Long-term care and lazy rotten kids

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February 2013

Abstract

This paper studies the determination of informal long-term care (family aid) to dependent elderly in a worst case scenario concerning the "harmony" of family relations. Children are purely selfish, and neither side can make credible commitments (which rules out efficient bargaining). The model is based on Becker's "rotten kid" specification except that it explicitly accounts for the sequence of decisions. In Becker's world, with a single good, this setting yields efficiency. We show that when family aid (and long-term care services in general) are introduced the outcome is likely to be inefficient. We identify these inefficiencies by comparing the *laissez-faire* (subgame perfect) equilibrium to the first-best allocation. We initially assume that families are identical *ex ante*. However, the case where dynasties differ in wealth is also considered. We study how the provision of long-term care (LTC) can be improved by public policies under various informational assumptions. Interestingly, crowding out of private aid by public LTC is *not* a problem in this setting. With an operational bequest motive, public LTC will have no impact on private aid. More amazingly still, when the bequest motive is (initially) not operational, public insurance may even enhance the provision of informal aid.

Keywords: Long-term care, optimal taxation

***JEL*-Classification:** D13, H21, I13

1 Introduction

Long-term care (LTC) concerns people who depend on help to carry out daily activities such as eating, bathing, dressing, going to bed, getting up or using the toilet. It is delivered informally by families – mainly spouses, daughters and step-daughters – and, to a lesser extent, formally by care assistants, who are paid under some form of employment contract. Formal care is given at home or in an institution (such as care centers and nursing homes). The governments of most industrialized countries are involved in either the provision or financing of LTC services, or often both, although the extent and nature of their involvement differs widely across countries.

In the future, the demand for formal LTC services by the population is likely to grow substantially. LTC needs start to rise exponentially from around the age of 80 years. The number of persons aged 80 years and above is growing faster than any other segment of the population. As a consequence, the number of dependent elderly at the European level (EU–27) is expected to grow from about 21 million people in 2007 to about 44 millions in 2060 (EC 2009). We thus anticipate increasing pressure on resources demanded to provide LTC services for the frail elderly, and this pressure will be on the three institutions currently financing and providing LTC services: the state, the market and the family.

These three institutions have their pluses and minuses. The family provides services that are warm and cheap. However, these services are restricted to each individual's family circle. Furthermore, some families are very poor and some dependent persons cannot count on family solidarity at all. The market can be expensive, particularly where it is thin and without public intervention, it only provides services to those who can afford it. The state is the only institution that is universal and redistributive, but quite often its information is limited and its means of financing are distortionary.

In assessing the adequacy of LTC financing and provision and in making projections, it is important to bear in mind the extent to which countries will be able in the future to rely on the informal provision of care. Most seniors with impairments reside in their home or that of their relatives, and they rely largely on volunteer care from family. These include seniors with severe impairments (unable to perform at least four activities of daily living). An important feature that is often neglected is the real motivation for family solidarity. For long, we have adopted the fairy tale view of children or spouses helping their dependent parents with joy and dedication, what we call pure altruism. We now increasingly realize that family solidarity is often based on forced altruism (social norm) or on strategic considerations (reciprocal altruism); see for instance Cox (1987).

Knowing the foundation of altruism is very important to see how family assistance will react to the emergence of private or public scheme of LTC insurance. For example, the introduction of LTC social insurance is expected to crowd out family solidarity based on pure altruism, where solidarity is based on strategic exchanges. The existing literature concentrates on strategic bequest type models with full commitment leading to efficient bargaining. In reality this appears to be a rather strong assumption; see Chami (1996).

In this paper, we study the determination of family aid in what can be considered a worst case scenario as to the "harmony" of family relations. Children act in a purely selfish way and neither side can make credible commitments (which would open the possibility of efficient bargaining as in the strategic bequest setting). The model we use is based on Becker's (1974) "rotten kid" specification (see also Bergstrom, 1989; 1996) except that we explicitly account for the sequence of decisions (like Bruce and Waldman, 1990). In Becker's world, with a single good, this setting yields an efficient outcome, even in the absence of commitment and when a child is purely selfish. We show that when family aid (and LTC services in general) are introduced the outcome is likely to be inefficient. This is particularly true when the parents value their children's care more than the market substitutes. We study how this inefficiency can be corrected by public policies under various informational assumptions. For most of the paper, we assume that families are identical *ex ante*. However, the case where dynasties differ in wealth is also considered.

The first justification for public intervention that appears is to overcome inefficiencies in the private insurance market. When – as is typically observed in reality – private LTC insurance involves significant loading costs, the *laissez-faire* solution implies insufficient insurance. Second, public policy has to account for the inefficiencies of informal aid. The conventional wisdom is that public policy often creates or at least enhances such inefficiencies through crowding out. In our setting, however, the relationship between public LTC and family aid is more complex. As long as the bequest motive is operational, the children do provide some informal aid to their parents, however its level is too low, except when the full impact of aid is captured by its monetary valuation in the parent's utility. Children's labor supply is then also inefficient, but this problem is not directly connected to the potential need for LTC. When the bequest motive is not operative, no family aid will be provided and the case for public intervention will be even stronger. Interestingly, this failure of family aid may effectively be related to private market inefficiencies. Particularly, an individual who cannot afford to buy insurance coverage to cover the potential monetary cost of LTC may be subject to a "double punishment". In case of dependency the individual will then not only run out of resources, but he can also not count on any family aid (since he has no resources to leave a bequest). Public aid may then even result in a positive bequest and thus bring about a positive level of aid. To sum up, crowding out of private aid by public LTC is not a problem in this setting. With an operational bequest motive, public LTC will have no impact on private aid. More amazingly still, when the bequest motive is (initially) not operational, public insurance may even enhance the provision of informal aid.

Either way, the effectiveness and the design of public LTC depend on the available instruments which is ultimately of course a question of information. We first study the implementation of the first-best (FB) under *full information.* Though of limited realism this is an interesting benchmark to show which instruments are necessary within this setting of multi-stage strategic interaction to achieve the efficient solution. We show that the FB can decentralized by a lump-sum transfer from the dependent to the healthy elderly supplemented by linear subsidies on labor incomes (of the young) and aid. Lump sum transfer are determined to mimic fair private insurance. Next we look at a second-best solution which is achieved when aid is not observable (and thus cannot not be subsidized). The set of instruments now consists of a lump-sum tax on the healthy old and linear taxes on child's income finance public LTC provision to dependent parents. We show that transfers are used to achieve full insurance of the old. The tax on labor, on the other hand, is not used to raise revenue, but because it increases informal aid (which becomes more attractive when market labor is taxed). The level of the tax is then set to strike a balance between the deadweight loss of the labor tax and the benefits associated with its effect on aid. Finally, we turn to a setting where individuals are heterogenous and dynasties differ in wealth. This adds an extra potential justification for public intervention, namely redistribution. It also makes a case where some individuals cannot afford private LTC coverage more plausible and we can have an initial equilibrium in which the bequest motive is operational for some individuals and not for others. We assume that the government does not observe wealth, but all other variables ad consider a two-types setting. We show that the second-best allocation achieved under this information structure implies a first-best tradeoff for labor supply and informal care. Furthermore, bequests in rich families are not distorted (neither taxed nor subsidized *at the margin*), but there is a downward distortion on bequests in poor families. In other words bequests left by low wealth parents are subject to a postive *marginal* tax. Finally, high wealth individuals are fully insured; the insurance decision of the low wealth parents, on the other hand is distorted (and the sign of this distortion does not appear to be unambiguous).

