The segregative properties of endogenous jurisdiction formation with a land market

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Abstract

This paper examines the segregative properties of Tiebout-like endogenous processes of jurisdiction formation in presence of a competitive land market. In the considered model, a continuum of households with different wealth levels and the same preferences for local public goods, private spending and housing chooses a location from a finite set. Each location has an initial endowment of housing that is priced competitively and that belongs to absentee landlords. Each location is also endowed with a specific technology for producing public goods.

Households’ preferences are assumed to be homothetically separable between local public goods on the one hand and private spending and housing on the other. Public goods provision is financed by a given, but unspecified, mixture of (linear) wealth and housing taxes. We show that stable jurisdiction structures will be segregated by wealth only if households have a Marshallian demand for any public good (conditionally on the quantities of the other public goods) that is a monotonic function of the price of private spending. We also show that if there are more than one public good, this condition is not sufficient for segregation unless households preferences are additively separable.

Since the GSC condition is necessary and sufficient for the segregation of stable jurisdiction structures without land market and with only one public good, these results suggest that introducing a land market does not affect the segregative properties of endogenous jurisdiction formation.

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1 Introduction

It is widely believed that endogenous processes of jurisdiction formation driven by perfectly mobile households who make a Tiebout (1956) trade-off between local taxes and local public provision are self-sorting or segregative. That is to say, such processes lead to the formation of homogenous jurisdictions inhabited by households with "similar" characteristics. Gravel and Thoron (2007) investigates the validity of this belief in the context of the classical model of endogenous jurisdiction formation due to Westhoff (1977). In this model, households with different wealth and the same preference for local public spending and private spending choose to locate in a finite set of possible places of residence. Households who choose the same location form a jurisdiction that decides local taxes (paid by households on their private wealth) and local public spending. The analysis focuses on stable jurisdiction structures. These are partitions of the set of households that are immune to individual deviations, under the assumption that an individual move has no effect on jurisdictions’ wealth, tax rates and public spending. Gravel and Thoron (2007) identify a condition on households’ preference - the GSC condition - that is necessary and sufficient to ensure the wealth-segregation of any such stable jurisdiction structure. As in Westhoff (1977), the definition of "segregation" used by Gravel and Thoron (2007) is that underlying the notion of consecutiveness (see also Greenberg and Weber (1986)). It defines as "wealth-segregated" a jurisdiction structure in which the richest household of a jurisdiction is weakly poorer than the poorest household of any other jurisdiction with a larger per capita wealth. The GSC condition requires households to consider local public spending to be either always a gross complement, or always a gross substitute, to the private good. In Biswas, Gravel, and Oddou (2012), the necessity and sufficiency of the GSC condition for segregation of stable jurisdiction structures is also established for a generalized version of the model that allows for the presence of a generalized-utilitarian redistributive central government, under the additional assumption that households preferences are additively separable. While the GSC condition is stringent, and may be violated even by additively separable preferences, it is certainly not an outlandish condition. For this reason, Gravel and Thoron (2007) and Biswas, Gravel, and Oddou (2012) results may be seen as providing support to the widespread intuition that endogenous processes of jurisdiction formation à la Tiebout are inherently segregative.

Yet these results are obtained in a model with only one public good and without a dwelling market. In such a model, households are assumed to have the option to settle for free in the jurisdiction that provide them with their favorite public good and tax package. This contrasts somewhat with actual processes of jurisdiction formation in which households must purchase (or rent) housing in order to have access to the public good and
tax package available at a particular place. Does the requirement for households to consume housing in order to benefit from local tax and public good package affect the segregative properties of endogenous processes of jurisdiction formation? More specifically, what are the conditions that households preferences must satisfy in order for any stable jurisdiction structure to be segregative when housing consumption is required for living in a jurisdiction? This is the main question examined in this paper.

A priori, the impact that the introduction of dwelling purchase (or rental) can have on segregation in not clear (see e.g. Schmidheiny (2006a) and Schmidheiny (2006b)). On the one hand, introducing land enlarges the dimension of choices by individual households. This would tend to reduce the congruence of the choices made by different households and, therefore, to mitigate the tendency for these households to self-sort. On the other hand, if housing acquisition is purchased on a market, the inequality of households with respect to their wealth may increase the segregative properties of the jurisdiction formation processes (for instance by excluding poor households from communities that are attractive with respect to their public goods and tax packages). Realism is only one reason for identifying the conditions on households preferences that are necessary and sufficient for the segregation of stable jurisdiction structure in presence of housing markets. Another motivation is that housing markets provide an easy way of testing the conditions through hedonic methods (see e.g. Rosen (1974), Bartik (1987) or, for more recent empirical investigation more connected to the subject matter of this paper, Epple, Romer, and Sieg (2001)).

Addressing the question requires of course a model of jurisdiction formation driven by households’ optimizing decisions with respect to local public good and taxes under perfect mobility that incorporates housing consumption. At least three approaches to this modeling have been proposed in the literature.

The first, explored notably by Rose-Ackerman (1979), Epple, Filimon, and Romer (1984), Epple, Filimon, and Romer (1993), assumes that housing, owned by absentee landlords, is a perfectly divisible good available in (possibly) different quantities in the different locations. Households have preference for housing, local public spending and non-housing private spending and can not consume a positive amount of housing at more than one place. Purchase of housing is made on competitive markets that equalize local supply and demand. Households who consume housing at the same location form a jurisdiction and choose by majority voting a property tax rate which, when applied to the (before tax) market value of local land, finances local public spending. An important issue discussed in this literature is the difficulty of establishing existence of stable jurisdiction structures. Epple, Filimon, and Romer (1984) and Epple, Filimon, and Romer (1993) have provided sufficient conditions on households’ preferences that are sufficient for the existence of stable jurisdiction structures. As shown by these au-
thors, the conditions also guarantee that the stable jurisdiction structures will be "segregated" in the same (consecutive) sense than above. Models that satisfy these assumptions have been the object of intensive empirical research in the last ten years or so (see e.g. Epple and Platt (1998), Epple and Sieg (1999), Epple, Romer, and Sieg (2001) and Epple, Peress, and Sieg (2011)). Yet this research has taken as given the sufficient conditions for existence (and segregation) of stable jurisdiction structures provided by Epple, Filimon, and Romer (1984) and Epple, Filimon, and Romer (1993) and has used these conditions to specify the empirical models that have been estimated or calibrated.

Another approach, explored by Denzau and Parks (1983), Greenberg and Shitovitz (1988) and Konishi (1996) among others, considers a similar setting than the previous stream but with the important difference that local public good provision is assumed to be financed by wealth taxation. To that extent, this setting is closer in spirit to that of Westhoff with which it is easy to compare. Another advantage of this setting is that it eases considerably the problem of existence of stable jurisdiction structures (see for instance Konishi (1996) for a quite general existence theorem). On the other hand, assuming the financing of local public spending by wealth taxation is at odd with what is observed in most institutional settings that we are aware, as property tax is by far the most widely used financing device for local public spending.

The third approach has been proposed by Nechyba (1997). It builds on the (largely unpublished) work of Dunz (1985), Dunz (1986) and Dunz (1989). It differs from the two previous ones in that it assumes that housing is an indivisible good that is available in various types in the various jurisdictions. In such a setting, Nechyba (1997) proves the existence of stable jurisdiction structure under both wealth and dwelling taxation. He also provides conditions that are sufficient for the segregation of stable jurisdiction structures.

In this paper, we stick to the perfectly divisible land (or housing) framework but we consider a financing scheme of local public good provision that combines wealth and dwelling taxation. Moreover, we allow for the possibility that jurisdictions produce several public goods rather than a single one. Yet, we adopt a more general view than the one typically taken in the literature with respect to the mechanism used by local jurisdictions to select public good provisions and taxes. Indeed, except for the linearity of taxation (on both housing and wealth) and the balancing of the local government budget, we do not make any assumption on the process by which taxes and local public good are chosen. By contrast, much of the literature assume that local taxes are chosen by some voting mechanism (e.g. tax rates are the favorite ones of the median individual). It is actually this voting assumption which, together with competitive pricing of lands, creates the existence problems alluded to above. By abstracting from the particular mechanism
used by jurisdictions to decide upon local public good provision and taxes, our analysis escapes from the difficulties raised by possible inexistence of stable jurisdiction structures.

While the abstraction from the particular intra-jurisdiction collective choice goes toward a generalization of the approach favoured in the literature, we conduct the analysis by making the (significantly) simplifying assumption that households preferences are homothetically separable in the sense of Blackorby, Primont, and Russel (1979) (3.4.2) between local public goods on the one hand and private spending (on housing and private consumption) on the other. This assumption is admittedly restrictive. Yet, it is not grossly inconsistent with the available empirical evidence (see for instance Davis and Ortalo-Magné (2010)) that indicates that the budget share devoted to housing is remarkably stable both across locations (who differ in local public good provision) and across households (who differ in their wealth). A stronger version of this assumption is used by Calabrese (2012).

