LIMITED RIGHTS AND SOCIAL CHOICE RULES

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Abstract. In 1970 Amartya Sen introduced within social choice theory the notion of *minimal liberty* and proved an impossibility result concerning social decision functions. In this paper, Sen's condition of (minimal) liberty is weakened within the framework of social choice rules. It is then shown that the same kind of impossibility obtains for social choice functions.

1. introduction

The origin of the tremendous development of studies on rights and freedom within social choice theory and normative economics can be traced back to the famous short paper of Amartya Sen published in 1970 (Sen 1970; see also his book published the same year (1970a), and Sen (1982, 2002)). In Sen's paper, it is shown in the framework of aggregation procedures that there is a conflict between collective rationality imbedded in the notion of social decision function (in terms of a transitivity-type of the social preference property—in fact, acyclicity of the asymmetric part of the social preference), Paretianism (a unanimity property) and some slight violation of neutrality (neutrality meaning that the names of options or social states are not to be taken into account) possibly combined with some slightly unequal distribution of power among individuals interpreted as an individual liberty property. Although, since then, rights have been considered within another paradigm, viz. game forms (see for instance Gärdenfors (1981, 2005), Gaertner, Pattanaik and Suzumura (1992), Peleg (1998, 1998a) and Suzumura (2006)), and freedom has been mainly analyzed in the context of opportunity sets following the pioneering paper of Pattanaik and Xu (1990) (see also the survey by Barbera, Bossert and Pattanaik (2004)), some authors (for instance Saari and Pétron (2006) and Igersheim (2006)) have recently revisited the foundational framework of Sen and Gibbard (1974) either by studying the informational structure of the aggregation procedure or by examining the consequences of taking a Cartesian structure to define the set of social states, consequences that take the form of a restriction of individual preferences. The purpose of my paper (Salles 2007) was different. I wished to formally study a weakening of the conditions associated with the notion of individual liberty. I have always considered that this condition was rather strong in Sen's original paper. In fact, the condition is quite strong in the mathematical framework and only the interpretation (the idea of personal sphere), to my view, makes it not only acceptable but obvious. However, there is nothing in the basic mathematical framework that guarantees this personal aspect (in contrast with a suitable Cartesian product structure). In this basic framework, it was however possible to weaken the liberalism conditions that amount to the existence of a kind of local dictators to local vetoers. Unfortunately I showed that this weakening did not suffice to offer an interesting escape route from Sen's negative result. Basically, I showed that with the weaker condition, we still had impossibility results, but for social welfare functions or some variants (where the

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social preference was an interval order or a semiorder)[1]. In the present paper, I explore the effect of a suitable weakening of the liberalism condition within the framework of social choice rules rather than aggregation functions where a choice function (rather than a social preference) is associated to a profile of individual preferences. [2] Although the mathematical derivations are quite simple, I initially expected to have to use properties of choice functions borrowed from the revealed preference literature as presented among others by Arrow (1959, 1984), Fishburn (1975), Jamison and Lau (1973), Sen (1971, 1982), Schwartz (1976) and Suzumura (1976, 1983), but, to my surprise, this was necessary only in a very specific case.

After introducing general definitions and recalling Sen's theorems, I will present new Sen-type impossibility theorems in the social choice rules setting.

2. Basic definitions and Sen's theorem

Let X be the set of social states. Nothing specific is assumed for this set unless it is clearly indicated that it is finite. A binary relation, a *preference*, over X is a subset of $X \times X$. It will be denoted by $\succeq$. I will write $x \succeq y$ rather than $(x, y) \in \succeq$. All binary relations considered in this chapter are supposed to be *complete* (for all x and $y \in X$, $x \succeq y$ or $y \succeq x$) and, consequently, *reflexive* (for all $x \in X$, $x \succeq x$). The *asymmetric* part of $\succeq$, denoted $\succ$ is defined (since $\succeq$ is complete) by $x \succ y$ if $\neg y \succeq x$. The *symmetric* part of $\succeq$ is defined by $x \sim y$ if $x \succeq y$ and $y \succeq x$. Intuitively, $x \succeq y$ will mean 'x is at least as good as y', $x \succ y$ will mean 'x is preferred to y' and $x \sim y$ will mean 'there is an indifference between x and y'. A preference $\succeq$ is *transitive* if for all x, y and $z \in X$, $x \succeq y$ and $y \succeq z \Rightarrow x \succeq z$. The asymmetric part of $\succeq$, $\succ$, is transitive if for all x, y and $z \in X$, $x \succ y$ and $y \succ z \Rightarrow x \succ z$. The symmetric part of $\succeq$, $\sim$, is transitive if for all x, y and $z \in X$, $x \sim y$ and $y \sim z \Rightarrow x \sim z$. If $x \succeq$ is transitive, $\succ$ and $\sim$ are transitive too. We will say that $\succ$ is *acyclic* if there is no finite subset of X, $\{x_1, ..., x_k\}$, for which $x_1 \succ x_2$, $x_2 \succ x_3$, ..., $x_{k-1} \succ x_k$ and $x_k \succ x_1$. A complete and transitive binary relation is a *complete preorder* (sometimes called 'weak ordering'). Let $\mathbb{P}$ denote the set of complete preorders over X, and $\mathbb{A}$ denote the set of complete binary relations over X whose asymmetric part is acyclic.