2 The model

Consider two generations consisting of a single parent (subscript 'p') and a single child (subscript 'c'). While the child is selfish, the parent is a pure altruist. The parent is retired and has accumulated wealth ω. He faces the probability π of becoming depending and needing long-term care. The need of LTC requires expenditures of amount L. LTC insurance coverage can be bought on the private market for a price $p \geq \pi$. For $p = \pi$ LTC insurance is actuarially fair. The parent decides how much LTC insurance coverage I to buy, while the child decides how much labor to supply and how much informal care to provide. Informal care not only reduces LTC expenditures by $h(a) < L$, where $h' > 0$ and $h'' < 0$, but may also reduce the (utility) loss $\gamma q(a) < 0$ the parent suffers from LTC (with $q' > 0$, $q'' < 0$). The strength of this second effect is measured by the parameter $\gamma \geq 0$. The parent can "enforce" informal care provision by a bequest b. The child, earns income $w\ell$ where w reflects the child's wage rate and ℓ labor supply. Labor supply as well as informal care provision come along with disutility captured by v, with $v' > 0, v'' > 0$. The altruistic parent maximizes the following welfare function

$$W_p = U_p + U_c,$$

where individual utility of the parent U_p is given by

$$U_p = \pi \left[u(\omega - pI + I - L + h(a) - b) + \gamma q(a) \right] + (1 - \pi) u(\omega - pI - \widehat{b}),$$

where a $\widehat{}$ indicates the state of staying healthy. Utility of the child is given by

$$U_c = \pi\left[u(w\ell + b) - v(\ell + a)\right] + (1-\pi)\left[u(w\widehat{\ell} + \widehat{b}) - v(\widehat{\ell})\right].$$

The timing of the model is as follows: first the government announces its policy. Then, the parent and the child play the following three stage game. In stage 1, the parent decides how much how much LTC insurance coverage I to buy. In stage 2, the state of nature is revealed, that is, the parent is either disabled or not. Then, the child decides how much labor to supply ℓ and $\widehat{\ell}$ and how much informal care a to provide if the parent is dependent. Finally, in stage 3 the parent decides the level of bequests $b \geq 0$ and $\widehat{b} \geq 0$. To determine the subgame perfect Nash equilibrium we solve this game by backward induction.

3 Laissez-faire allocation

3.1 Stage 3: optimal bequests

The parent is either healthy or dependent. In both states of nature he observes the child's labor income and informal care provision (in case he requires LTC). The parent chooses his optimal bequests by maximizing equation (1) when he is healthy and equation (2) when he requires LTC:

$$\max_{\widehat{b}} \quad \widehat{W_p} = u(\omega - pI - \widehat{b}) + u(w\widehat{\ell} + \widehat{b}) - v(\widehat{\ell}) \tag{1}$$

$$\max_{b} \quad W_p = u(\omega + (1-p)I - L + h(a) - b) + \gamma q(a) + u(w\ell + b) - v(\ell + a). \tag{2}$$

Assuming and interior solution, the optimal bequests in each state of nature are implicitly given by

$$\frac{\partial \widehat{W_p}}{\partial \widehat{b}} = -u'(d_p) + u'(d_c) = 0, \tag{3}$$

$$\frac{\partial W_p}{\partial b} = -u'(m_p) + u'(m_c) = 0. \tag{4}$$

Recall that bequests are restricted to be nonnegative, and one obtains from (1) and (2)

$$\widehat{b} > 0 \iff \omega - pI > w\widehat{\ell},$$

$$b > 0 \iff \omega + (1-p)I - L + h(a) > w\ell.$$

In words, the net resources of the parents (including LTC cost and the monetary value of informal aid, if any) must be larger than that of the children. Let $\widehat{b}^* \equiv \widehat{b}(I, \widehat{\ell})$ and

$b^* \equiv b(I,\ell,a)$ denote the optimal bequest levels. When the solution is interior, the derivatives with respect to LTC insurance coverage, labor supply and informal care are as follows

$$\frac{\partial b^*}{\partial a} = \frac{u''(m_p)h'(a)}{u''(m_p)+u''(m_c)} = \frac{h'(a)}{2} \tag{5}$$

$$\frac{\partial b^*}{\partial \ell} = \frac{-u''(m_c)w}{u''(m_p)+u''(m_c)} = -\frac{w}{2} \equiv \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}} \tag{6}$$

$$\frac{\partial b^*}{\partial I} = \frac{(1-p)u''(m_p)}{u''(m_p)+u''(m_c)} = \frac{1-p}{2}, \quad \frac{\partial \widehat{b}^*}{\partial I} = -\frac{p}{2}. \tag{7}$$

When the child increases his LTC provision the parent increases his bequest by half of the return. On the other hand, if the child increase his labor supply and thereby his income, then half of this additional income is taxed away by a reduction in the parent's bequest. The costs or net return of LTC insurance coverage are equally divided between the parent and the child.

3.2 Stage 2: optimal labor supply and informal care provision

The child takes into account the bequest he gets from the parent and chooses labor supply and informal care by maximizing

$$\max_{\widehat{\ell}} \quad \widehat{U}_c = u(w\widehat{\ell} + \widehat{b}(I,\widehat{\ell})) - v(\widehat{\ell}), \tag{8}$$

$$\max_{\ell,a} \quad U_c = u(w\ell + b(I,\ell,a)) - v(\ell + a). \tag{9}$$

The FOCs with respect $\widehat{\ell}$, ℓ and a are given by

$$\frac{\partial \widehat{U}_c}{\partial \widehat{\ell}} = u'(d_c)\left(w + \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}}\right) - v'(\widehat{\ell}) = 0 \tag{10}$$

$$\frac{\partial U_c}{\partial \ell} = u'(m_c)\left(w + \frac{\partial b^*}{\partial \ell}\right) - v'(\ell + a) = 0 \tag{11}$$

$$\frac{\partial U_c}{\partial a} = u'(m_c)\frac{\partial b^*}{\partial a} - v'(\ell + a) = 0 \tag{12}$$

Equations (10) and (11) show that the anticipation of a positive bequest reduces the child's marginal benefits of labor supply. This is because the parent lowers the bequest as child's income increase. In other words, part of the child's extra revenue is "taxed away" by the parents. Equation (12) implies that without bequest, it is never optimal for the selfish child to provide LTC. If the bequest motive is operative (b is determined by an interior solution) then the amount bequeathed increases with informal care provision since informal care reduces the monetary costs of LTC; see equation (5).

When the bequest motive is operational equations (5) to (7) imply

$$h'(a) = w \tag{13}$$

$$u'(m_c)\frac{w}{2} = v'(\ell + a) \tag{14}$$

$$u'(d_c)\frac{w}{2} = v'(\widehat{\ell}). \tag{15}$$

These expressions implicitly determine labor supply in both states of nature, as functions of I (set in the previous stage of the game): $\ell^* \equiv \ell(I)$ and $\widehat{\ell^*} \equiv \widehat{\ell}(I)$. Equation (13) determines informal care a^* which is independent of LTC insurance coverage. Since the child acts completely selfish when determining his informal care provision, he only takes into consideration its effect on bequests. Recall that $\partial b^*/\partial a = h'(a)/2$ which means that the child will receive half of the monetary value his aid represents to the parent. However, since wage income is also "taxed away" at 50% the tradeoff represented by equation (13) is effectively the efficient one as far the monetary value of aid is concerned. The non-monetary value of aid (that arises when $\gamma > 0$) is not taken into account.