In this framework, we show that the segregative features stable jurisdiction structures are not affected by the introduction of the housing market. For we show that a suitable generalization, to multiple public goods, of the GSC condition remains necessary and sufficient for any stable jurisdiction to be segregated. While we interpret this result as indicative of a somewhat robust connection between the GSC condition and the segregative properties of endogenous jurisdiction formation, we emphasize that the assumption of separable homotheticity is crucial for the result. We emphasize this importance by providing an example showing that the GSC condition is not sufficient for the wealth segregation of stable jurisdiction structure when preferences are not homothetically separable. Moreover, our analysis establishes also that the GSC condition may not be sufficient for segregation if preferences are not additively separable and the number of public goods is larger than one. This suggests, somewhat intuitively, that increasing the number of public goods by which jurisdictions can be distinguished mitigates the segregative feature of endogenous jurisdiction formation.

The plan of the rest of this paper is as follows. In the next section, we introduce the main notation and concepts. Section 3 provides the results and the examples showing that the GSC condition is not sufficient for the segregation of stable jurisdiction structure if the number of public goods is larger than one (if preference are not additively separable) or if preferences are not homothetically separable. Section 4 provides some conclusion.

2 The model

As in the literature, we consider economies with a continuum of households represented by the [0, 1] interval, and we denote by λ the Lebesgue measure
defined over all (Lebesgue measurable) subsets of \([0, 1]\). For any Lebesgue measurable subset \(S\) of \([0, 1]\), we interpret \(\lambda(S)\) as “the fraction of households” in the set \(S\). An economy is made of the three following ingredients.

First, there is a wealth distribution modeled as a Lebesgue measurable, increasing and bounded from above function \(\omega : [0, 1] \to \mathbb{R}_+\) that associates to every household \(i \in [0, 1]\) its strictly positive private wealth \(\omega_i\). Limiting attention to increasing functions is a convention according to which households are ordered by their wealth \((i \leq i' \iff \omega_i \leq \omega_{i'})\).

The second ingredient in the description of an economy is a specification of the households’ preferences, taken to be the same for all households. These preferences are defined over \(k\) local public good quantities (denoted by the vector \(Z = (Z_1, \ldots, Z_k)\)), private spending \((x)\) and housing \((h)\). They are represented by a twice differentiable, strictly increasing and strictly quasi-concave\(^1\) utility function \(U : \mathbb{R}_{++}^{k+2} \to \mathbb{R}\). We sometimes focus on a particular public good \(j\). On such occasions, we may find convenient to write a bundle \(Z\) of public goods as \(Z = (Z_j; Z_{-j})\) where the bundle \(Z_{-j}\) of the \(k - 1\) other public goods is defined by \(Z_{-jh} = Z_h\) if \(h < j\) and \(Z_{-jh} = Z_{h+1}\) if \(h > j\). All preferences considered in this paper are also assumed to satisfy the following first regularity condition.

**Condition 1** If \(k \geq 2\), then, for any bundle \((x, h) \in \mathbb{R}_+^2\) of private goods and any two bundles \(Z\) and \(Z' \in \mathbb{R}_+^k\) of public goods, there exists a public good \(j \in \{1, \ldots, k\}\) for which a quantity \(Z_j \in \mathbb{R}_+\) can be found such that \(U(Z_j; Z_{-j}, x, h) = U(Z', x, h)\).

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\(^1\)A function \(f : A \to \mathbb{R}\) (where \(A\) is some convex subset of \(\mathbb{R}^l\)) is strictly quasi-concave if, for every \(a, b, x \in A\), with \(a \neq b\) and every \(\alpha \in [0, 1]\), \(f(\alpha a + (1 - \alpha)b) > f(x)\) and \(f(b) \geq f(x)\) imply \(f(\alpha a + (1 - \alpha)b) > f(x)\).
This regularity condition, that applies only if there are more than one public good, rules out the possibility for indifference surfaces in the public goods space (conditional on any bundle of private goods) to have "vertical or horizontal" asymptotes in the interior of $\mathbb{R}^k_+$. If indifference surfaces in the public goods space have vertical or horizontal asymptotes, then these asymptotes must be the axes of the plane. For instance, preferences that would generate indifference curves in $\mathbb{R}^2_+$ as in figure 1 above, are ruled out by this condition.

As mentioned above, we also assume that the households preferences are homothetically separable, in the sense of Blackorby, Primont, and Russel (1979) (3.4.2) between the $k$ public goods on the one hand and the two private goods on the other. This assumption amounts to say that, for all $Z \in \mathbb{R}^k_+$ and $(x, h) \in \mathbb{R}^2_+$, $U$ can be written as:

$$U(Z, x, h) = G(Z, \Phi(x, h))$$

(1)

for some twice continuously differentiable increasing and strictly quasi-concave function $G : \mathbb{R}^{k+1}_+ \rightarrow \mathbb{R}$ and some twice continuously differentiable, increasing and homogenous of degree 1 function $\Phi : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+$. For proving the sufficiency of the GSC condition when the number of public goods is larger than 1, we shall assume that households preference are not only homothetically separable but are, also, additively separable so that the function $G$ of expression (1) can be written, for any $Z \in \mathbb{R}^k_+$ and $\phi \in \mathbb{R}_+$, as $G(Z, \phi) = g(Z) + \Gamma(\phi)$ for some increasing and strictly quasi-concave function $g : \mathbb{R}^k_+ \rightarrow \mathbb{R}_+$. and some continuous and increasing function $\Gamma : \mathbb{R}_+ \rightarrow \mathbb{R}$.

We denote by $Z^M_j(p^Z; p_x, p_h, R)$, $x^M_j(p^Z; p_x, p_h, R)$ and $h^M_j(p^Z, p_x, p_h, R)$ the household's Marshallian demands for public good $j$ (for $j = 1, ..., k$), private consumption and housing (respectively) when the prices of local public goods are $p^Z = (p^Z_1, ..., p^Z_k)$, the prices of private spending and housing are $p_x$ and $p_h$ and income is $R$. These Marshallian demands are the - unique under our assumptions - solution of the program:

$$\max_{Z, x, h} U(Z, x, h) \text{ subject to } p^Z Z + p_x x + p_h h \leq R$$

(2)

As such they are (under our assumptions) differentiable functions of their arguments (except, possibly, at the boundary of $\mathbb{R}^{k+2}_+$). We emphasize that we view Marshallian demand functions as a dual way of representing households preference for the $k + 2$ goods rather than as a behavioral description of households behavior. After all real households rarely purchase local public goods on competitive markets.

For any given vector $Z \in \mathbb{R}^k_+$ of public goods, we denote by $V^Z$ the conditional (upon $Z$) indirect utility function defined, for any $(p_x, p_h, y) \in \mathbb{R}^2_+$
\[ V^Z(p_x, p_h, y) = \max_{x,h} U(Z, x, h) \text{ subject to } p_x x + p_h h \leq y \]  

This conditional indirect utility function plays some role in the analysis. It describes indeed the maximal utility achieved by a household endowed with a (net of tax) wealth \( y/p_x \) when living in a jurisdiction offering quantities \( Z \in \mathbb{R}^k_+ \) of the local public goods and (net of tax) dwelling price \( p_h/p_x \). We denote by \( hZ^M \) and \( xZ^M \) the conditional (upon \( Z \)) Marshallian demand functions of the two private goods that solve program (3). Under our assumptions, these conditional Marshallian demands are continuous functions of their three arguments and satisfy all the usual properties of Marshallian demands.

We note also that, thanks to homothetic separability, one can describe program (2) by means of a two-step budgeting procedure (see e.g. Blackorby, Primont, and Russell (1979), ch. 5).

The first step of the procedure is described by the program:

\[ \max_{(Z, \phi) \in \mathbb{R}^k_{+}^2} G(Z, \phi) \text{ s. t. } p^Z \cdot Z + E(p_x, p_h) \phi \leq R. \]  

where the function \( E : \mathbb{R}^2_+ \) is defined by the dual program:

\[ E(p_x, p_h) \phi = E(p_x, p_h, \phi) = \min_{x,h} p_x x + p_h h \text{ s. t. } \Phi(x, h) \geq \phi \]  

As is well-known indeed, the expenditure function associated to a homogeneous utility function is linear in utility. As is also well-known from standard microeconomic theory, the function \( E \) is continuous, homogeneous of degree 1, increasing and concave. Hence, in this first step, the household is depicted as allocating its wealth \( R \) between the local public goods \( Z \) and the "utility of the private goods" (measured by \( \phi = \Phi(x, h) \)), taking as given the public price vector \( p^Z \) and the "aggregate price" \( p^X = E(p_x, p_h) > 0 \) of (the utility of) the private goods. We notice that the function \( E \) is also involved in the definition of the conditional indirect utility function \( V^Z \) defined in program (3). Indeed, it is immediate to verify that \( V^Z \) writes:

\[ V^Z(p_x, p_h, R) = G(Z, \frac{R}{E(p_x, p_h)}) \]  

