the regions are claimed. Now you and the person you are matched up with have to bid for each of the four regions at the same time. However, any amount you bid on a region is deducted from your earnings regardless of whether or not you get to claim the region. Since you have to pay what you bid, the sum of your bids for the four regions cannot exceed the amount of earnings you have when placing your bids.

Bidding for a region works in the following way. The chance that you claim a region is proportional to how much you bid relative to the total amount bid for that region. For example, suppose that Yellow bid $6 for the North and Green bid $2 for the North then the chance that Yellow would claim North is $6/(6+2) = 6/8 = 75\%$. The chance that Green would claim North is $2/(6+2) = 2/8 = 25\%$.

As another example, suppose that Yellow bid $0 for the North and Green bid $0.25 for the North then the chance that Yellow would claim North is $0/(0+0.25) = 0\%$. The chance that Green would claim North is $0.25/(0+0.25) = 100\%$.

If both bidders bid $0 then each would claim the region with a 50\% chance.

You and the person you are matched with will both privately and simultaneously place your bids for all four regions at one time. The computer will then determine who claims each region based upon the probabilities associated with the bids. Each region will turn Yellow or Green to indicate who claimed that region.

As before, whoever completes a path will receive the value for it and have that value added to their earnings. Suppose Yellow bid a total of $23 on the four regions and Green bid a total of $18 for the four regions. If Yellow completes a path then Yellow will earn $48 - $23 = $25 and Green will earn -$18. Thus $25 will be added to Yellow’s earnings and Green will have $18 subtracted from their earnings. However, if Green completes a path then Yellow will earn -$23 and Yellow will earn $48 - $18 = $30.

[Complementarities Phase 3]

Part 3 of this experiment is just like part 2, except that there are now 16 regions on the map. Regions will be claimed in the same way, a completed path is still worth $48, your color will be the same and you will be randomly and anonymously rematched with someone in the opposite role for each task.