Let N be the set of individuals. Individual $i \in N$ has her preference given by a complete preorder $\succeq_i$ over X. A *profile* π is a function from N to $\mathbb{P}'$, $\pi : i \mapsto \succeq_i$, where $\mathbb{P}' \subseteq \mathbb{P}$ with $\mathbb{P}' \neq \emptyset$. Let Π' be the set of profiles when the $\succeq_i$'s are in $\mathbb{P}'$ and Π be the set of all profiles (when the $\succeq_i$'s are in $\mathbb{P}$). When N is finite and $\#N = n$, a profile is a n-list $(\succeq_1, ..., \succeq_n)$ with each $\succeq_i$ in $\mathbb{P}'$. Then $\Pi' = \mathbb{P}'^n$ and $\Pi = \mathbb{P}^n$ ($\mathbb{P}'^n$ and $\mathbb{P}^n$ are n-times Cartesian products of $\mathbb{P}'$ and $\mathbb{P}$). Let 2^X be the set of subsets of X and $2^X_{-\emptyset}$ be the set of nonempty subsets of X.

Definition 1. A *choice function* is a function $C : 2^X_{-\emptyset} \to 2^X_{-\emptyset}$ such that for all $S \in 2^X_{-\emptyset}$, $C(S) \subseteq S$.

Given a subset S of elements in X, the choice set $C(S)$ can be interpreted as the elements that are chosen with the assumption that at least one element is chosen and all chosen elements belong to S. Given X, let $\mathbb{C}$ be the set of choice functions as defined above.

Definition 2. A *social choice function* is a function $f : \Pi' \to \mathbb{C}$.

A social choice function associates a unique choice function, that is a way to select options in each subset of options, to individual preferences (one preference for each individual).

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[1]For these concepts see Fishburn (1985) and Suppes et al. (1989)

[2]This social choice rule approach probably originates in Suzumura's works (Suzumura 1983).

Given a social choice function f, and two (distinct) social states x and $y \in X$,[3] we will say that individual $i \in N$ is (x, y)-c-decisive if for all $\pi \in \Pi'$ and all $S \in 2^X_{-\emptyset}$, $x \succ_i y$ and $x \in S \Rightarrow y \notin C(S)$, where $C = f(\pi)$.

This means that whenever individual i prefers x to y and x is available, y cannot be socially chosen.

Definition 3. An individual who is (x, y)-c-decisive and (y, x)-c-decisive will be said to be $\{x, y\}$*-c-decisive* or a $\{x, y\}$*c-strong vetoer.*[4]

I can now define Sen's two liberalism conditions. Let f be a social choice function.

Definition 4. (c-Liberalism, general 2-cSV) For all $i \in N$, *there exist* a_i *and* $b_i \in X$ such that i is a $\{a_i, b_i\}$-c-strong vetoer.

It should be noticed that $\mathbb{P}'$ must be large enough to have a non trivial satisfaction of general 2-cSV: for each individual i it must be possible to have both $a_i \succ_i b_i$ and $b_i \succ_i a_i$. Also, it should be outlined that the condition is rather fair since each individual is endowed with the same kind of power. The theorem can be proved by using a weaker form of the foregoing condition.