Denote now by \( Z^*(p^Z, E(p_x, p_h), R) \) and \( \phi^*(p^Z, E(p_x, p_h), R) \) the (unique under our assumptions) solution to program (4). Denote also by \( e(p^Z, p_x, p_h, R) \) the optimal expenditure on the "utility of the private goods" that results from program (4) and that is defined by:

\[ e(p^Z, p_x, p_h, R) = E(p_x, p_h) \phi^*(p^Z, E(p_x, p_h), R) \]
The second step of the procedure consists in solving the program:

$$\max_{x, h} \Phi(x, h) \text{ s. t. } p_xx + p_h h \leq c(p^Z, p_x, p_h, R)$$

(8)

Denote by $x^*(p_x, p_h, c(p^Z, p_x, p_h, R))$ and $h^*(p_x, p_h, c(p^Z, p_x, p_h, R))$ the solution of program (8). Thanks to the homotheticity of the (separable from the public goods) preferences for the private goods represented by the function $\Phi$, we know from standard microeconomic theory that the functions $x^*$ and $h^*$ so defined can be written as:

$$x^*(p_x, p_h, y) = F^x(p_x, p_h)y$$

and

$$h^*(p_x, p_h, y) = F^h(p_x, p_h)y$$

where, thanks to Sheppard’s lemma, $F^x$ and $F^h$ can be written as:

$$F^x(p_x, p_h) = \frac{\partial E(p_x, p_h)/\partial p_x}{E(p_x, p_h)}$$

(9)

and:

$$F^h(p_x, p_h) = \frac{\partial E(p_x, p_h)/\partial p_h}{E(p_x, p_h)}$$

(10)

Moreover, one has:

$$h^{ZM}(p_x, p_h, y) = h^*(p_x, p_h, y)$$

and

$$x^{ZM}(p_x, p_h, y) = x^*(p_x, p_h, y)$$

for any bundle $Z$ of local public goods (Marshallian demands of housing and private spending are independent from public good provision). As a result of theorem 5.8 in Blackorby, Primont, and Rassoul (1979), it follows that:

$$Z^M(p^Z, p_x, p_h, R) = Z^*(p^Z, E(p_x, p_h), R)$$

(11)

$$x^M(p^Z, p_x, p_h, R) = F^x(p_x, p_h)c(p^Z, p_x, p_h, R)$$

(12)

$$h^M(p^Z, p_x, p_h, R) = F^h(p_x, p_h)c(p^Z, p_x, p_h, R)$$

(13)

These results will be used later on.

The definition of the generalized GSC condition requires also that some notation pertaining to the conditional Marshallian demands for public goods be introduced. Specifically or any public good $j = 1, \ldots, k$, and any given specification $\mathbf{Z}_{-j} = (\mathbf{Z}_1, \ldots, \mathbf{Z}_{j-1}, \mathbf{Z}_{j+1}, \ldots, \mathbf{Z}_k) \in \mathbb{R}_+^{k-1}$ of the quantities of the $k - 1$ other public goods, we denote by $Z^M_j(\mathbf{Z}_{-j}; p^Z_j; p_x, p_h, R)$, $x^M_j(\mathbf{Z}_{-j}; p^Z_j; p_x, p_h, R)$ and $h^M_j(\mathbf{Z}_{-j}; p^Z_j; p_x, p_h, R)$ the demands for public good $j$, private spending and housing (respectively) conditional upon $\mathbf{Z}_{-j}$.
These demands are defined as the (unique under our assumptions) solution of program (2) for the conditional utility function \( U^{\mathbf{Z}-j} : \mathbb{R}^3_+ \to \mathbb{R} \) defined, for any \((Z_j,h,x) \in \mathbb{R}^3_+\), by:

\[
U^{\mathbf{Z}-j}(Z_j, x, h) = U(Z_j; \mathbf{Z}_{-j}, x, h)
\]  

(14)

It can be checked that the aforementioned properties of \( U \), including homothetic separability, are also possessed by \( U^{\mathbf{Z}-j} \) for any \( \mathbf{Z}_{-j} \in \mathbb{R}^{k-1}_+ \). We accordingly denote by \( G^{\mathbf{Z}-j} \) the (conditional) specification of the function \( G \) of expression (1) above when \( U \) is replaced by \( U^{\mathbf{Z}-j} \). As in Gravel and Thoron (2007) and Biswas, Gravel, and Oddou (2012), we impose the following second regularity condition on preferences.

**Condition 2** For every public good \( j \in \{1, ..., k\} \) and specification \( \mathbf{Z}_{-j} = (Z_1, ..., Z_{j-1}, Z_{j+1}, ..., Z_k) \in \mathbb{R}^{k-1}_+ \) of the quantities of the \( k - 1 \) other public goods, if there are prices \( p_j^Z \) and \( p_h \) of public good \( j \) and duelling, wealth level \( R \) and a non-degenerate interval \( I \) of strictly positive real numbers for which \( Z_j^M(\mathbf{Z}_{-j}; p_j^Z, p_x, p_h, R) = Z_j^M(\mathbf{Z}_{-j}; p_j^Z, p_x', p_h, R) \) for all prices \( p_x' \) and \( p_x \) in \( I \), then \( Z_j^M(\mathbf{Z}_{-j}; p_j^Z, p_x, p_h, R) = \alpha(p_j^Z, p_h, R) \) for some function \( \alpha : \mathbb{R}^3_+ \to \mathbb{R}_+ \).

This condition is a variant of the regularity condition discussed in Gravel and Thoron (2007). It requires that the only local invariance that the Marshallian demand for any public good \( j \) - conditional upon the quantities of the other public goods - can have with respect to the price of private spending be global. That is, if the conditional (upon the other public goods) Marshallian demand for a public good is locally invariant with respect to the price of private good on some non-degenerate interval of variation of that price, then this conditional demand is in fact everywhere independent from that price.

The last two elements of our description of an economy are a common finite set \( L \) of \( I \) locations available to households together with a specification, for each location \( l \in L \), of the amount of land \( L^l \in \mathbb{R}_+ \) available at \( l \), and, for each public good \( j = 1, ..., k \), of a cost function \( C_j^l : \mathbb{R}_+ \to \mathbb{R}_+ \) of producing public good \( j \) at \( l \). We assume that these cost functions satisfy \( C_j^l(0) = 0 \) and are increasing at every location \( l \) and for every public good \( j \). Allowing the cost of producing a given public good to differ across locations seems natural to us (it is more costly to provide a given access to a sand beach in Paris than in Miami). We assume throughout that the endowments of land belong to absentee landlords that play no role in the economy.

We denote by \( \mathbb{D} \) the domain of all economies \((\omega, U, L, \{L^l, C_j^1, ..., C_j^L\}_{l \in L})\) that satisfy all these assumptions and by \( \mathbb{D}^A \) the subset of \( \mathbb{D} \) made of those economies for which households preferences are additively separable.
A jurisdiction structure for the economy \((\omega, U, L, \{L^l, C^l_1, ..., C^l_k\}_{l \in L})\) is a list \((j, \{p^l, t^l_h, t^l_w, Z^l\}_{l \in L})\) where:

- \(j : [0, 1] \rightarrow L \times \mathbb{R}_{++}\) is a Lebesgue measurable function that assigns to each household \(i\) in \([0, 1]\) a unique combination \(j(i) = (l, h^l_i)\) of a place of residence and a housing consumption at that place of residence and where, for every \(l \in L\):
  - \(p^l \in \mathbb{R}_{++}\) is the (before tax) housing price at location \(l\).
  - \(t^l_h \in \mathbb{R}_{++}\) is the housing tax rate prevailing at location \(l\)
  - \(t^l_w \in [0, 1]\) is the wealth tax rate prevailing at location \(l\)
  - \(Z^l \in \mathbb{R}^k\) is the bundle of local public goods available at location \(l\).

For any such jurisdiction structure, and any \(l \in L\), we denote by \(j^l = \{i \in [0, 1] : j(i) = (l, a)\}\) for some \(a > 0\). Hence \(j^l\) is the (Lebesgue measurable) set of all households who are located at \(l\) in the considered jurisdiction structure. The possibility that \(\lambda(j^l) = 0\) (\(l\) is a "desert" jurisdiction) is of course not ruled out. We restrict attention throughout to feasible jurisdiction structures that satisfy, for every location \(l\), the additional conditions that:

\[
\int_{j^l} h^l_i \, d\lambda \leq L^l \quad \text{and} 
\]

(aggregate housing consumption does not exceed the total amount of land locally available) and:

\[
t^l_h p^l \int_{j^l} h^l_i \, d\lambda + t^l_w \int_{j^l} \omega_i \, d\lambda \geq \sum_{h=1}^k C^l_h(Z^l_h) 
\]

(local tax revenues are sufficient to cover the cost of local public goods provision).