The comparative statics of labor supply and informal care with respect to I (under an operational bequest motive) are given by

$$\frac{\partial \ell^*}{\partial I} = \frac{-u''(m_c)\left(w + \frac{\partial b^*}{\partial \ell}\right)\frac{\partial b^*}{\partial I}}{u''(m_c)\left(w + \frac{\partial b^*}{\partial \ell}\right)^2 - v''(\ell + a)} = \frac{-u''(m_c)(1-p)w}{u''(m_c)w^2 - 4v''(\ell + a)} < 0 \tag{16}$$

$$\frac{\partial \widehat{\ell^*}}{\partial I} = \frac{-u''(d_c)\left(w + \frac{\partial \widehat{b^*}}{\partial \widehat{\ell}}\right)\frac{\partial \widehat{b^*}}{\partial I}}{u''(d_c)\left(w + \frac{\partial \widehat{b^*}}{\partial \widehat{\ell}}\right)^2 - v''(\widehat{\ell})} = \frac{u''(d_c)pw}{u''(d_c)w^2 - 4v''(\widehat{\ell})} > 0. \tag{17}$$

Since LTC insurance is a net benefit for the parent in the state of dependency it increases the parent's bequest an thereby reduces labor supply incentives of the child. In the state of being healthy, by contrast, LTC insurance is only a cost which in turn reduces the bequest and the child becomes less rotten.

Finally, when the bequest motive is not operational, we obtain

$$a = 0, \tag{18}$$

$$u'(m_c)w = v'(\ell + a), \tag{19}$$

$$u'(d_c)w = v'(\widehat{\ell}). \tag{20}$$

In words, there is no aid, but labor supply decisions are efficient (there is no implicit tax anymore). Interestingly, once the bequest motive becomes operational, its actual level is of no relevance for the level of family aid; see equation (13).

3.3 Stage 1: optimal LTC insurance coverage

We now turn to the first stage in which the parent chooses LTC insurance to maximize the following welfare function

$$\begin{aligned}\max_{I} \quad W_p =& \pi\left[u(\omega + (1-p)I - L + h(a^*) - b^*) + \gamma q(a^*) + u(w\ell^* + b^*) - v(\ell^* + a^*)\right] \\ &+ (1-\pi)\left[u\left(\omega - pI - \widehat{b}^*\right) + u\left(w\widehat{\ell}^* + \widehat{b}^*\right) - v\left(\widehat{\ell}^*\right)\right]. \end{aligned} \tag{21}$$

In this function a^*, b^*, ℓ^*, $\widehat{b}^*$ and $\widehat{\ell}^*$ are determined at the equilibrium of the subsequent stages which, as described in the previous subsections, is contingent on the level of I set in the first stage. The FOC of (21) with respect to I is given by

$$\begin{aligned}\frac{\partial W_p}{\partial I} =& \pi u'(m_p)\left[1 - p - \frac{\partial b^*}{\partial I} - \frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^*}{\partial I}\right] + (1-\pi)u'(d_p)\left[-p - \frac{\partial \widehat{b}^*}{\partial I} - \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^*}{\partial I}\right] \\ &+ \pi\left[u'(m_c)\left[w\frac{\partial \ell^*}{\partial I} + \frac{\partial b^*}{\partial I} + \frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^*}{\partial I}\right] - v'(\ell^* + a^*)\frac{\partial \ell^*}{\partial I}\right] \\ &+ (1-\pi)\left[u'(d_c)\left[w\frac{\partial \widehat{\ell}^*}{\partial I} + \frac{\partial \widehat{b}^*}{\partial I} + \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^*}{\partial I}\right] - v'(\widehat{\ell}^*)\frac{\partial \widehat{\ell}^*}{\partial I}\right] \end{aligned} \tag{22}$$

With the envelope theorem, this expression reduces to

$$\begin{aligned}\frac{\partial W_p}{\partial I} &= \pi u'(m_p)\left[1 - p - \frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^*}{\partial I}\right] - (1-\pi)u'(d_p)\left[p + \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^*}{\partial I}\right] = 0 \\ \Leftrightarrow \quad \frac{u'(m_p)}{u'(d_p)} &= \frac{(1-\pi)\left(p + \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^*}{\partial I}\right)}{\pi\left(1 - p - \frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^*}{\partial I}\right)} \end{aligned} \tag{23}$$

Interestingly, equation (23) implies full insurance ($m_p = m_c$) when private insurance is fair ($\pi = p$). To see this observe that under full insurance (14)–(15) yield $\ell + a = \widehat{\ell}$. It then follows immediately from (6)–(7) together with (16)–(17) that full insurance is a solution to equation (23). This result is not particularly surprising; obtaining full insurance in a fair market is a rather common result. On the other hand, in our setting it is not obvious at first glance because of the multi-stage nature of the game. Full insurance is only optimal (and for that matter feasible) when the symmetry between states of nature sought in the first stage is not destroyed by the subsequent strategic interactions. This happens to be the case in our setting in particular because total labor supply (market labor plus aid) will be the same in both states of nature.

When $\pi > p$, we obtain $m_p < d_p$ implying less than full insurance. To see this we take equation (23) and evaluate it at full insurance $u'(d_p) = u'(m_p)$

$$\left.\frac{\partial W_p}{\partial I}\right|_{I:=u'(m_p)=u'(d_p)} = (\pi - p)u'(m_p)\left[1 - \frac{u''(m_c)w^2}{2u''(m_c)w^2 - 8v''(\ell + a)}\right] < 0$$

which is negative since the first term in brackets is negative for $\pi < p$ and the second term in brackets is positive. In other words, a marginal reduction in I, from its full insurance level increases welfare. Consequently, the parents' optimal choice involves lower than full insurance.[1]

In order to put this *laissez-faire* solution in perspective we now turn to the first-best solution.

4 First-best solution

With *ex ante* identical families, we can define the optimal allocation as the one maximizing the expected utility of a representative dynasty. This problem can be written as

$$\begin{aligned} \max \;\; & W^{fb} = \pi\left[u(m_p) + \gamma q(a) + u(m_c) - v(\ell + a)\right] + (1 - \pi)\left[u(d_p) + u(d_c) - v(\widehat{\ell})\right] \\ \text{s.t.} \;\; & \omega + (1 - \pi)w\ell + \pi\left(w\widehat{\ell} + h(a)\right) = \pi(m_p + m_c - L) + (1 - \pi)(d_p + d_c), \end{aligned} \tag{24}$$

where the decision variables are m_p, m_c, d_p, d_c, a, ℓ and $\widehat{\ell}$. We set all variables directly, assuming full information and disregarding the multi-stage structure of the game. However, the specification of the game will of course be relevant below when we study the decentralization of the optimum. Denoting the Lagrange multiplier associated with the resource constraint (24) by μ the FOC characterizing the solution can be written as

[1] As long as W_p is a concave function of I (and given that in stage 1, the parent's problem is single dimensional).

follows

$$u'(m_p^{fb}) = u'(m_c^{fb}) = u'(d_p^{fb}) = u'(d_c^{fb}) = \mu, \tag{25}$$

$$w = \frac{v'(\ell^{fb} + a^{fb})}{\mu} = \frac{v'(\ell^{fb} + a^{fb})}{u'(m_c^{fb})}, \tag{26}$$

$$w = h'(a^{fb}) + \frac{\gamma q'(a^{fb})}{\mu}, \tag{27}$$

$$w = \frac{v'(\widehat{\ell}^{fb})}{\mu} = \frac{v'(\widehat{\ell}^{fb})}{u'(d_c^{fb})}. \tag{28}$$

These expressions are pretty much self-explanatory. Equation (25) states the equality of marginal utilities of incomes across individuals and states of nature (full insurance). Equations (26) and (28) are the usual conditions describing the efficient choice of labor supply. We obtain a similar condition for a, except that this variable involves both monetary benefits $h'(a)$ and a utility gain $\gamma q'(a)$, which translates into the marginal rate of substitution term on the RHS of equation (27).