Definition 5. (c-Minimal liberalism, minimal 2-cSV) *There exist two individuals* i *and* $j \in N$, and a, b, c, $d \in X$ such that i is a $\{a, b\}$-c-strong vetoer and j is a $\{c, d\}$-c-strong vetoer. [5]

Of course, the fairness property disappeared. The options are to be 'interpreted' as being specific to the concerned individual, i.e., a and b are specific to individual i; a and b can even be 'interpreted' as perfectly identical social states except for some features that are personal to individual i. Clearly general 2-cSV implies minimal 2-cSV.

As previously mentioned, the domain of the social choice function f must be rich enough. This will be taken care of (with some excess) by the following condition U.

Definition 6. (Universality, U) Let f be a social choice function. *Universality* requires that $\mathbb{P}' = \mathbb{P}$.

This means that an individual preference can be any complete preorder. There is no restriction imposed by some kind of upper rationality or the existence of inter-individual constraints. The last condition (condition P) is a weak form of unanimity (Pareto principle).

Definition 7. (Pareto principle, P) Let f be a social choice function, $\pi \in \Pi'$, x, $y \in X$, and $S \in 2^X_{-\emptyset}$. [6] *If for all* $i \in N$, $x \succ_i y$, *and* $x \in S$, *then* $y \notin C(S)$, *where* $C = f(\pi)$.

Sen's theorem is obtained in this setting without introducing any further properties of social choice functions. In Sen's original setting the theorem was obtained within a large

[3]They have to be distinct so that saying it is superfluous since we consider that $x \succ_i y$ and $\succ_i$ is asymmetric.

[4]The letter 'c', as in c-decisive and c-vetoer etc. indicates that decisiveness, vetoer etc. are defined in the social choice functions setting rather than in the more standard aggregation functions setting where the values taken by the aggregation functions are social preferences.

[5]I will justify in Section 4 the 'vetoer' labelling rather than the term 'dictator' I used in my previous paper (Salles 2007).

[6]Again, x and y are necessarily distinct.

class of aggregation functions (Sen called them *social decision functions*), viz., $\mathbb{A}$-valued aggregation functions that include among others social welfare functions à la Arrow (1950, 1951, 1963). (See Salles (2007).)

The collective rationality (acyclicity of the asymmetric part of the social preference relation) imposed in the case of a social decision function is rather weak. It has an interesting consequence on the non-emptiness of the set of maximal elements in any finite subset of X (or since we are considering complete binary relations on the non-emptiness of maximum elements or –possible– choices).

Theorem 1. *If there are at least two individuals and if $\#X \geq 2$, there is no social choice function satisfying minimal 2-cSV, U and P.*

An immediate corollary is:

Corollary 1. *If there are at least two individuals and if $\#X \geq 2$, there is no social choice satisfying general 2-cSV, U and P.* [7]

3. Partial weak veto and Sen-type theorems

As mentioned in Salles (2007), it is in reading Pattanaik's paper (1996) that I got the impetus to work on this topic. In particular in this paper, Pattanaik discusses Sen's possible views regarding a distinction between a conception of rights as the ability to prevent something and a conception of rights as the obligation to prevent something which seems to be endowed in the liberalism conditions. Although I wished to devote some time to introduce modal theoretic techniques to deal with this distinction, I will be in this paper more modest and will consider a weakening of the liberalism conditions. It is however obvious that this weakening is not a real response to the ability-obligation problem. Nevertheless, at least from a semantical point of view, having a (partial) veto power weaker than the strong veto defined above corresponds rather well to the idea of an ability to prevent something. I will then introduce the notion of partial weak veto and will show how extremely robust Sen's theorem is.

Given a social choice function f, and two (distinct) social states x and $y \in X$, we will say that individual $i \in N$ is (x, y)-c-semi-decisive if for all $\pi \in \Pi'$ and all $S \in 2^X_{-\emptyset}$, $x \succ_i y$ and $x \in S \Rightarrow (y \in C(S) \Rightarrow x \in C(S))$, where $C = f(\pi)$.

This means that whenever individual i prefers x to y and x is available, if y happens to be socially chosen, then x has to be socially chosen too.

Definition 8. An individual who is (x, y)-c-semi-decisive and (y, x)-c-semi-decisive will be said to be $\{x, y\}$*-c-semi-decisive* or a $\{x, y\}$*-c-weak vetoer.*

I can now define weak versions of liberalism.

