Risk Taking under Heterogenous Revenue Sharing

Mohamed Belhaj and Frédéric Deroïan

April 22, 2010

Abstract

We examine the impact of informal risk-sharing on risk taking incentives when transfers are organized on a social network. We find that individual risk level are in general differentiated and related to the Bonacich measure of a network representing the transfer rule. Second, more revenue sharing enhances risk taking on average, although some agents may lower their risk level. Last, we provide conditions under which agents under or over invest with respect to the risk allocation maximizing the sum of utilities, and we find that under investment might often be observed.

JEL Classification Numbers: C72, D81, D85

Keywords: Risk Taking, Revenue Sharing, Social Networks, Systematic Risk, Strategic Substitutes

Authors’ affiliations. Mohamed Belhaj is at GREQAM and Ecole Centrale de Marseille; e-mail: mbelhaj@ec-marseille.fr. Frédéric Deroïan is at GREQAM; e-mail: frederic.deroian@univmed.fr.
1 Introduction

In economic theory, risk averse agents need to insure against income fluctuations. While markets for insurance exist in developed countries, developing villages often have no formal institution to make insurance mechanisms operational. In such a context, it has been now widely documented that households use social networks as an informal insurance. Typically, more wealthy households make transfers to others. In general, facing idiosyncratic income fluctuations, risk averse agents should put all their income in a common pool and share the pool equally. However, the empirical literature has stressed that villagers do not proceed to full equal sharing of incomes (Rosenzweig [1988], Townsend [1994], Udry [1994]). Further, relevant networks of transfers are composed of relatives and friends\(^1\). This non anonymity makes transfers heterogenous: the shares of revenues transferred to neighbors may differ across households; two households facing the same adverse shock may not receive the same amount of transfers from a common neighbor. While sharing revenues allows agents to reduce income volatility, it also makes agents exposed to the risk taken by their neighbors. Then, individual risk-taking decisions will be interdependent. Moreover, heterogeneity of transfers may differentiate individual risk levels.

In this paper, we analyze the impact of the organization of transfers on risk taking. To proceed formally, we consider a society of risk-averse agents with mean-variance utility function. Each agent has one unit to invest in a project through a portfolio of two technologies. One is risk-free, the other is more profitable but risky. The returns of the risky technology are positively correlated across projects. Each agent chooses the share to invest in the risky technology, that we interpret as individual level of risk. After income realizations, agents proceed to transfers. We consider a simple linear transfer

\(^1\)For instance, Fafchamps and co-authors (cited thereafter) collected in villages the entire relevant network of transfers. It appears that these networks are generally not completely connected, and that agents occupy asymmetric positions on the network. In Fafchamps and Lund (2003), each household of some village of rural Philippine was asked to identify a number of individuals on which it could rely in case of need or to whom the respondent gives help when called upon to do so. Respondents listed on average 4.6 individuals, with a minimum of 1 and a maximum of 8. In Dercon and De Weerdt (2006), households of Tanzanian village mentioned between 2 and 22 intra-village network partners in their interviews, with a mean equal to 6.5.
rule, where for each pair of neighbors, the richer gives a fixed share of the difference between their revenues. Equivalently, this transfer rule can be viewed as a bilateral exchange, in which neighbors exchange a fixed share of their revenue. The structure of transfers can be represented as a network of exchanges, in which the value of the connection $ij$ is the share of the realization of agent $j$ that she gives to agent $i$. We call own share the part that each individual keeps for herself.

In our setting, the existence of positive correlations among projects generates strategic substitutability$^2$ in risk levels. Through transfers, agents are exposed to the risk levels of their neighbors. Strategic interaction arises from correlated projects. As the return of neighbors’ project are correlated, when some agent increases risk, her neighbors reduce risk.

Then we begin our analysis with societies in which each agent keeps a same own share. Proposition 1 shows that equilibrium risk levels are homogenous, and depend only on own shares. Further, risk levels are decreasing in the value of own share. That is, more revenue sharing enhances risk taking. The economic intuition is as follows. Two factors shape incentives to take risk. First, when one agent exchanges more, she is less exposed to her own risk, thus she increases risk. Second, when neighbors increase risk, she is incited to reduce her own risk by strategic substitutability. Actually, when own shares are identical, the first effect dominates. A direct consequence of proposition 1 is that agents take more risk than under autarky regime. Turning to profits, proposition 2 confirms the usual view that a society in which individuals share their realized incomes equally, i.e. that offers maximal diversification, exhibits maximal utility for each agent. This is conform to the intuition, although our result obtains with endogenous risk choice.