The following proposition summarizes the main results of Sections 3 and 4 and specifically compares the equilibrium to the optimal allocation.[2]

Proposition 1 *The laissez-faire solution (subgame perfect equilibrium) of the three stage game with altruistic parents and selfish children has the following properties*

(i) When private insurance is fair ($\pi = p$) there is full insurance ($m_p = m_c$); otherwise insurance is less than full ($m_p < d_p$).

(ii) When the bequest motive is operational

(a) we have $a > 0$ and defined by $h'(a) = w$. Consequently, informal aid is efficient when $\gamma = 0$; otherwise it is too low because the utility benefits ($\gamma q'(a)/u'(m_p)$ valued by parents are not accounted for.

(b) children's market labor supply decisions in both states of nature are inefficient; there is a downward distortion because children face an implicit tax of 50% on their labor income (via a reduction in bequests).

(iii) When the bequest motive is not operational (so that $b = \widehat{b} = 0$)

(a) we have $a = 0$; no informal aid is provided. Consequently aid provision is always inefficient.

(b) children's market labor supply decisions are efficient in both states of nature; they no longer face any implicit tax on their labor incomes.

[2]The comparison follows directly form expression (13)–(15), (18)–(20) and (25)–(28).

4.1 The case for public LTC policy

The results summarized in Proposition 1 provide us with a basis on which we can build to assess the opportunity and the design of public LTC policy. The first justification for public intervention that appears is to overcome inefficiencies in the private insurance market. When, as is typically observed in reality, $\pi > p$ (see, *e.g.* Brown and Finkelstein, 2007) the *laissez-faire* solution implies insufficient insurance. As long as the bequest motive is operational, the children do provide some informal aid to their parents, however its level is too low, except when $\gamma = 0$ (so that the full impact of aid is captured by its monetary valuation in the parent's utility). Children's labor supply is then also inefficient, but this problem is not directly connected to the potential need for LTC. When the bequest motive is not operative, no family aid will be provided and the case for public intervention will be even stronger. Interestingly this failure of family aid may effectively be related to private market inefficiencies. To see this assume that $pL > \omega > \pi L$. In that case the individual cannot afford to buy insurance coverage to cover the potential monetary cost of LTC. In case of dependency the individual will then not only run out of resources but he can also not count on any family aid (since he has no resources to leave a bequest). However, as long as $\omega > \pi L$ the individual can afford public coverage at a fair rate. Interestingly, this may even result in a positive bequest and thus bring about a positive level of aid. To sum up, crowding out of private aid by public LTC is not a problem in this setting. With an operational bequest motive, public LTC will have no impact on private aid. More amazingly, when the bequest motive is (initially) not operational, public insurance may even enhance the provision of informal aid.

Either way, the effectiveness and the design of public LTC depend on the available instruments which is ultimately of course a question of information. In the remainder of the paper we shall first study the implementation of the FB under full information. Though of limited realism this is an interesting benchmark to show which instruments are necessary within this setting of multi-stage strategic interaction to achieve the efficient solution. Next, we look at a second-best solution which is achieved when a is not observable (and thus cannot be subsidized). Finally, we turn to a setting where individuals are heterogenous and dynasties differ in ω. This adds an extra potential justification for public intervention, namely redistribution. It also makes the case where some individuals cannot afford private LTC coverage more plausible and we can have an initial equilibrium in which the bequest motive is operational for some individuals and not for others.

5 Decentralization of the first-best allocation

Assume for the time being that there is no asymmetry of information so that all relevant variables including informal aid are publicly observable. The following proposition (which is established in the Appendix) shows how this FB allocation within our multi-stage setting can be decentralized by a lump-sum transfer from the dependent to the healthy elderly supplemented by linear subsidies on labor incomes (of the young) and aid.

Proposition 2 *Under full information, the FB allocation can be decentralized by a lump-sum transfer from the dependent to the healthy elderly supplemented by linear subsidies on labor incomes (of the young) and aid. To achieve this the instruments are set at the following levels:*

(i) the rates of subsidies on $w\ell$ and $w\widehat{\ell}$ denoted τ_y and $\widehat{\tau}_y$ are given by

$$\tau_y = \widehat{\tau}_y = 1 \tag{29}$$

(ii) informal care is subsidized at rate τ_a given by

$$\tau_a = h'(a^{fb}) + 2\frac{\gamma q'(a^{fb})}{u'(m_c^{fb})} \tag{30}$$

(iii) the lump-sum payment to dependent elderly, D and the lump-sum tax imposed on the healthy elderly are given by

$$D = (1-\pi)\left[w\widehat{\ell}^{fb} - (w\ell^{fb} + h(a^{fb}) - L)\right] - \tau_a a^{fb} - \tau_y w\ell^{fb} \gtreqless 0 \tag{31}$$

$$\widehat{D} = \pi\left[w\widehat{\ell}^{fb} - (w\ell^{fb} + h(a^{fb}) - L)\right] + \widehat{\tau}_y w\widehat{\ell}^{fb} > 0. \tag{32}$$

The intuition behind these conditions is as follows. Condition (29) is the most straightforward: since the children face a 50% implicit tax on their labor incomes (via the reduction in bequests), we have to subsidize them at rate 1. In other words the total income is multiplied by 2 of which the children receive half so that we get the correct tradeoff. Expression (30) is also quite intuitive, except that the factor 2 may appear to be surprising at first. The sole benefit children get from a is $h'(a)/2$; see equation (5). Consequently, the remaining social benefits, namely

$$\frac{h'(a^{fb})}{2} + \frac{\gamma q'(a^{fb})}{u'(m_c^{fb})},$$

are not taken into account. This can be compensated by a subsidy. However since half of the subsidy will be lost due to the reduction in bequests, τ_a must be set at a level of twice the unaccounted social benefits, which yields (30).

Turning to D and $\widehat{D}$, the dependent old get from the share $1-\pi$ of the population the monetary loss of dependency less (which is exactly the net benefit a fair private insurance would give) less the subsidies to their children. The healthy old finance the monetary loss of dependent families (who are of share π) plus the subsidies to their own children. In sum, since parents finance the subsidies to their own children, these payments do not involve any transfers between families (i.e., accross states of nature). Such transfers are not necessary because the lump sum transfers between the elderly are already designed to achieve full insurance.

This first-best decentralization provides an interesting benchmark. However, in reality some of the relevant variables are likely not to be publicly observable which, in turn will restrict the available policy instruments. We shall now study the policy design in second-best settings where information is no longer complete. We start by a setting in which informal aid is not observable so that it cannot be subsidized.