Definition 9. (c-Weak liberalism, general 2-cWV) For all $i \in N$, *there exist a_i and $b_i \in X$* such that i is a $\{a_i, b_i\}$*-c-weak vetoer.*

Definition 10. (c-Minimal weak liberalism, minimal 2-cWV). *There exist two individuals i and $j \in N$*, and a, b, c, $d \in X$ such that i is a $\{a, b\}$*-c-weak vetoer* and j is a $\{c, d\}$*-c-weak*

[7] For a proof see Suzumura (1983) or Pattanaik (1994).

vetoer.[8]

I can now stated the main result of this paper.

Theorem 2. *If there are at least two individuals, there is no social choice function satisfying U, P and minimal 2-cWV, provided that, in the definition of minimal 2-cWV,* $\{a,b\} \neq \{c,d\}$ *and provided that* $\#X \geq 4$.

Proof. Let f be a social choice function satisfying U, P and 2-cWV. Suppose first that $\{a,b\} \neq \{c,d\}$, but $\{a,b\} \cap \{c,d\} \neq \emptyset$. Without loss of generality, assume that $a = d$. Consider a profile π such that $a \succ_i b$, $c \succ_j a$, and for all $k \in N$, $b \succ_k e$ and $e \succ_k c$, where e is a fourth social state. Note that $a \succ_i b \succ_i e \succ_i c$ and $b \succ_j e \succ_j c \succ_j a$. Let $T = \{a,b,c,e\}$. Then, by condition P, $e \notin C(T)$ and $c \notin C(T)$. Since j is c-semi-decisive over $\{a,c\}$ and $c \succ_j a$, if $a \in C(T)$ then $c \in C(T)$. Since $c \notin C(T)$, $a \notin C(T)$. One is left with b, i.e., $\{b\} = C(T)$. But since i is c-semi-decisive over $\{a,b\}$ and $a \succ_i b$, if $b \in C(T)$, we have also $a \in C(T)$, a contradiction.

Now suppose $\{a,b\} \cap \{c,d\} = \emptyset$, i.e., a, b, c, and d are distinct options. Consider a profile π such that $a \succ_i b$, $c \succ_j d$, and for all $k \in N$, $b \succ_k c$ and $d \succ_k a$. Note that $d \succ_i a \succ_i b \succ_i c$ and $b \succ_j c \succ_j d \succ_j a$. Let $T = \{a,b,c,d\}$. Then, by condition P, $a \notin C(T)$ and $c \notin C(T)$. Either b or $d \in C(T)$. But since i is c-semi-decisive over $\{a,b\}$ and $a \succ_i b$, if $b \in C(T)$, then a also $\in C(T)$, a contradiction. Then $\{d\} = C(T)$. Since j is c-semi-decisive over $\{c,d\}$ and $c \succ_j d$, $c \in C(T)$, a contradiction.

Although we need four options, this does not seem to be a very constraining assumption. For instance with a basic set of two elements α and β, if we have two individuals 1 and 2, a set of options X in a Cartesian product setting could be given by $\{(\alpha,\alpha),(\alpha,\beta),(\beta,\alpha),(\beta,\beta)\}$, a four-option set.

If $\{a,b\} = \{c,d\}$, the result does not apply. Given a profile π and a nonempty subset S of X, let us define the set of Pareto-dominated options, denoted $D_{Par}(S)$, by $D_{Par}(S) = \{y \in S : \text{there is an } x \in S \text{ such that for all } i \in N,\ x \succ_i y\}$. Let us suppose that $\{a,b\} = \{c,d\}$. Assume X is finite. We define a social choice function, f_1, in the following way. For a profile π:

$C(\{a,b\} = \{a\}$ if for all $k \in N$, $a \succ_k b$,
$C(\{a,b\} = \{b\}$ if for all $k \in N$, $b \succ_k a$, and
$C(\{a,b\} = \{a,b\}$ otherwise.

For $S \neq \{a,b\}$, if (i) a and $b \in S$, (ii) there is a third alternative in S, say z, such that for all $k \in N$, $z \succ_k a$ (respectively $z \succ_k b$), and (iii) $a \succ_i b$ or $a \succ_j b$ (respectively $b \succ_i a$ or $b \succ_j b$), then

$C(S) = S - (D_{Par}(S) \cup \{b\})$ (respectively $C(S) = S - (D_{Par}(S) \cup \{a\})$, and
$C(S) - D_{Par}(S)$ otherwise.