Given an economy \((\omega, U, L, \{L^l, C^l_1, ..., C^l_k\}_{l \in L})\) and a jurisdiction structure \((j, \{p^l, t^l_h, t^l_w, Z^l\}_{l \in L})\) for this economy, we denote by \(H^l = \int_{j^l} h^l_i \, d\lambda\) and \(\omega^l = \int_{j^l} \omega_i \, d\lambda\) the aggregate consumption of land and wealth (respectively) at location \(l\).

We remark that our definition of jurisdiction structures is quite general and covers several models of endogenous jurisdiction formation with a housing market examined in the literature. It covers in particular models like Konishi (1996) where local public good provision is assumed to be financed by (linear) wealth taxation as well as models such as Rose-Ackerman (1979),
Epple, Filimon, and Romer (1984), Epple, Filimon, and Romer (1993), Epple and Romer (1991), Epple and Platt (1998), Epple and Sieg (1999), Epple, Romer, and Sieg (2001), Epple, Peress, and Sieg (2011)) in which local public goods are financed by dwelling (or property) tax only. We believe that allowing for both types of taxation is consistent with what is observed in several real-world institutional settings. For instance several states in the US have "property tax relief" features that reduce the tax burden of specific categories of taxpayers on the basis of their income. Similarly in France, many households are exempt from the so-called "taxe d’habitation" (dwelling tax) on the basis of their income.

We now turn to our definition of a stable jurisdiction structure, which we formally state as follows.

**Definition 1** A feasible jurisdiction structure \((j, \{p_l, t_{h_l}, t_{w_l}, Z_l\}_{l \in L})\) for the economy \((\omega, U, L, \{L^l, C^l_1, ..., C^l_4\}_{l \in L})\) is stable if, for every \(l\) and \(l' \in L\), and every \(i \in j', U(Z', \omega_i(1 - t_{w_l}) - p_l(1 + t_{h_l}^l)h_i^l, h_i^l) \geq VZ'(1, p_l'(1 + t_{h_l}^l), \omega_i(1 - t_{w_l}^l)).\)

In words, a feasible jurisdiction structure is stable if, for every household, the bundle of local public goods, land and private spending obtained in its jurisdiction of residence is preferred to any bundle that it could achieve anywhere else given its wealth, and the prevailing land prices, dwelling taxes, wealth taxes, and local public good provision. We notice that this definition of stability does not assume any specific mechanism for choosing local public good provision and tax rate. By contrast, much of the literature dealing with endogenous formation of jurisdictions assumes that local jurisdiction choose their tax rate and/or public good provision by some voting mechanism. It is well-known that combining a voting mechanism with a competitive pricing of the land may raise serious existence problem, that are exacerbated if voting concerns the housing tax rate (see e.g. Rose-Ackerman (1979), Epple, Filimon, and Romer (1984), Epple, Filimon, and Romer (1993), Epple and Romer (1991) and Epple and Platt (1998)). These existence problems are alleviated here by avoiding the requirement for the tax rate to be result of a voting procedure. For instance the (grand) jurisdiction structure in which all households are put into a single jurisdiction will be stable if a Inada’s condition on one of the public good is assumed (as no household would want to unilaterally move to a desert jurisdiction with zero public goods in that case, even if land is free).

This paper identifies a condition on households preferences under which any stable jurisdiction structure as per definition 1 will be wealth-segregated. This requires of course a definition of wealth-segregation that we take, along with much of the literature, to be the following.
Definition 2 A feasible jurisdiction structure \((j, \{p^l, t_w^l, t_h^l, \mathbf{Z}^l\}_{l \in L})\) for the economy \((\omega, U, \mathbb{I}, \{L_i, C_1^l, ..., C_k^l\}_{l \in L})\) is wealth-segregated if, for every households \(h, i\) and \(k \in [0, 1]\) such that \(\omega_h < \omega_i < \omega_k\), \(h \in j^l, k \in j^l\) and \(i \in j''\) for some \(l' \neq l\) imply that \(V^{\mathbf{Z}^l}(1, p^l(1 + t^l_g), \omega_g(1 - t^l_w)) = V^{\mathbf{Z}^l'}(1, p^{l'}(1 + t^{l'}_g), \omega_g(1 - t^{l'}_w))\) for every \(g \in j^l \cup j^{l'}\).

In words, a jurisdiction structure is wealth-segregated if any jurisdiction \(j^l\) containing two households with strictly different levels of wealth also contains any household with wealth in between of those two or, if it does not contain this household, it is because it resides in another jurisdiction \(j''\) that is "identical" to jurisdiction \(j^l\) in the sense that all households living in the two jurisdiction are indifferent between the two.

In Gravel and Thoron (2007), in a model without housing and with one public good, it was established that the Gross Substitutability/Complementarity (GSC) condition according to which the (non-symmetric) relation of gross substitutability or complementarity (as it may be) of the unique public good vis-à-vis private consumption is independent from all prices is necessary and sufficient for the wealth segregation of any stable jurisdiction structure. In the current context where land is present and where there are, many public goods, it happens that the GSC condition applied to the conditional Marshallian demand of every local public is necessary and sufficient for securing the wealth-segregation - as per definition 2 - of any stable jurisdiction structure as per definition 1. The formal statement of this GSC condition is as follows.

Condition 3 (Generalized GSC) The household’s preference satisfies the generalized GSC condition if, for every local public good \(j\), and every quantities \(\mathbf{Z}_{-j} \in \mathbb{R}_+^{k-1}\) of the other public goods, the function \(Z^M_j(\mathbf{Z}_{-j}; ..)\) is monotonic with respect to \(p_x\).

We notice that, thanks to (11), one can write
\[
Z^M_j(\mathbf{Z}_{-j}; p^Z_j, p_x, p_h, R) = Z^{\mathbf{Z}_{-j}}(p^Z_j; \mathbf{E}^{\mathbf{Z}_{-j}}(p_x, p_h), R) \tag{17}
\]
where the functions \(Z^{\mathbf{Z}_{-j}}\) and \(\mathbf{E}^{\mathbf{Z}_{-j}}\) are nothing else than the functions \(Z^*\) and \(\mathbf{E}\) defined for the utility function \(U^{\mathbf{Z}_{-j}}\). Since the function \(\mathbf{E}^{\mathbf{Z}_{-j}}\) is increasing in its two arguments, requiring the function \(Z^M_j(\mathbf{Z}_{-j}; p^Z_j, p_x, p_h, R)\) to be monotonic with respect to \(p_x\) is equivalent to requiring the function \(Z^{\mathbf{Z}_{-j}}\) to be monotonic with respect to the aggregate private goods price index \(p^Z_{X^*} = \mathbf{E}^{\mathbf{Z}_{-j}}(p_x, p_h)\).

3 Results and examples

We now establish that the generalized GSC condition is necessary and sufficient for the wealth segregation of any stable jurisdiction structure. As it
turns out, this condition will not be sufficient in the most general version of the model presented here. It will be sufficient either if we make the assumption that there is only one local public good, or that the household’s preferences are additively separable between local public goods on the one hand and the private goods on the other.

Yet, as established in the following proposition, the condition will be necessary for segregation.

**Proposition 1** Suppose \((\omega, U, \mathbb{L}, \{L^i, C^i_l, \ldots, C^i_k\}_{i \in \mathbb{L}})\) is an economy in \(\mathbb{D}\) for which all feasible and stable jurisdiction structures are wealth-segregated. Then the utility function \(U\) satisfies the generalized GSC condition.

**Proof.** By contraposition, suppose that \(U\) is a utility function that satisfies the properties required for the economy to belong to \(\mathbb{D}\) but assume that \(U\) violates the generalized GSC condition. This means that there exists a public good \(j \in \{1, \ldots, k\}\), some private spending prices \(p^0_x, p^1_x\) and \(p^2_x\) satisfying \(0 < p^0_x < p^1_x < p^2_x\) for which one has:

\[
Z^M_j((\overline{Z}_{-j}; p^0_j, p^x, p_h, R)) = Z^M_j((\overline{Z}_{-j}; p^2_j, p^x, p_h, R)) > Z^M_j((\overline{Z}_{-j}; p^1_j, p^x, p_h, R))
\]

(18)

(the argument would be similar if we assumed instead \(Z^M_j((\overline{Z}_{-j}; p^2_j, p^x, p_h, R)) = Z^M_j((\overline{Z}_{-j}; p^1_j, p^x, p_h, R))\) for some public good \(j\) price \(p^2_j \in \mathbb{R}^+\), housing price \(p_h \in \mathbb{R}^+\), some vector \(\overline{Z}_{-j} \in \mathbb{R}^{k-1}\) of quantities of the other public goods and some income \(R > 0\). Denote by \(Z^i\) and \(Z^2\) the vectors of public goods defined by:

\[
Z^i = (Z^M_j((\overline{Z}_{-j}; p^j, p^x, p_h, R); \overline{Z}_{-j}) \]

and:

\[
Z^2 = (Z^M_j((\overline{Z}_{-j}; p^2_j, p^x, p_h, R); \overline{Z}_{-j}) = (Z^M_j((\overline{Z}_{-j}; p^1_j, p^x, p_h, R); \overline{Z}_{-j})
\]