6 Second-best: unobservable aid

Assume the government employs a lump-sum tax on the healthy old, $\widehat{D}$ and taxes child's income at a proportional rate t to finance public LTC provision, D to dependent parents. The optimization problem is then characterized by

$$\begin{aligned}\max_{t,D,\widehat{D}} \quad W(t,D,\widehat{D}) =& \pi\left[u(\omega+(1-p)I^*+D-L+h(a^*)-b^*)+\gamma q(a^*)\right]\\ &+\pi\left[u((1-t)w\ell^*+b^*)-v(\ell^*+a^*)\right]\\ &+(1-\pi)\left[u(\omega-pI^*-\widehat{D}-\widehat{b}^*)+u\left((1-t)w\widehat{\ell}^*+\widehat{b}^*\right)-v\left(\widehat{\ell}^*\right)\right]\\ \text{s.t.} \quad & \pi tw\ell^*+(1-\pi)tw\widehat{\ell}^*+(1-\pi)\widehat{D}=\pi D\end{aligned}$$

The FOC wrt t, D and $\widehat{D}$ are given by

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial t} =&\pi u'(m_p)\left[h'(a^*)\frac{\partial a^*}{\partial t}-\frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^*}{\partial t}-\frac{\partial b^*}{\partial a^*}\frac{\partial a^*}{\partial t}\right]+\pi\gamma q'(a^*)\frac{\partial a^*}{\partial t}\\ &-\pi u'(m_c)w\ell^*-(1-\pi)u'(d_p)\frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^*}{\partial t}-(1-\pi)u'(d_c)w\widehat{\ell}^*\\ &+\mu\left[\pi\left(w\ell^*+tw\frac{\partial \ell^*}{\partial t}\right)+(1-\pi)\left(w\widehat{\ell}^*+tw\frac{\partial \widehat{\ell}^*}{\partial t}\right)\right]=0 \qquad (33)\\ \frac{\partial\mathcal{L}}{\partial D} =&\pi u'(m_p)\left(1-\frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^*}{\partial D}\right)-\mu\pi\left[1-tw\frac{\partial \ell^*}{\partial D}\right]=0 \qquad (34)\\ \frac{\partial\mathcal{L}}{\partial \widehat{D}} =&-(1-\pi)u'(d_p)\left(1+\frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^*}{\partial \widehat{D}}\right)+\mu(1-\pi)\left[1+tw\frac{\partial \widehat{\ell}^*}{\partial \widehat{D}}\right]=0 \qquad (35)\end{aligned}$$

Appendix B shows that rearranging and combining these FOCs and defining the compensated effects as follows[3]

$$\frac{\partial \ell^{*c}}{\partial t} = \frac{\partial \ell^*}{\partial t}+w\ell^*\frac{\partial \ell^*}{\partial D}, \qquad (36)$$

$$\frac{\partial \widehat{\ell}^{*c}}{\partial t} = \frac{\partial \widehat{\ell}^*}{\partial t}-w\widehat{\ell}^*\frac{\partial \widehat{\ell}^*}{\partial D}, \qquad (37)$$

yields the following proposition

Proposition 3 *Assume that informal aid a is not observable and that policy instruments are restricted to public LTC provision to dependent parents, D, financed by a lump-sum tax on the healthy old, $\widehat{D}$ and a linear tax on the children's labor income at rate t. The optimal policy is characterized by*

(i) $u'(d_p) = u'(m_p)$: the transfers are used to achieve full insurance which (for an operational bequest motive) also implies full insurance for the children, $u'(d_c) = u'(m_c)$.

(ii) we have

$$t=\frac{\pi\left[u'(m_p)h'(a^*)+\gamma q'(a^*)\right]\frac{\partial a^*}{\partial t}-\left[\pi u'(m_p)\left[\frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^{*c}}{\partial t}+\frac{\partial b^*}{\partial a^*}\frac{\partial a^*}{\partial t}\right]+(1-\pi)u'(d_p)\frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^{*c}}{\partial t}\right]}{-\mu w\left[\pi\frac{\partial \ell^{*c}}{\partial t}+(1-\pi)\frac{\partial \widehat{\ell}^{*c}}{\partial t}\right]} \qquad (38)$$

To explain these results, let us first interpret the expression for t. The numerator can be interpreted as Pigouvian terms. The first one is a direct effect which resembles

[3] The sign of the last term changes because one is a transfer and the other a tax so the compensation goes (to remain at the same utility level) in opposite directions. We expect these compensated effects to be negative.

the Pigouvian expression derived in the previous section. The child does not consider the positive externality of informal care provision on parent's utility. This calls for a positive income tax which is effectively a subsidy on aid. However, the labor and aid variation induced by the tax also has indirect effects on the parent's utility. These effects are negative and operate via the adjustment in bequests. Both more aid and lower labor supply increase bequests and thus reduce parent's utility which counteracts the first positive effect of higher income taxes. The term in the denominator represents the deadweight loss of income taxation.

To sum up, transfers are used to achieve full insurance of the old. The tax on labor, on the other hand, is not used to raise revenue, but because it increases informal aid (which becomes more attractive when market labor is taxed). The level of t is then set to strike a balance between the deadweight loss of the labor tax (the denominator of (38)) against the net benefits associated with its effect on aid (which arise provided that the direct effects in the numerator (38) dominate the negative ones).

7 Second-best: heterogenous agents

So far, families were identical. In this section we introduce parents who differ in their wealth ω_i ($i = l, h$). This brings in another important justification for government intervention namely redistribution between families. It also makes a case where some parents cannot afford private LTC coverage more plausible and we can have an initial equilibrium in which the bequest motive is operational for some families and not for others. We assume that ω_i is unobservable to the government. All other variables, that is, bequests $(b_i, \widehat{b}_i)$, labor supplies $(\ell_i, \widehat{\ell}_i)$, informal care (a_i) and LTC care insurance coverage (I_i) can be observed. The children's consumption levels are thus effectively known by the government. We proceed as follows. First, we restate the parent's and child's optimization problem given the transfer scheme the government has at its disposal. Then, we determine the second-best allocation and show how the transfer scheme must be designed to implement this allocation.

7.1 Individual optimization reconsidered

The transfer scheme for parents and children in each state of nature is: $D(I_i, b_i)$, $\widehat{D}(I_i, \widehat{b}_i)$ and $T(w\ell_i, a_i)$, $T(w\widehat{\ell}_i)$ respectively. The transfer scheme for the elderly may also include a lump-sum transfer. Utility of the parent and the child are then given by

$$\begin{aligned} W_i^p =& \pi \left[u(\omega_i + (1-p)I_i - L + h(a_i) - b_i + D(I_i, b_i)) + \gamma q(a_i)\right] \\ &+ (1-\pi) u\left(\omega_i - pI_i - \widehat{b}_i + \widehat{D}(I_i, \widehat{b}_i)\right) + U_i^c \end{aligned} \tag{39}$$

$$\begin{aligned} U_i^c =& \pi \left[u(w\ell_i + b_i + T(w\ell_i, a_i)) - v\left(\ell_i + a_i\right)\right] \\ &+ (1-\pi)\left[u\left(w\widehat{\ell}_i + \widehat{b}_i + T(w\widehat{\ell}_i)\right) - v\left(\widehat{\ell}_i\right)\right] \end{aligned} \tag{40}$$

The optimal bequests in both states of nature are implicitly given by

$$\frac{u'(m_p^i)}{u'(m_c^i)} = \frac{1}{1 - D_b^i} \quad \text{and} \quad \frac{u'(d_p^i)}{u'(d_c^i)} = \frac{1}{1 - D_{\widehat{b}}^i} \tag{41}$$

From the above equations we get

$$\frac{\partial b_i}{\partial a_i} = \frac{(1 - D_b^i) u''(m_p^i) h'(a_i) - u''(m_c^i) T_a^i}{u''(m_c^i) + (1 - D_b^i) u''(m_p^i)} \tag{42}$$

$$\frac{\partial b_i}{\partial \ell_i} = \frac{-u''(m_c^i)(1 + T_\ell^i) w}{u''(m_c^i) + (1 - D_b^i) u''(m_p^i)} \tag{43}$$

$$\frac{\partial \widehat{b}_i}{\partial \widehat{\ell}_i} = \frac{-u''(d_c^i)(1 + T_{\widehat{\ell}}^i) w}{u''(d_c^i) + (1 - \widehat{D}_{\widehat{b}}^i) u''(d_p^i)}. \tag{44}$$