One can verify that $C(S) \neq \emptyset$ for all nonempty S and the conditions P and minimal 2-cWV are satified.

[8]Karni (1974) was probably the first author to propose this kind of weakening of the liberalism condition in a working paper that is still unpublished. In Karni's paper the condition applies to subsets of alternatives (and in the aggregation functions framework). This working paper also includes important comments about the Cartesian product structure and unconditional preferences.

Now if $\#X = 2$, let $X = \{a, b\}$. We can define a social choice function, f_2, exactly in the same way as in the previous case when $\{a, b\} = \{c, d\}$, i.e., for a profile π:

$C(\{a, b\} = \{a\}$ if for all $k \in N$, $a \succ_k b$,
$C(\{a, b\} = \{b\}$ if for all $k \in N$, $b \succ_k a$, and
$C(\{a, b\} = \{a, b\}$ otherwise.

If $\#X = 3$, let $X = \{a, b, c\}$. If i and j are c-semi-decisive over the same subset of two options, we can use f_1. If i and j are c-semi-decisive over different subsets of two options, say, i is c-semi-decisive over $\{a, b\}$ and j is c-semi-decisive over $\{b, c\}$, we can define a social choice function, f_3, in the following way. For a profile π:

$C(\{a, b\} = \{a\}$ if for all $k \in N$, $a \succ_k b$,
$C(\{a, b\} = \{b\}$ if for all $k \in N$, $b \succ_k a$, and
$C(\{a, b\} = \{a, b\}$ otherwise;
$C(\{b, c\} = \{b\}$ if for all $k \in N$, $b \succ_k c$,
$C(\{b, c\} = \{c\}$ if for all $k \in N$, $c \succ_k b$, and
$C(\{b, c\} = \{b, c\}$ otherwise;
$C(\{a, c\} = \{a, c\} - P_{Par}(\{a, c\})$.

If either for all $k \in N$, $c \succ_k a$, or for all $k \in N$, $c \succ_k b$,

$C(\{a, b, c\}) = \{c\}$ and,
$C(\{a, b, c\}) = \{a, b, c\} - P_{Par}(\{a, b, c\})$ otherwise.

One can verify that $C(S) \neq \emptyset$ for all nonempty S and the conditions P and minimal 2-cWV are satified.

Our next theorem is, however, negative with exactly three options, if we want that the choice functions satisfy a rationality property borrowed from the revealed preference literature. A choice function satisfies the weak axiom of revealed preference (WARP) if for S and $T \in 2^X_{-\emptyset}$ with $S \subseteq T$, $C(T) \cap S \neq \emptyset \Rightarrow C(S) = C(T) \cap S$ (see, for instance, Arrow (1959, 1984) and Schwartz (1976)).

Definition 11. (Weak Axiom of Revealed Preference, WARP) A social choice function, $f : \Pi' \to \mathbb{C}$, is said to satisfy WARP if all choice functions $C \in \mathbb{C}$ satisfy WARP.

Theorem 3. *Let us assume that $X = \{a, b, c\}$. Then there is no social choice function satisfying U, P, minimal 2-cWV and WARP, provided that the individuals i and j of the definition of c-2WV are not c-semi-decisive over the same subset of options.*

Proof. Let f be a social choice function satisfying U, P, 2-cWV and WARP. Assume that i is c-semi-decisive over $\{a, b\}$ and j is c-semi-decisive over $\{b, c\}$. Consider a profile π such that $a \succ_i b$, $b \succ_j c$ and for all $k \in N$, $c \succ_k a$. Observe that $c \succ_i a \succ_i b$ and $b \succ_j c \succ_j a$. Let $T = \{a, b, c\}$. Since for all $k \in N$, $c \succ_k a$, by condition P, $a \notin C(T)$. Then $b \in C(T)$ or $c \in C(T)$. If $b \in C(T)$, since $a \succ_i b$ and i is c-semi-decisive over $\{a, b\}$, $a \in C(T)$, a contradiction. As a consequence, $C(T) = \{c\}$. But $\{c\} = C(T) \cap \{b, c\}$. By WARP, one should have $C(\{b, c\}) = \{c\}$. Since j is c-semi-decisive over $\{b, c\}$ and $b \succ_j c$, $c \in C(\{b, c\}) \Rightarrow b \in C(\{b, c\})$, a contradiction.