Using (17) and homogeneity of degree 0 of Marshallian demands, expression (18) can be written as:

\[
\overline{Z}^j_{-j}^* (q^2_j; \tilde{E}(p^0_x, p_h), 1) = \overline{Z}^j_{-j}^* (q^1_j; \tilde{E}(p^2_x, p_h), 1) > \overline{Z}^j_{-j}^* (q^1_j; \tilde{E}(p^1_x, p_h), 1)
\]

(19)

where:

\[
q^2_j = p^2_j / R
\]

and

\[
\tilde{E}(p^0_x, p_h) = \overline{Z}^j_{-j}^* (p^1_x, p_h) / R
\]

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for \( j = 0, 1, 2 \). Of course, since \( 0 < p_0^0 < p_1^1 < p_2^2 \) and the function \( E^{Z-1} \) is increasing with respect to its arguments, one has \( 0 < E(p_0^0, p_h) < E(p_1^1, p_h) < E(p_2^2, p_h) \). We need to show that we can find an economy in \( \mathbb{D} \) for which a feasible and stable jurisdiction structure can be non-segregated. For this sake, set \( \mathbb{L} = \{1, 2\} \) and let the wealth distribution function \( \omega \) be such that there are \( \alpha \) and \( \beta \in [0, 1] \) satisfying \( \alpha < \beta \) for which one has:

\[
\omega_i = \omega_0 = 1/E(p^0_i, p_h) \quad \text{for all } i \in \{0, \alpha\},
\]

\[
\omega_i = \omega_1 = 1/E(p^1_i, p_h)/R \quad \text{for all } i \in [\alpha, \beta],
\]

\[
\omega_i = \omega_2 = 1/E(p^2_i, p_h)/R \quad \text{for all } i \in [\beta, 1],
\]

and let there be a mass \( \mu_t > 0 \) of household of type \( t \) (for \( t = 0, 1, 2 \)) with the masses chosen in such a way as to satisfy:

\[
\mu_0 \omega_0 + \mu_2 \omega_2 = \frac{1}{q^2_j} = \mu_1 \omega_1 \tag{20}
\]

It is clearly possible to find such positive real numbers \( \mu_t \) (for \( t = 0, 1, 2 \)). One simply set

\[
\mu_1 = \frac{1}{\omega_1 q^2_j} > 0 \tag{21}
\]

and observe that, for any given strictly positive values of \( \omega_0, \omega_1 \) and \( q^2_j \), there are several pairs of positive values of \( \mu_0 \) and \( \mu_2 \) that satisfy:

\[
\mu_2 = \frac{1}{\omega_2} \left( \frac{1}{q^2_j} - \omega_0 \mu_0 \right)
\]

Consider any increasing cost functions \( C^1_g \) (for \( g = 1, \ldots, k \), \( g \neq j \) and \( l = 1, 2 \)) such that \( \sum_{g \neq j} C^1_g(Z_g) = \sum_{g \neq j} C^2_g(Z_g) = c \) for some non-negative real number \( c \) that we leave, for the moment, unspecified. Let also \( C^1_j = C^2_j = C \) for some strictly increasing cost function \( C \) satisfying:

\[
C(Z^Z-1_j(q^2_j; E(p^2_x, p_h), 1)) < \\
C(Z^Z-1_j(q^2_j; E(p^0_x, p_h), 1)) = C(Z^Z-1_j(q^2_j; E(p^0_x, p_h), 1)) < \mu_0 \omega_0 + \mu_2 \omega_2 \tag{22}
\]

Consider the jurisdiction structure \( \{ j, \{p^l, t^l_h, t^l_w, Z^l \}_{i=1,2} \} \) defined by:

\[
\begin{align*}
\gamma(i) &= (2, F^h(p^0_x, p_h) e^{Z-1}(q^2_j, p^0_x, p_h, 1)) \quad \text{for } i \in [0, \alpha], \\
\gamma(i) &= (1, F^h(p^1_x, p_h) e^{Z-1}(q^1_j, p^1_x, p_h, 1)) \quad \text{for } i \in [\alpha, \beta], \\
\gamma(i) &= (2, F^h(p^2_x, p_h) e^{Z-1}(q^2_j, p^2_x, p_h, 1)) \quad \text{for } i \in [\beta, 1],
\end{align*}
\]
\[ p^1(1 + t^1_h) = p^2(1 + t^1_l) = p_h \]  
\[ t^2_w = q^2_j C(Z_j^{-j}(q^2_j; \bar{E}(p^2, p_h), 1)) \]  
and
\[ t^1_w = q^1_j C(Z_j^{-j}(q^1_j; \bar{E}(p^1, p_h), 1)) \]

where the function \( e^{Z^{-j}} \) is the analogue, for the utility function defined by (14), to the function \( e \) defined in expression (7) for program (4). Observe that, thanks to (20) and (22), one has \( t_w \in [0, 1] \) for \( l = 1, 2 \). Observe also that equation (26) leaves complete freedom for choosing before-tax housing price and dwelling tax rates \( p^l \) and \( t^l_h \) satisfying \( p^l(1 + t^l_h) = p_h \) for \( l = 1, 2 \). Set now the quantities of land \( L^1 \) and \( L^2 \) in such a way that:

\[ \mu_1 F^h(p^1, p_h) e^{Z^{-j}(q^1_j; p^1_x, p_h, 1)} = L^1 \]  
and:

\[ \mu_0 F^h(p^2, p_h) e^{Z^{-j}(q^2_j; p^2_x, p_h, 1)} + \mu_2 F^h(p^2, p_h) e^{Z^{-j}(q^2_j; p^2_x, p_h, 1)} = L^2 \]  

Given \( L^l \), set the yet undetermined positive real numbers \( t^l_h \) and \( p^l \) so that:

\[ t^l_h p^l_h L^l = c \]

(for \( l = 1, 2 \)). It is clear that this equality, which says that the cost of producing the public goods other than \( j \) in the two jurisdiction is financed by housing taxation, requires to set \( t^l_h = \frac{c}{p^l_h L^l} \) for \( l = 1, 2 \). Substituting this back into equation (26) yields:

\[ p^l = p_h - \frac{c}{L^l} \]

for \( l = 1, 2 \). It is clear that the before tax housing price \( p^l \) so defined will be positive if the cost functions \( C^l_g \) (for \( g = 1, \ldots, k \) and \( l = 1, 2 \)) are chosen in such a way that \( c \) is sufficiently small. Let us show that this non-segregated jurisdiction structure is stable. Our choice of \( L^1 \) and \( L^2 \) already guarantees (equations (29)-(30)) that land markets clear at the two locations. Since, as was just established, the cost of producing public goods other than \( j \) in the two jurisdictions is exactly covered by housing tax revenues, equations (27)-(28) imply that the total tax raised in each of the two jurisdictions covers exactly the cost of public good provision. This establishes that this jurisdiction structure is feasible. We only need to show that it is stable so that no household has incentive to modify its consumption of public good and private goods either at its location or at the other. We provide the argument for a household of type 1 living at jurisdiction 1, leaving to the reader the task of verifying that the same argument holds for a type 0 and type 2 household living at jurisdiction 2. Consider therefore a household
with private wealth $\omega_1 = 1/\tilde{E}(p^1_2, p_h)$ and located at 1 where it consumes
the bundle $Z^1 = (Z_j^{Z-1}(q_j^2; \tilde{E}(p^1_2, p_h), 1), \bar{Z}_{-j})$ of public goods and has
$\omega_1(1 - t^1_w)$ to spend on housing and private spending. Yet:

$$\omega_1(1 - t^1_w) = \omega_1[1 - q_j^Z C(Z_j^{Z-1}(q_j^2; \tilde{E}(p^1_2, p_h), 1))]$$

$$= \frac{[1 - q_j^Z C(Z_j^{Z-1}(q_j^2; \tilde{E}(p^1_2, p_h), 1))]}{\tilde{E}(p^1_2, p_h)}$$

$$\phi^{Z-1}(q_j^Z; \tilde{E}(p^1_2, p_h), 1)$$

(31)

thanks to the budget constraint associated to program (4) applied to the
conditional utility function (14). Now, since a type 1 household consumes
$F^h(p^1_2, p_h)e^{Z-1}(q_j^2, p^1_2, p_h, 1)$ units of housing (equation (23) purchased at
price $p_h$, such a household has

$$\omega_1(1 - t^1_w) - p_h F^h(p^1_2, p_h)e^{Z-1}(q_j^Z, p^1_2, p_h, 1)$$

available for private spending. Yet we know that:

$$\omega_1(1 - t^1_w) - p_h F^h(p^1_2, p_h)e^{Z-1}(q_j^Z, p^1_2, p_h, 1)$$

$$= \phi^{Z-1}(q_j^Z; \tilde{E}(p^1_2, p_h), 1) - p_h F^h(p^1_2, p_h)(e^{Z-1}(q_j^Z, p^1_2, p_h, 1))$$

$$= p^1_2 F^x(p^1_2, p_h)(e^{Z-1}(q_j^Z, p^1_2, p_h, 1))$$

Since $(F^h(p^1_2, p_h)e^{Z-1}(q_j^2, p^1_2, p_h, 1), F^x(p^1_2, p_h)e^{Z-1}(q_j^2, p^1_2, p_h, 1))$ solves pro-
gram (8), a type 1 - household can not find a bundle of private spending
x and housing h satisfying $p_h h + x \leq \phi^{Z-1}(q_j^Z; \tilde{E}(p^1_2, p_h), 1)) = \omega_1(1 - t^1_w)$
that is strictly preferred to:

$$(F^h(p^1_2, p_h)e^{Z-1}(q_j^2, p^1_2, p_h, 1), p^1_2 F^x(p^1_2, p_h)e^{Z-1}(q_j^2, p^1_2, p_h, 1))$$