Then, we pursue the analysis with the case of heterogenous own shares. In general, an equilibrium exists and is unique. As a first result, theorem 1 states that, under a reasonable condition, assumption 2 thereafter, risk levels are differentiated and related to the structure of transfers in the society. More precisely, individual risk is an affine function of a Bonacich measure defined over a slight transformation of the network of

$^2$By strategic substitutability we mean that the return of a marginal increase of risk level is a decreasing function of the risk level of others.
transfers. Assumption 2 states that own shares exceed a threshold that depends on the intensity of the correlation; a simple and realistic sufficient condition is that every own share exceeds one half. Second, we address some comparative analysis with regard to the volume of transfers. Theorem 2 states that more revenue sharing generates more risk taking on average. That is, with regard to proposition 1, a sufficiently low volume of exchanges guarantees that, on average, the positive impact of the reduction of the exposition to own project on risk dominates the impact of strategic interaction. This result does not prevent some agents to decrease their risk as a response to more revenue sharing. In contrast with the above results, if strategic interaction is high (i.e. own shares are low enough, violating assumption 2), heterogeneity of transfers may harm the result. A simple three-player example shows that risk levels can be lower than under autarky regime, and that average risk taking can be lower under increased volume of transfers. We insist that without heterogeneity in own shares, strategic interaction cannot produce such consequences. We also make some comparative statics with respect to correlation parameter, which contributes to the intensity of interaction. Proposition 3 states that average risk taking is decreasing in correlation parameter, confirming that strategic substitutability lowers risk taking. Third, we examine agents’ participation constraints. We determine conditions under which individuals are better off in a society with transfers than under autarky. This issue is important since our linear rule of transfer is not derived from an optimal transfer scheme. Proposition 4 shows that this participation constraint is valid under realistic conditions. To illustrate, one simple sufficient condition is that all own shares exceed one half.

Last, we explore efficiency issue. Our game exhibits both positive and negative externalities. This arises from a simple tradeoff: when some agent increases investment in the risky technology, this raises both the expected return and the volatility of the future transfer to her neighbors. Proposition 5 characterizes the efficient risk profile as an affine function of a Bonacich measure defined over a network which aggregates all

---

This measure has been introduced in Bonacich (1987). Ballester, Calvo and Zènou (2006) renewed the idea in the field of economics. Some recent papers relate optimal decisions on networks to Bonacich centrality. See Ghiglino and Goyal (2008) for a model of a pure exchange economy with a positional good; see Corbo, Calvo and Parkes (2007) and Bramoullé et al. (2008) for models of local public goods, Bloch and Quérou (2008) for a model of oligopoly with local externalities among consumers. In the two latter works, as well as in ours, actions are strategic substitutes.
externalities. At equilibrium, agents may either under invest or over invest. When risk levels are low (resp. high), the externalities are positive (resp. negative), the return effect dominates (is dominated by) the variance one, thus agents under (resp. over) invest. Corollary 1 relates under or over investment to the structure of transfers. To illustrate, one simple sufficient condition for under investment to occur is that every own share exceeds $\frac{1}{\sqrt{2}} \simeq .7$.

Related literature. A recent theoretical literature about revenue sharing in developing economies examines the formation of risk-sharing networks. Bramoullé and Kranston (2006, 2007) examine the formation of risk-sharing networks under equal revenue sharing, and discuss stability/efficiency dilemma of the social network. Given that real world does not exhibit full equal sharing, some papers explain partial risk-sharing by self-enforcing mechanisms on networks (Ambrus, Mobius and Szeidl [2007], Bloch, Genicot and Ray [2008]). These models consider contracts shaped by social norms. Hence, transfers are not optimal, and possibly heterogenous. Broadly speaking, these models relate the maximal volume of transfers that forbids hold up problems to network properties. That is, in these models, the social network is endogenous, but the rules that shape transfers on these links are kept exogenous. With regard to this literature, we let both the transfer rules and the social network exogenous and we examine the link between the structure of transfers and risk-taking incentives. Further, we precise conditions under which individuals prefer sharing revenues than being isolated.

The empirical literature on risk-sharing in village economies has emphasized some features of informal insurance. First, Townsend (1994) rejects the full equal sharing hypothesis in Indian villages$^4$. Second, the importance of social networks as relevant channels for informal insurance as been attested, opening the scope for transfers’ heterogeneity. Rosenzweig (1988) and Udry (1994) documented that the majority of transfers takes place only between neighbors and relatives. More recently, some works have confirmed this finding by collecting the whole social network in villages (Fafchamps and Lund [2003], Dercon and De Weerdt [2006], Fafchamps and Gubert [2007], De Weerdt and Fafchamps [2007]). These works suggest that households share risk within confined

$^4$Some economic literature has proposed as a possible explanation to this finding that limited commitment may be due to enforcement issues (Coate and Ravaillon [1993], Ligon, Thomas and Worrall [2002], Dubois, Jullien and Magnac [2008]), or to moral hazard issues (REFERENCE)
networks of family and friends. Importantly, the identity based nature of networks of
transfers indicates that they are presumably not formed for the unique objective of
sharing revenue. That is, these networks are at least partially exogenous to optimal
contracting decisions. Following the finding of this recent empirical literature, our
model assumes that the network of transfers is exogenous to agents’ decisions. To
describe transfers, we present a simple linear sharing rule that incorporates transfers’
heterogeneity.