Of course, as in Sen's result, it is straightforward to establish corollaries to Theorems 2 and 3 by replacing minimal 2-cWV by general 2-cWV. These corollaries are omitted.

4. Discussion

In this section, I wish to make some remarks and comments.

First, with Sen-type theorems there is no need to assume that N is finite. Of course, giving practical meaning to N infinite may be hazardous, although in mathematical economics this kind of assumption (even non countable infinite) is routinely made, for instance to model perfect competition.

Second, compared with the framework in Salles (2007), it is interesting to discuss the notions of 2-dictatorship or 2-vetoers. For aggregation functions, Sen's notions of liberalism request that two individuals (or all individuals) are decisive over a subset of two options. This means that, for individual i, for two options a and b, a is socially preferred to b whenever i prefers a to b, and b is socially preferred to a if (s)he prefers b to a. Then, there is some compulsory effect on both a and b at the level of the social preference. The preference, be it individual or social, is a kind of global notion. On the other hand, in the social choice function setting, b is not chosen when a is available whenever i prefers a to b (and a is not chosen when a is available whenever i prefers b to a). This means that one option is rejected, but this does not say that one option is chosen. This is why I refrained from using the word 'dictator' in this context, but preferred 'strong veto'. Of course, when one considers the subset $S = \{a, b\}$, $C(S) = \{a\}$ whenever i prefers a to b. Only in this case, one has something that reminds a local dictatorship. In other cases, the status of a vis à vis the choice set remains uncertain. In the present paper, I introduced the notion of 'weak veto' as a weakening of 'strong veto'. With weak veto, b is not necessarily rejected when a is available whenever i prefers a to b. But if b is not rejected (that is, if b is chosen), then a has to be chosen too. In the aggregation function setting, the weakening of the liberalism condition consists in requesting that a be socially at least as good as b whenever i prefers a to b etc. Again, in the social choice function framework, it is only in the case of $S = \{a, b\}$ that we have either $C(S) = \{a, b\}$ or $C(S) = \{a\}$ whenever i prefers a to b. In the other cases the status of a is linked to the status of b. If b is not chosen, the status of a is again uncertain.

An important feature of the main result obtained in the present paper is that this result is independent of any properties of the choice functions and is, accordingly, exactly of the same kind as the social choice functions version of Sen's Theorem (with the exception of the assumptions on the number of options and on the subsets over which the individuals have a limited weak veto, but I believe that these assumptions are not very constraining). In Salles (2007), the new results were obtained for specific aggregation functions (social welfare functions, or functions whose values were semi-orders or interval orders). These functions request that the social preferences are 'more rational' than the social preferences obtained in the case of Sen's social decision function (incidentally, they are even more rational than the aggregation functions whose values are quasi-transitive social preferences, aggregation functions for which a Pareto extension function gives a kind of counter-example with every individual being a—non limited—vetoer). It is however rather surprising that the assumptions on the number of options and on the subsets over which the individuals have a limited weak veto in Theorem 2 of the present paper are identical to the assumptions made in Theorem 4 in Salles (2007) (for semi-ordered social preferences) and that when $\#X \geq 3$ (this case was dealt with social welfare functions) we need (for $\#X = 3$) a specific assumption on social choice functions (viz. WARP) that plays a major rôle in the rationalizability by complete preorders.

In Salles (2007), I tried to compare Sen-type theorems and Arrovian theorems. I cannot establish this kind of comparison in the present paper, because, Arrovian theorems in the social choice functions setting have recourse to properties of choice functions (path-independence, Chernoff condition, base-acyclicity etc.) [9]

I will not discuss the non-welfaristic aspects of the new results since the comments that one can find in many papers by Amartya Sen, by Pattanaik (1994) and Salles (2007) do apply to these results.

5. Conclusion

In this paper, Sen's liberalism conditions have been weakened in a social choice functions setting where choice functions are obtained, rather than social preferences that are obtained in an aggregation function framework. The social choice functions setting can be justified to some extent by a wish to consider rights as the possibility to prevent something to happen. The proposed weakening does not take us very far since impossibilities will occur for exactly the same kind of functions. In fact Theorem 2 of the present paper can appear as a kind of generalization of Sen's Theorem (Theorem 1 of the present paper), noting however the specific conditions on the number of options and the subsets of options over which two individuals have a weak veto.

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[9]See Blair et al. (1976) and Suzumura (1983).

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