Optimal labor supplies in each state of nature and informal care provision are implicitly given by

$$\frac{v'(\ell_i + a_i)}{u'(m_i^c)} = (1 + T_\ell^i) w + \frac{\partial b_i}{\partial \ell_i} \quad \text{and} \quad \frac{v'(\widehat{\ell}_i)}{u'(d_i^c)} = (1 + T_{\widehat{\ell}}^i) w + \frac{\partial \widehat{b}_i}{\partial \widehat{\ell}_i} \tag{45}$$

$$\frac{v'(\ell_i + a_i)}{u'(m_i^c)} = \frac{\partial b_i}{\partial a_i} + T_a^i \tag{46}$$

7.2 General solution

This section characterizes the optimal allocation constrained by the information structure just sketched. The optimization problem of the government is given by

$$\max_{I_i, D_i, b_i, a_i, \ell_i, \widehat{D}_i, \widehat{b}_i, \widehat{\ell}_i} \quad W = \sum_i \phi_i \Big\{ \pi \left[u(\omega_i + (1-p)I_i + D_i - L + h(a_i) - b_i) + \gamma q(a_i)\right]$$

$$+ \pi \left[u(w\ell_i + b_i) - v(\ell_i + a_i)\right] + (1-\pi)\left[u\left(\omega_i + \widehat{D}_i - pI_i - \widehat{b}_i\right) + u\left(w\widehat{\ell}_i + \widehat{b}_i\right) - v(\widehat{\ell}_i)\right] \Big\}$$

subject to the resource constraint

$$\sum_i \phi_i \left\{\pi D_i + (1-\pi) \widehat{D}_i\right\} = 0 \tag{47}$$

and the following incentive constraints

$$\begin{aligned}
&\pi\left[u(\omega_i+(1-p)I_i+D_i-L+h(a_i)-b_i)+\gamma q(a_i)+u(w\ell_i+b_i)-v(\ell_i+a_i)\right]\\
&+(1-\pi)\left[u\left(\omega_i+\widehat{D}_i-pI_i-\widehat{b}_i\right)+u\left(w\widehat{\ell}_i+\widehat{b}_i\right)-v(\widehat{\ell}_i)\right]\geq\\
&\pi\left[u(\omega_i+(1-p)I_j+D_j-L+h(a_j)-b_j)+\gamma q(a_j)+u(w\ell_j+b_j)-v\left(\ell_j+a_j\right)\right]\\
&+(1-\pi)\left[u\left(\omega_i+\widehat{D}_j-pI_j-\widehat{b}_j\right)+u\left(w\widehat{\ell}_j+\widehat{b}_j\right)-v\left(\widehat{\ell}_j\right)\right]\quad\forall\, i\neq j. \qquad (48)
\end{aligned}$$

Denote λ_{ij} the Lagrange multiplier associated with the self-selection constraint from type-i to type-j and μ the one associated with the resource constraint. The FOCs of this problem are stated in Appendix C. Rearranging the FOCs wrt b_i and $\widehat{b}_i$ yields the following marginal rates of substitution between parent and child utility

$$\frac{u'(m_p^i)}{u'(m_c^i)}=\frac{\phi_i+\lambda_{ij}-\lambda_{ji}\frac{u'(m_c^{ji})}{u'(m_c^i)}\left[1-\frac{u'(m_p^{ji})}{u'(m_c^{ji})}\right]}{\phi_i+\lambda_{ij}}\quad\text{and} \qquad (49)$$

$$\frac{u'(d_p^i)}{u'(d_c^i)}=\frac{\phi_i+\lambda_{ij}-\lambda_{ji}\frac{u'(d_c^{ji})}{u'(d_c^i)}\left[1-\frac{u'(d_p^{ji})}{u'(d_c^{ji})}\right]}{\phi_i+\lambda_{ij}}. \qquad (50)$$

Consider the "top" family, that is, the rich family who is not mimicked implying $\lambda_{lh}=0$. For such a family equations (49) and (50) are equal to one. In other words, rich families' bequests are not taxed at the margin which is the traditional "no distortion at the top" result. With (41) we thus have $D_b^h=D_{\widehat{b}}^h=0$. Now consider poor families. These are mimicked by rich families $\lambda_{hl}>0$. Since the consumption of children is effectively observed, we have $u'(m_c^{ji})=u'(m_c^i)$ and $u'(d_c^{ji})=u'(d_c^i)$. However, rich families who mimic the poor no longer equalize marginal utilities between parents and children, but $u'(m_p^{ji})<u'(m_c^{ji})$ and $u'(d_p^{ji})<u'(d_c^{ji})$ due to the parents' higher wealth. With (41), we have for the poor

$$\frac{1}{1-D_b^l}=\frac{\phi_l-\lambda_{hl}\left[1-\frac{u'(m_p^{hl})}{u'(m_c^{hl})}\right]}{\phi_l}<1. \qquad (51)$$

In other words, the poor face a downward distortion on their bequests. A tax on the poors' bequests relaxes a binding incentive constraint; since type-h families want to bequest more to their children due to their higher wealth, they are also hurt more by a tax on these transfers.

Rearranging the FOCs wrt ℓ_i and $\widehat{\ell}_i$, we get the following marginal rates of substitution for labor supplies

$$\frac{v'(\ell_i + a_i)}{u'(m_c^i)} = \frac{\left[\phi_i + \lambda_{ij} - \lambda_{ji}\frac{u'(m_c^{ji})}{u'(m_c^i)}\right] w}{\phi_i + \lambda_{ij} - \lambda_{ji}} = w \quad \forall\, i \quad \text{and} \tag{52}$$

$$\frac{v'(\widehat{\ell}_i)}{u'(d_c^i)} = \frac{\left[\phi_i + \lambda_{ij} - \lambda_{ji}\frac{u'(d_c^{ji})}{u'(d_c^i)}\right] w}{\phi_i + \lambda_{ij} - \lambda_{ji}} = w \quad \forall\, i. \tag{53}$$

Since consumption of the children is effectively observed, the trade-off that both rich and poor families face in the second-best is the same as in the first-best. Given the multi-stage nature of our problem this however does not imply a marginal tax rate equal to zero. From equation (45), it can be seen that

$$1 + T_\ell^i = \frac{w - \frac{\partial b_i}{\partial \ell_i}}{w} \quad \text{and} \quad 1 + \widehat{T}_{\widehat{\ell}}^i = \frac{w - \frac{\partial \widehat{b}_i}{\partial \widehat{\ell}_i}}{w} \quad \forall\, i. \tag{54}$$

That is, the tax on labor is chosen to offset the downward distortion of bequests on labor supply. Since the rich face no distortion on bequests this yields the first-best marginal tax rates on labor supply: $T_\ell^h = \widehat{T}_{\widehat{\ell}}^h = 1$ (see Proposition 2). For the poor, we have

$$T_\ell^l = \frac{u''(m_c^l)}{(1 - D_b^l)u''(m_p^l)} > 0 \quad \text{and} \quad T_{\widehat{\ell}}^l = \frac{u''(d_c^l)}{(1 - D_{\widehat{b}}^l)u''(d_p^l)} > 0 \tag{55}$$

implying a subsidy on labor supplies. Whether these subsidies are smaller or larger than one can no longer be determined.