Hence a type 1 household has no incentive to change its private consumption
pattern within its jurisdiction. It has also no incentive to move to location
2. Indeed, if it were to move there, it would obtain the bundle:

$Z^2 = (Z_j^{Z-1}(q_j^2; \tilde{E}(p^0_2, p_h), 1), \bar{Z}_{-j}) = (Z_j^{Z-1}(q_j^2; \tilde{E}(p^2_2, p_h), 1), \bar{Z}_{-j})$

for which it would pay $t^2_w \omega_1$ amount of tax and would have $\omega_1(1 - t^2_w)$ units
of numéraire to spend on private matters. Observe now that, thanks to (27)
and (31):

$$[t^1_w + (1 - t^1_w)] \omega_1 = C(Z_j^{Z-1}(q_j^Z; \tilde{E}(p^1_2, p_h), 1)) / \mu_1 + \phi^{Z-1}(q_j^Z; \tilde{E}(p^1_2, p_h), 1)$$

$$= [t^2_w + (1 - t^2_w)] \omega_1$$

$$= C(Z_j^{Z-1}(q_j^Z; \tilde{E}(p^2_2, p_h), 1)) / \mu_1 + (1 - t^2_w) \omega_1$$

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Define now the function $G^{Z^-j} : \mathbb{R}_+^k \rightarrow \mathbb{R}_+^k$ by:

$$G^{Z^-j}(c, \phi) = G^{Z^-j}(C^{-1}(c), \phi)$$

where $C^{-1}$ is the inverse cost function. This function is well-defined if $C$ is strictly increasing. Since $(\bar{Z}_j^{Z^-j*}(q_j^Z; \bar{E}(p_{x_j}^1, p_{h}), 1), \phi^{Z^-j*}(q_j^Z; \bar{E}(p_{x_j}^1, p_{h}), 1))$ solves the program:

$$\max_{(Z, \phi)i \in \mathbb{R}_+^k} G^{Z^-j}(Z_j, \phi) \text{ s.t. } Z_j/\mu_1 + \phi \leq \omega_1.$$ 

it follows from a standard revealed preference argument that

$$G^{Z^-j}(\bar{Z}_j^{Z^-j*}(q_j^Z; \bar{E}(p_{x_j}^1, p_{h}), 1), \phi^{Z^-j*}(q_j^Z; \bar{E}(p_{x_j}^1, p_{h}), 1)) \geq G^{Z^-j}(Z_j^{Z^-j*}(q_j^Z; \bar{E}(p_{x_j}^1, p_{h}), 1))/\mu_1 + (1 - t_{h})/\omega_1.$$ 

Hence type 1 household prefer staying in 1 than moving to 2.

**Remark 1** Conditions 1 and 2 play no role in the proof. Hence the necessity of the generalized GSC condition for the segregation of any stable feasible structure does not ride on these assumptions.

We now turn to the question of the sufficiency of the GSC condition for the segregation of any stable jurisdiction structure. As in Gravel and Thoron (2007) or Westhoff (1977), doing this analysis requires some knowledge of the households preferences defined in the space of all parameters that affect its choice of place of residence. These preferences are described by the conditional indirect utility function defined in (3). Under homothetic separability, we know from (6) that we can write this conditional indirect utility function $V^Z$ as:

$$V^Z(1, q_h, \omega_i(1 - t_{x_j})) = G(Z, \omega_i(1 - t_{x_j}))/\bar{E}(q_h)$$

where $\bar{E}(q_h) = E(1, q_h)$. Suppose now that we focus on some public good $j$ and that we fix the quantities $Z^{-j} \in \mathbb{R}_+^k$ of the other $k - 1$ public goods. We can then represent a typical indifference curve of a household with wealth $\omega_i$ in the space $[0, 1] \times \mathbb{R}_+$ of all combinations of wealth tax rate and public good $j$ by means of the implicit function $z^I : [0, 1] \rightarrow \mathbb{R}$ defined by

$$G^{Z^-j}(z^I(t, \omega_i), \omega_i(1 - t_{x_j}))/\bar{E}(q_h) \equiv a$$

for some $a$. Since $G^{Z^-j}$ is a twice differentiable concave and strictly increasing function of its two arguments, it is clear that the implicit function $z^I$ is
well-defined and twice differentiable. Differentiating (32) with respect to \( t \) yields:

\[
\frac{\partial z^t(\bar{t}, \bar{\omega}_t)}{\partial t} \equiv \frac{\omega_t}{\widehat{E}(q_h) G_{Z_j}^{-1}} \tag{33}
\]

If we now differentiate (33) with respect to \( \omega_t \), we obtain:

\[
\frac{\partial^2 z^t(\bar{t}, \bar{\omega}_t)}{\partial t \partial \omega_t} = \frac{G_{Z_j}^{-1}}{\widehat{E}(q_h) G_{\phi^{-1}}^{-1}} \cdot \frac{1 - \omega_t}{\left(\frac{\omega_t q_{Z_j}}{\widehat{E}(q_h)} - G_{Z_j}^{-1} G_{\phi^{-1}}^{-1}\right)^2} \tag{34}
\]

Notice also that the Marshallian demand for public good \( j \) conditional upon the quantities \( \bar{Z}^{-j} \) of the other \( k - 1 \) public goods can be defined to be the solution of the following program:

\[
\max_{Z_j} G_{Z_j}^{-1}(Z_j, \omega_t - p^Z Z_j) \tag{35}
\]

that is characterized (under our conditions) by the first order condition:

\[
\frac{G_{Z_j}^{-1}}{\widehat{E}(q_h)} \equiv \frac{p^Z_j}{\widehat{E}(q_h)} \tag{36}
\]

If we differentiate identity (36) with respect to \( \widehat{E}(q_h) = p_X \), we obtain (upon manipulations):

\[
\frac{\partial Z^{-j*}(p^Z_j; p_X, \omega_t)}{\partial p_X} = \frac{G_{Z_j}^{-1}}{\widehat{E}(q_h)} \left[ G_{Z_j}^{-1} \frac{1}{\widehat{E}(q_h)} - \frac{\omega_t - p^Z Z_j}{\left(\frac{\omega_t q_{Z_j}}{\widehat{E}(q_h)} - G_{Z_j}^{-1} G_{\phi^{-1}}^{-1}\right)^2} \left(G_{\phi^{-1}}^{-1} G_{Z_j}^{-1} - G_{Z_j}^{-1} G_{\phi^{-1}}^{-1}\right)\right] \tag{37}
\]

If both the generalized GSC and the second regularity condition hold, the sign of the left hand side of identity (37) is the same for all values of \( (p^Z_j; p_X, \omega_t) \) and all quantities \( \bar{Z}^{-j} \) of the other public goods. As the denominator of the right hand side of (37) is negative thanks to the second order condition of program (35), the sign of the right hand side is completely determined by the sign of

\[
\frac{G_{Z_j}^{-1}}{G_{\phi^{-1}}^{-1}} \cdot \frac{\omega_t - p^Z Z_j}{\left(\frac{\omega_t q_{Z_j}}{\widehat{E}(q_h)} - G_{Z_j}^{-1} G_{\phi^{-1}}^{-1}\right)^2} \left(G_{\phi^{-1}}^{-1} G_{Z_j}^{-1} - G_{Z_j}^{-1} G_{\phi^{-1}}^{-1}\right) \tag{38}
\]
which must be constant under the GSC condition. Moreover, the sign of expression (38) also determines the sign of the change in the slope of an indifference curve as per (33) brought about by a change in the household’s wealth. Hence the slope of the indifference curve as per (33) is monotonic with respect to \( \omega_i \). This monotonicity property, which implies that any two indifference curves belonging to households with different wealth can cross at most once, and which is illustrated in figure 2, will play a key role in the proof of proposition 2 below.

![Figure 2](image)

We now establish that, if we restrict attention to economies in \( \mathbb{D}^A \) in which households have additively separable preferences, or if we assume that there is only one local public good, then the generalized GSC condition is sufficient for the wealth segregation of any stable jurisdiction structure.

**Proposition 2** If households preferences satisfy the generalized GSC condition, then any stable jurisdiction structure associated to an economy in \( \mathbb{D}^A \), or to an economy in \( \mathbb{D} \) if \( k = 1 \), is wealth-segregated.