The marginal rate of substitution for informal care provision can be written as

$$\frac{v'(\ell_i + a_i)}{u'(m_c^i)} = \frac{\left[\phi_i + \lambda_{ij} - \lambda_{ji}\frac{u'(m_c^{ji})}{u'(m_c^i)}\right] h'(a_i)}{\phi_i + \lambda_{ij} - \lambda_{ji}} + \frac{\gamma q'(a_i)}{u'(m_c^i)} = h'(a_i) + \frac{\gamma q'(a_i)}{u'(m_c^i)}. \tag{56}$$

As for labor supply the trade-off for informal care provision in the second-best is the same as in the first-best. As for labor supply this does not imply zero marginal tax rates. With equation (46), we get

$$\frac{\partial b_i}{\partial a_i} + T_a^i = h'(a_i) + \frac{\gamma q'(a_i)}{u'(m_c^i)} \tag{57}$$

Informal care in the *laissez-faire* is inefficiently low, that is, to achieve the first-best trade-off, informal care provision must be subsidized at the margin (see Proposition 2). For rich families this amounts to

$$T_a^h = h'(a_h) + 2\frac{\gamma q'(a_h)}{u'(m_c^h)} \tag{58}$$

since they face no distortion on bequests. Poor families, by contrast, face a distortion on their bequests and to achieve the first-best trade-off for informal care, the subsidy on aid is given by

$$T_a^l = \frac{u''(m_c^i)h'(a_l) + [u''(m_c^l) + (1 - D_b^l)u''(m_p^l)]\frac{\gamma q'(a_l)}{u'(m_c^l)}}{(1 - D_b^l)u''(m_p^l)} > 0. \tag{59}$$

Now lets turn our attention to the optimal lump-sum transfers. Note that D can be interpreted as public LTC provision. Rearranging and combining the FOCs wrt D_i and $\widehat{D}_i$, we get

$$\frac{u'(d_p^i)}{u'(m_p^i)} = \frac{\phi_i + \lambda_{ij} - \lambda_{ji}\frac{u'(m_p^{ji})}{u'(m_p^i)}\left[1 - \frac{u'(d_p^{ji})}{u'(m_p^{ji})}\right]}{\phi_i + \lambda_{ij}} \tag{60}$$

Rich agents are fully insured implying $d_p^h = m_p^h$. For poor individuals, we have $\lambda_{hl} > 0$ and $u'(m_p^{hl})/u'(m_p^l) < 1$. However, we can not determine whether the expression in brackets is smaller or larger than one, so we can have both more or less than full insurance for the poor.

We summarize our results in the following proposition

Proposition 4 *When parents differ in wealth ω_i (with $i = l, h$) which is – as opposed to all other variables – unobservable to the government, the optimal second-best policy is characterized by*

(i) the first-best tradeoff for labor supply and informal care,

(ii) no distortion of bequests for rich families and a downward distortion of bequests for poor families,

(iii) full public LTC insurance for rich families and less or more than full insurance for poor families.

8 Summary and conclusion

This paper has studied family aid and the demand for LTC in a model of family decision making which is based on Becker's (1974) rotten kid specification. While in Becker's world, with a single good, this setting yields an efficient outcome, we showed that when family aid (and LTC services in general) is introduced the outcome is likely to be inefficient. This was particularly true when the parents value their children's care more than the market substitutes. As long as the bequest motive was operational, the children did provide some informal aid to their parents, however its level was too low. Additionally, children's labor supply was inefficient. We have studied how this inefficiencies can be corrected by public policies under various informational assumptions. Public intervention concerning LTC was needed to overcome inefficiencies in the private LTC insurance market and to correct for inefficiencies in informal care provision by children.

Appendix

A Proof of Proposition 2

To determine the levels of the different instruments we have to revisit the different stages of the game.

A.1 Stage 3

The optimal bequest in each state of nature is now determined by maximization of

$$\begin{aligned}
\max_{\widehat{b}} \quad \widehat{W}_p =& u(\omega - pI - \widehat{b} - \widehat{D}) + u((1+\widehat{\tau}_y)w\widehat{\ell} + \widehat{b}) - v(\widehat{\ell}) \\
\max_{b} \quad W_p =& u(\omega + (1-p)I - L + h(a) + D - b) + \gamma q(a) \\
& + u((1+\tau_y)w\ell + b + \tau_a a) - v(\ell + a)
\end{aligned}$$

The FOCs (3)–(4) continue to apply. Consequently, as long as there is an interior solution for b and $\widehat{b}$ we will automatically have

$$m_p = m_c \quad \text{and} \quad d_p = d_c. \tag{61}$$

However, the comparative statics change since now we have $\widehat{b}^* \equiv \widehat{b}(I, \widehat{\ell}, \widehat{\tau}_y, \widehat{D})$ and $b^* \equiv b(I, \ell, a, \tau_y, \tau_a, D)$

$$\frac{\partial b^*}{\partial a} = \frac{h'(a) - \tau_a}{2}, \quad \frac{\partial b^*}{\partial \ell} = -\frac{w(1+\tau_y)}{2}, \quad \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}} = -\frac{w(1+\widehat{\tau}_y)}{2}. \tag{62}$$

A.2 Stage 2

The child solves

$$\max_{\widehat{\ell}} \quad \widehat{U}_c = u\left((1+\widehat{\tau}_y)w\widehat{\ell} + \widehat{b}^*\right) - v\left(\widehat{\ell}\right) \tag{63}$$

$$\max_{\ell, a} \quad U_c = u\left((1+\tau_y)w\ell + b^* + \tau_a\right) - v\left(\ell + a\right). \tag{64}$$

The FOCs with respect to $\widehat{\ell}$, ℓ and a amount to

$$\frac{\partial \widehat{U}_c}{\partial \widehat{\ell}} = u'(d_c)\left(w(1+\widehat{\tau}_y) + \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}}\right) - v'(\widehat{\ell}) = 0 \tag{65}$$

$$\frac{\partial U_c}{\partial \ell} = u'(m_c)\left(w(1+\tau_y) + \frac{\partial b^*}{\partial \ell}\right) - v'(\ell + a) = 0 \tag{66}$$

$$\frac{\partial U_c}{\partial a} = u'(m_c)\left(\frac{\partial b^*}{\partial a} + \tau_a a\right) - v'(\ell + a) = 0 \tag{67}$$

Comparing (65) and (66) to the corresponding fist-best conditions (26) and (28) shows that the first-best can be decentralized with $\tau_y = \widehat{\tau}_y = 1$, which establishes (29).

Turning to the subsidy on aid, combining (67) with the first-best allocations (27) and (26) shows that τ_a must be chosen such that

$$u'(m_c)\left(\frac{\partial b^*}{\partial a} + \tau_a\right) = u'(m_c^{fb})h'(a^{fb}) + \gamma q'(a^{fb}) \quad \Leftrightarrow \quad \tau_a = h'(a^{fb}) + 2\frac{\gamma q'(a^{fb})}{u'(m_c^{fb})},$$

which establishes (30).

We are now also in a position to determine the levels of D and $\widehat{D}$. Transfers must be designed so that $m_p + m_c = d_p + d_c$, which along with the third stage equilibrium condition (61) implies $m_p = m_c = d_p = d_c$ and thus (25). This requires

$$\omega - L + h(a) + D + (1+\tau_y)w\ell + \tau_a a = \omega - \widehat{D} + (1+\widehat{\tau}_y)w\widehat{\ell}, \tag{68}$$

In addition, the budget constraint requires

$$\pi[D + \tau_y w\ell + \tau_a a] + (1-\pi)\widehat{\tau}_y w\widehat{\ell} = (1-\pi)\widehat{D}, \tag{69}$$

where ℓ, $\widehat{\ell}$ and a are set at the FB level (but superscripts are dropped at this stage to simplify notation). The budget constraint can be rewritten as

$$\widehat{D} = \frac{\pi}{1-\pi}[D + \tau_y w\ell + \tau_a a] + \widehat{\tau}_y w\widehat{\ell}. \tag{70}$$

Substituting into (68) and rearranging successively yields

$$\begin{aligned} D =&(1-\pi)\left\{L + (1+\widehat{\tau}_y)w\widehat{\ell} - [(1+\tau_y)w\ell + h(a) + \tau_a a]\right\} \\ &- \pi[\tau_y w\ell + \tau_a a] - (1-\pi)\widehat{\tau}_y w\widehat{\ell}. \end{aligned} \tag{71}$$

Rearraning (71) and using (70) then establishes (31) and (32).