**Proof.** Consider first any economy \( (\omega, U, L, \{L^1, C_1^1, ..., C^1_L, ... \}) \) in \( \mathbb{D}^A \) and a jurisdiction structure \( (j, \{p^j, t^j_h, t^j_{w'}, Z^j_k \}) \) for this economy and denote by \( q^j \) the after-tax dwelling price in jurisdiction \( l \) defined by \( q^j = p^j(1 + t^j_h) \). Proceed by contraposition and assume that the jurisdiction structure is not wealth-segregated. This means that there are households \( a, b \) and \( c \) in \([0, 1]\) endowed with private wealth \( \omega_a < \omega_b < \omega_c \), and two jurisdictions \( l \) and \( l' \), with \( a, c \in j^l \) and \( b \in j^{l'} \) between which not every resident is indifferent. Either this jurisdiction structure is not stable, and the proof is over, or it is stable. If it is stable, then one must have (exploiting the additive separability
of the preferences):

\[ g(\mathbf{Z}') + \Gamma(E(q^*)'(1 - t_{\omega}^*)\omega_a) \geq g(\mathbf{Z}''') + \Gamma(E(q^*)'(1 - t_{\omega}^*)\omega_a) \quad (39) \]
\[ g(\mathbf{Z}') + \Gamma(E(q^*)'(1 - t_{\omega}^*)\omega_b) \leq g(\mathbf{Z}''') + \Gamma(E(q^*)'(1 - t_{\omega}^*)\omega_b) \quad (40) \]
\[ g(\mathbf{Z}') + \Gamma(E(q^*)'(1 - t_{\omega}^*)\omega_c) \geq g(\mathbf{Z}''') + \Gamma(E(q^*)'(1 - t_{\omega}^*)\omega_c) \quad (41) \]

for some continuous and increasing function \( \Gamma : \mathbb{R}_+ \rightarrow \mathbb{R} \) with at least one inequality being strict (because otherwise, this would imply universal indifference between the two locations). Suppose that \( q^* \geq q'' \) (the proof being symmetric if \( q^* < q'' \)). Since both the functions \( \Gamma \) and \( E \) are continuous and increasing, we know from the intermediate value theorem that there exists some \( \bar{t}_{\omega} \in [0; 1] \) such that \( \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})) = \Gamma(\bar{E}(q'')'(1 - \bar{t}_{\omega})) \) It follows that:

\[ g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_a) = g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_a) \quad (42) \]
\[ g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_b) = g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_b) \quad (43) \]
\[ g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_c) = g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_c) \quad (44) \]

Using the first regularity condition on preferences, let \( \bar{Z}_j \) be the amount of some public good \( j \) such that \( g(\mathbf{Z}''_{-j}, \bar{Z}_j) = g(\mathbf{Z}') \). Hence one has:

\[ g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_a) = g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_a) \]
\[ g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_b) = g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_b) \]
\[ g(\mathbf{Z}') + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_c) = g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_c) \]

and, as a result, inequalities (39)-(41) write:

\[ g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_a) \geq g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_a) \quad (45) \]
\[ g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_b) \leq g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_b) \quad (46) \]
\[ g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_c) \geq g(\mathbf{Z}''_{-j}, \bar{Z}_j) + \Gamma(\bar{E}(q^*)'(1 - \bar{t}_{\omega})\omega_c) \quad (47) \]

which violates the single-crossing implication of the generalized GSC condition in the plane of all housing tax rates and quantity of public good \( j \).

We leave to the reader the task of verifying that the same argument can be established under separability only if there is only one public good. \[ \blacksquare \]

Additive separability (along with regularity) plays a key role in the proof if the number of public goods is larger than one. We further emphasize this by providing an example of an economy in \( \mathbb{D} \) (but not in \( \mathbb{D}^4 \)) where a stable jurisdiction structure can be non-segregated even when the GSC condition holds. Hence, the GSC condition is not sufficient for segregation if there are several public goods and if preferences are homothetically separable - but not additively so.
Example 1 Let $k = 2$ and consider an economy $(\omega, U, \mathbb{L}, \{L^l, C^l_1, C^l_2\}_{l \in \mathbb{L}})$ in $\mathbb{D}$ where $L = \{1, 2\}$, $C^l_1(x) = C^l_1(x) = x$ and $C^l_2(x) = C^l_2(x) = x^2/5000$ for all $x \in \mathbb{R}_+$ and $l = 1, 2$ and $U$ is defined by:

$$U(Z_1, Z_2, x, h) = Z_1 + Z_2 \ln(1 + 2(xh)^{\frac{1}{2}})$$

Such a utility function is continuous, increasing and strictly-quasi concave with respect to all its arguments. Furthermore, the function is homothetically separable - but not additively separable - between the 2 public goods on one hand and the two private goods on the other. As a result, one can use the two-step budgeting procedure to establish that maximizing the utility function subject to the budget constraint is equivalent to solving the program:

$$\max_{(Z_1; Z_2; \phi) \in \mathbb{R}_+^3} Z_1 + Z_2 \ln(1 + \phi) \text{ s. t. } p_1^Z Z_1 + p_2^Z Z_2 + p_\phi \phi \leq R \quad (48)$$

where $p_\phi = \bar{E}(p_x, p_h)$. For any amount $\bar{Z}_2$ of public good 2, the Marshallian demand for $Z_1$ conditional on that amount is given by:

$$Z^M_1(\bar{Z}_2; p^Z_1, p_\phi, R) = \frac{R + p_\phi}{p_1^Z} - \bar{Z}_2 \text{ if } \frac{R + p_\phi}{p_1^Z} > \bar{Z}_2$$

$$Z^M_1(\bar{Z}_2; p^Z_1, p_\phi, R) = 0 \text{ otherwise}$$

which is always (weakly) increasing with respect to $p_\phi$. Even though it is difficult to provide an explicit definition of the Marshallian demand for public good 2 (equal to the conditional demand for that public good thanks to additive separability between the two public goods), we can prove that it is always decreasing with respect to $p_\phi$. Indeed, the Marshallian demand for public good 2 conditional upon quantity $\bar{Z}_1$ of public good 1 is the solution of the program:

$$\max_{\bar{Z}_2} Z_2 \ln(p_\phi + R - p_2^Z \bar{Z}_2)$$

and is characterized therefore by the 1st order condition:

$$\ln(p_\phi + R - p_2^Z \bar{Z}_2^M(\bar{Z}_1; \cdot)) - \frac{p_2^Z \bar{Z}_2^M(\bar{Z}_1; \cdot)}{p_\phi + R - p_2^Z \bar{Z}_2^M(\bar{Z}_1; \cdot)} = 0$$

Differentiating this identity with respect to $p_\phi$ and rearranging yields:

$$\frac{\partial \bar{Z}_2^M(\bar{Z}_1; \cdot)}{\partial p_\phi} = \frac{-p_2^Z \bar{Z}_2^M(\bar{Z}_1; \cdot)}{p_\phi + R - p_2^Z \bar{Z}_2^M(\bar{Z}_1; \cdot)} \leq 0$$

Hence, the Marshallian demand for public good 2 conditional on public good 1 is decreasing with respect to the price of the private good and the preference represented by this utility function satisfies the generalized GSC condition.

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Let us construct a stable but yet non-segregated jurisdiction structure for this economy. For this sake, set \( Z_1 = 1/10, Z_2 = 2, p^l = 1 = p^l, \) for \( l = 1, 2, \)
\( t^l_h = 0 = t^l_w, t^l_w = 7/10, Z_1^2 = 0, Z_2^2 = 1 \) and \( t^2_w = 1/1000 \) and define the wealth distribution function \( \omega \) by:

\[
\omega_i = \begin{cases} 
1/10 & \text{if } i \in [0, a[ \\
1 & \text{if } i \in [a, b] \\
4 & \text{if } i \in ]b, 1] 
\end{cases}
\]

where \( a \) and \( b \) (with \( a < b \)) are (for the moment) unspecified elements of the \([0,1]\) interval. Notice that:

\[
Z_1^1 + Z_2^1 \ln(1 + (1 - t^1_w)\omega_i) = \frac{1}{10} + 2\ln(1 + \frac{3}{100}) \approx 0.15912 \\
> 0.095219 \approx \ln(1 + \frac{999}{10000}) \\
= Z_1^2 + Z_2^2 \ln(1 + (1 - t^2_w)\omega_i)
\]

for every \( i \in [0, a[. \) Hence any such household prefers the public goods and tax package offered at 1 to that offered at 2. Any household \( i \in ]b, 1] \) would have just the same preference since for such households:

\[
Z_1^1 + Z_2^1 \ln(1 + (1 - t^1_w)\omega_i) = \frac{1}{10} + 2\ln(1 + \frac{12}{10}) \approx 1.6769 \\
> 1.6086. \approx \ln(1 + \frac{3996}{1000}) \\
= Z_1^2 + Z_2^2 \ln(1 + (1 - t^2_w)\omega_i)
\]

However a household \( i \in [a, b] \) with wealth \( \omega_i = 1 \) would have just the opposite preference because the inequality:

\[
Z_1^1 + Z_2^1 \ln(1 + (1 - t^1_w)\omega_i) = \frac{1}{10} + 2\ln(1 + \frac{3}{10}) \approx 0.62473 \\
< 0.69265. \approx \ln(1 + \frac{999}{1000}) \\
= Z_1^2 + Z_2^2 \ln(1 + (1 - t^2_w)\omega_i)
\]

holds for such a household. Hence the unsegregated jurisdiction structure where \( j(i) = (1, h^*(1, 1, \frac{3}{10})) = (1, \frac{3}{n^2}) \) for \( i \in [0, a[ \), \( j(i) = (2, h^*(1, 1, \frac{999}{1000}) = (2, \frac{999}{2000}) \) for \( i \in [a, b] \) and \( j(i) = (1, h^*(1, 1, \frac{17}{18})) = (1, \frac{3}{2}) \) for \( i \in ]b, 1] \) would be stable if it was feasible. To make it feasible as per equations (15) and (16), one just needs to choose \( a \) and \( b \) so that:

\[
\frac{7a}{100} + \frac{28(1 - b)}{10} = \frac{14(1 - b)}{5} + \frac{7a}{100} = C_1^1(\frac{1}{10}) + C_2^1(2) = \frac{63}{625}
\]

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and 

\[ \frac{b - a}{1000} = C_2^2(1) = \frac{1}{5000} \]

so that \( a = 0.78359 \) and \( b = 0.98359 \) does the trick. As for the land market clearing condition, one simply needs to endow the two regions with aggregate land \( L_1 \) and \( L_2 \) in such a way that the price \( p_h^1 = p_h^2 = 1 \) clear the land market in each of the two regions. That is, \( L_1 \) and \( L_2 \) must be such that:

\[ ah^*(1, 1, \frac{3}{100}) + (1 - b)h^*(1, 1, \frac{12}{10}) = L_1 \]

\[ 0.78359 \times \frac{3}{200} + 0.01641 \times \frac{12}{10} = 0.031446 = L_1 \]

and:

\[ (b - a)h^*(1, 1, \frac{999}{1000}) = L_2 \]

\[ 0.2 \times \frac{999}{2000} = 0.0999 = L_2 \]

In the next - and last - example, the importance of the homothetic separability assumption under which the analysis is conducted is illustrated. Specifically this example shows that the generalized GSC condition is not sufficient for the wealth segregation of any stable jurisdiction structure if preferences are not homothetically separable.

**Example 2** Let \( k = 1 \) and consider an economy \((\omega, U, L, \{L_i, C^i\}_{i \in L})\) in \( \mathbb{D} \) where \( L = \{1, 2\} \), \( C^1 \) and \( C^2 \) are cost functions such that \( C^1(8) = 1 - b + a \) and \( C^2(5) = a - b \) for some real numbers \( a \) and \( b \) satisfying \( 0 < a < b < 1 \) and \( U \) is defined by:

\[ U(Z, x, h) = \ln Z + \ln(1 + x) + \ln h \]

Such a utility function is continuous, increasing and strictly-quasi concave with respect to all its arguments. While this function is (additively) separable between the unique public good and the two private goods, it is not homothetically separable. It is easy to see that Marshallian demands are given by:

\[ Z^M(p_Z, p_x, p_h, R) = \frac{R + p_x}{3p_Z} \]

\[ x^M(p_Z, p_x, p_h, R) = \frac{R - 2p_x}{3p_x} \]

\[ h^M(p_Z, p_x, p_h, R) = \frac{R + 3p_x}{3p_h} \]
if $R \geq 2p_x$ and by:

$$Z^M(p^Z_x, p_x, ph, R) = \frac{R}{2p_x}$$

$$x^M(p^Z_x, p_x, ph, R) = 0$$

$$h^M(p^Z_x, p_x, ph, R) = \frac{R}{2ph}$$

otherwise. Hence the local public good is always a gross substitute to private spending ($Z^M$ is everywhere increasing with respect to $p_x$) so that the generalized GSC condition holds. Let us nonetheless construct a stable but yet non-segregated jurisdiction structure for this economy. For this sake, set $Z^1 = 8$, $Z^2 = 5$, $p^1_h = 2$, $p^2_h = 1 = p^1_x = p^2_x$, $t^1_h = 0 = t^2_h$, $t^1_\omega = 2/5$ and $t^2_\omega = 1/2$, and define the wealth distribution function $\omega$ by:

$$\omega_i = \begin{cases} 
\frac{17}{10} & \text{if } i \in [0, a]\n 
2 & \text{if } i \in [a, b]\n 
5 & \text{if } i \in [b, 1]
\end{cases}$$

with the numbers $a$ and $b$ defined above. Let us show that the unsegregated jurisdiction structure $j(i) = (1, \frac{\omega_i(1-t_i)+1}{2p^i_h}) = (1, \frac{101}{200})$ for $i \in [0, a]$, $j(i) = (2, \frac{\omega_i(1-t_i)+1}{2p^i_h}) = (2, 1)$ for $i \in [a, b]$ and $j(i) = (1, \frac{\omega_i(1-t_i)+1}{2p^i_h}) = (1, 1)$ for $i \in [b, 1]$ would be stable if it was feasible. Indeed one can notice that at this jurisdiction structure, people are consuming land just as per their Marshallian demand conditional upon their public good provision. Notice that for $\omega_i = \frac{17}{10}$:

$$V^Z_1(p^1_x, p^1_h, (1-t^1_\omega)\frac{17}{10}) = \ln 8 + \ln 101 + \ln 101 = 1.4062$$

$$V^Z_2(p^2_x, p^2_h, (1-t^2_\omega)\frac{17}{10}) = \ln 5 + \ln 17 = 0.75377$$

Hence any household in the interval $[0, a]$ prefers living at 1 than moving to 2. A household in the interval $[b, 1]$ also prefers living in 1 than living in 2 since:

$$V^Z_1(p^1_x, p^1_h, (1-t^1_\omega)5) = \ln 8 + \ln 2 + \ln 1 \approx 2.7726$$

$$V^Z_2(p^2_x, p^2_h, (1-t^2_\omega)5) = \ln 5 + \ln 7 + \ln \frac{7}{4} \approx 2.7287$$

However a household belonging to the $[a, b]$ interval and having a wealth of
2 would prefer living in jurisdiction 2. Indeed
\[ VZ_1(p^2_x, p^1_h, (1 - t^1_\omega)^2) = \ln 8 + \ln \frac{11}{20} + \ln \frac{11}{10} \approx 1.5769 \]
\[ < VZ_1(p^1_x, p^1_h, (1 - t^1_\omega)^2) \]
\[ = \ln 5 + \ln 1 + \ln 1 \approx 1.6094 \]

Since the measures of households belonging to the \([0, a\), \([a, b]\) and \([b, 1]\) are \(a, b - a\) and \(1 - b\) respectively, and that \(C^1(8) = (b - 1)5 + a \frac{17}{10} \) and \(C^5(5) = (b - a)2\), this jurisdiction structure satisfies condition (16) of the definition of feasibility. As for condition (15) of the definition of feasibility, just choose initial endowments of land \(L_1\) and \(L_2\) for which the land market clearing conditions:
\[ a \frac{101}{100p^1_h} + (1 - b) \frac{2}{p^1_h} = L_1 \]
and:
\[ (a - b) \frac{1 + 1}{2p^1_h} = L_2 \]
are solved for \(p^1_h = 2\) and \(p^2_h = 1\). This yields:
\[ \frac{101a + (1 - b)200}{200} = L_1 \]
and:
\[ (a - b) = L_2 \]

4 Conclusion

The main lesson of this paper holds in one sentence. If a continuum of households with differing wealth but with the same regular and homothetically additively separable preferences for local public goods, private spending and housing are free to choose their favorite combination of dwelling tax rates, wealth tax rates and local public good provision, then any stable jurisdiction structure that results from such a free choices will involve perfect wealth stratification of those households if and only if the households preferences satisfies the generalized GSC condition. Put plainly, for homothetically separable preferences, introducing land does not affect the segregative properties of endogenous process of jurisdiction formation. Yet, as illustrated by the examples, the generalization of the GSC condition is not sufficient to ensure the segregation of any stable jurisdiction structure if there is more than one public good and preferences are homothetically separable but not additively so (example 1), or if preferences are not homothetically separable (example 2). While we believe this main lesson to be of some interest, it is clear that more work needs to be done for identifying the (stronger) condition is necessary or sufficient for segregation in the case where households
preferences are not homothetically separable. It would also be important to test whether the GSC condition is actually verified by households who populate the jurisdictions of the real world. The fact that we have provided such condition in a model with competitive land market opens up the way for empirical testing using the housing market, using a similar methodology than the pioneering one proposed by Eppele, Peress, and Sieg (2011). We plan to provide these empirical tests in our future work.

References