Finally note that this solution implies full insurance, it is plain that no additional private insurance will be bought.

B Proof of Proposition 3

Making use of equation (16) (which coincides with $\partial\ell^*/\partial D$ for $p = 0$) and (17) (which coincides with $\partial\widehat{\ell}^*/\partial\widehat{D}$ for $p = 1$), equations (34) and (35) reduce to

$$\frac{u'(d_p)}{u'(m_p)} = \left(\frac{1 - \frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^*}{\partial D}}{1 + \frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^*}{\partial \widehat{D}}}\right)\left(\frac{1 + tw\frac{\partial \widehat{\ell}^*}{\partial \widehat{D}}}{1 - tw\frac{\partial \ell^*}{\partial D}}\right) \quad \Leftrightarrow \quad u'(d_p) = u'(m_p),$$

which establishes *(i)*.

To establish *(ii)* first multiply (34) by $w\ell^*$ and (35) by $-w\widehat{\ell}^*$ to obtain

$$\pi u'(m_p)\left(w\ell^* - w\ell^*\frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^*}{\partial D}\right) - \mu\pi\left[w\ell^* - tw^2\ell^*\frac{\partial \ell^*}{\partial D}\right] = 0 \tag{72}$$

$$(1-\pi)u'(d_p)\left(w\widehat{\ell}^* + w\widehat{\ell}^*\frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^*}{\partial \widehat{D}}\right) - \mu(1-\pi)\left[w\widehat{\ell}^* + tw^2\widehat{\ell}^*\frac{\partial \widehat{\ell}^*}{\partial \widehat{D}}\right] = 0 \tag{73}$$

Adding (72), (73) and (33) and simplifying by using (36) and (37) yields

$$\pi\left[u'(m_p)\left[h'(a^*) - \frac{\partial b^*}{\partial a^*}\right] + \gamma q'(a^*)\right]\frac{\partial a^*}{\partial t} - \left[\pi u'(m_p)\frac{\partial b^*}{\partial \ell^*}\frac{\partial \ell^{*c}}{\partial t} + (1-\pi)u'(d_p)\frac{\partial \widehat{b}^*}{\partial \widehat{\ell}^*}\frac{\partial \widehat{\ell}^{*c}}{\partial t}\right]$$

$$+\mu\left[\pi tw\frac{\partial \ell^{*c}}{\partial t} + (1-\pi)tw\frac{\partial \widehat{\ell}^{*c}}{\partial t}\right] = 0.$$

Factoring out t and rearranging then yields (38).

C Second-best: first-order conditions

Denoting $\mathcal{L}$ the Langrangean function of this problem, the FOCs are given by

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial I_i} =& (\phi_i + \lambda_{ij})\left[\pi(1-p)u'(m_p^i) - (1-\pi)pu'(d_p^i)\right] \\ &- \lambda_{ji}\left[\pi(1-p)u'(m_p^{ji}) - (1-\pi)pu'(d_p^{ji})\right] \le 0 \end{aligned}\tag{74}$$

$$\frac{\partial \mathcal{L}}{\partial b_i} = (\phi_i + \lambda_{ij})\left[u'(m_c^i) - u'(m_p^i)\right] - \lambda_{ji}\left[u'(m_c^{ji}) - u'(m_p^{ji})\right] = 0 \tag{75}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial a_i} =& (\phi_i + \lambda_{ij})\left[u'(m_p^i)h'(a_i) + \gamma q'(a_i) - v'(\ell_i + a_i)\right] \\ &- \lambda_{ji}\left[u'(m_p^{ji})h'(a_i) + \gamma q'(a_i) - v'(\ell_i + a_i)\right] = 0.\end{aligned}\tag{76}$$

$$\frac{\partial \mathcal{L}}{\partial \ell_i} = (\phi_i + \lambda_{ij})\left[u'(m_c^i)w - v'(\ell_i + a_i)\right] - \lambda_{ji}\left[u'(m_c^{ji})w - v'(\ell_i + a_i)\right] = 0 \tag{77}$$

$$\frac{\partial \mathcal{L}}{\partial \widehat{b}_i} = (\phi_i + \lambda_{ij})\left[u'(d_c^i) - u'(d_p^i)\right] - \lambda_{ji}\left[u'(d_c^{ji}) - u'(d_p^{ji})\right] = 0 \tag{78}$$

$$\frac{\partial \mathcal{L}}{\partial \widehat{\ell}_i} = (\phi_i + \lambda_{ij})\left[u'(d_c^i)w - v'(\widehat{\ell}_i)\right] - \lambda_{ji}\left[u'(d_c^{ji})w - v'(\widehat{\ell}_i)\right] = 0 \tag{79}$$

$$\frac{\partial \mathcal{L}}{\partial D_i} = (\phi_i + \lambda_{ij})u'(m_p^i) - \lambda_{ji}u'(m_p^{ji}) - \mu\phi_i = 0 \tag{80}$$

$$\frac{\partial \mathcal{L}}{\partial \widehat{D}_i} = (\phi_i + \lambda_{ij})u'(d_p^i) - \lambda_{ji}u'(d_p^{ji}) - \mu\phi_i = 0, \tag{81}$$

Inserting equations (80) and (81) in equation (74) yields

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial I_i} =& \pi(1-p)\left[\lambda_{ji}u'(m_p^{ji}) + \mu\phi_i\right] - (1-\pi)p\left[\lambda_{ji}u'(d_p^{ji}) + \mu\phi_i\right] \\ &- \lambda_{ji}\left[\pi(1-p)u'(m_p^{ji}) - (1-\pi)pu'(d_p^{ji})\right] = \pi(1-p) - (1-\pi)p \le 0\end{aligned}\tag{82}$$

Thats is, when $\pi < p$, we have $I_i = 0$ for $i = l, h$.

References

[1] **Becker, G.S.**, "A Treatise on the Family," *Harvard University Press*, Cambridge.

[2] **Bergstrom, T.C.**, "A Fresh Look at the Rotten-Kid Theorem – and Other Household Mysteries," *Journal of Political Economy*, 1989, *97*, 1138–1159.

[3] **Bergstrom, T.C.**, "Economics in a Family Way," *Journal of Economic Literature*, 1996, *34*, 1903–1934.

[4] **Brown, J.R., and A. Finkelstein**, "Why is the Market for Long-Term Care Insurance so Small?," *Journal of Public Economics*, 2007, *91*, 1967–1991.

[5] **Bruce, N., and M. Waldman**, "The Rotten-Kid Theorem Meets the Samaritan's Dilemma," *Quarterly Journal of Economics*, 1990, *105*, 155–165.

[6] **Chami, R.**, "King Lear's Dilemma: Precommitment Versus the Last Word," *Economics Letters*, 1996, *52*, 171–176.

[7] **Cox, D.**, "Motives for Private Income Transfers," *Journal of Political Economy*, 1987, *95*, 508–546.