We finish with a literature on risk-taking. In many economic contexts, a redistri-
bution of incomes in a society of risk averse agents enhances risk taking incentives.
For instance, in labor markets, unemployment insurance encourages workers to seek
higher productivity jobs because they are more willing to endure the possibility of
unemployment (Acemoglu and Shimer [1999, 2002]). Similarly, redistributive taxation
can enhance entrepreneurship (Mayshar [1977], Kanbur [1981], Boadway, Marchand
and Pestiau [1991], Sinn [1996], Garcia-Penalosa and Wen [2008]). The economic in-
tuition behind this result is that redistribution reduces agents exposure to individual
risk. With regard to this literature, our model incorporates strategic interaction in
risk-taking decisions.

The article is organized as follows. Section 2 describes the model. Section 3 analyzes
Nash equilibria and offers some comparative statics. Section 4 examines efficiency issue,
and the last section concludes. All proofs are presented in the appendix.

2 The model

We consider a game in which, first, agents invest in a risky project, second, incomes are
realized and agents make transfers. The society contains a finite set \( N = \{1, 2, \cdots, n\} \)
of risk-averse agents.

The network of transfers. To reduce income fluctuations, agents share parts of
their realized incomes with their neighbors. The economic literature generally assumes

\[\text{However, many factors related to risk issue may explain why revenue sharing is heterogenous across households. To cite a few, self-enforcing mechanisms and trust (social sanctions may be heterogenous), heterogeneity in information flows, in income correlations, in geographic costs, increasing costs to group size (see Murgai et al. [2002]).}\]
that agents set up optimal contracts, contingent on the states of nature. As mentioned before, full equal sharing is not observed, and the recent literature emphasized the role of social networks as channels for transfers. The network aspect is hardly compatible with optimal contracts, that require verification of all realizations. Typically, agents may observe the realizations of their neighbors, but not the ones of the neighbors of their neighbors. This may create asymmetric information problems, preventing potentially optimal contracts. We opt here for a simple approach, in which transfers do not depend on the realizations of third parties. However, our rule is state-contingent and compatible with heterogeneous transfers. We consider a linear transfer rule, i.e. for each pair of neighbors, the richer gives a fixed share of the difference between the two revenues\(^6\). This transfer rule is equivalent to exchanging the same share of revenue. Particularly, an unlucky agent transfers part of her income only to more unlucky agents, while a lucky agent receives transfers only from more lucky ones (note that this also happens under full equal sharing).

Formally, in a society with \(n\) agents, transfers are represented by a symmetric\(^7\) matrix of exchanges \(\Lambda = [\lambda_{ij}]\), where \(\lambda_{ij} \in [0, 1]\) represents the share of agent \(j\)'s income that she gives to agent \(i\), and \(\lambda_{ii}\) represents agent \(i\)'s own share. By budget constraint, \(\lambda_{ii} + \sum_{j \neq i} \lambda_{ji} = 1\) for all \(i\).

To illustrate, figure 1 presents two tree-player networks. Players are depicted by nodes and links by edges, the numbers above nodes (resp. links) represent own shares (resp. symmetric exchanged shares). The left figure represents a completely connected society with full equal sharing, the right one a two-link society with heterogeneous sharing.

**Investments.** If \(Y\) is the uncertain individual income, the utility of a risk-averse agent is \(U(Y) = E(Y) - \frac{\kappa}{2} \text{Var}(Y)\), with \(\kappa > 0\) denoting the coefficient of individual

\(^6\)Bloch, Genicot and Ray (2008) consider bilateral norms that aggregate third-party obligations. In our model, the transfer rule does not take care of third-party obligations. In Ambrus, Mobius and Szeidl (2007), transfer arrangements are possibly heterogenous, and they are limited by commitment issues. In their model, connections generate intrinsic utilities, like friendship. In our model, social links do not intrinsically generate intrinsic utility (except by conducting transfers).

\(^7\)Technically, our results are only based on row-stochasticity, which is implied by symmetry. Our proofs are presented in this more general setting.

\(^8\)Benefits as well as losses are concerned with transfers. The literature on insurance considers mutualization of losses only.
risk aversion. Each agent has one unit to invest in a specific project. The project can be developed through two technologies $A$ and $B$. Technology $B$ is risk-free and has expected mean normalized to 1. The return $Y^A$ of technology $A$ is random with expected mean $\mu > 1$ and variance $\sigma^2$. Let $\rho \sigma^2$ be the covariance between the returns of two distinct projects that use technology $A$, with $\rho \in [0,1]$. Economically, correlations are related to systematic risks, while the variance $\sigma^2$ incorporates both specific and systematic risk. Define $x_i \in [0,1]$ be the share that agent $i$ invests in technology $A$. We interpret $x_i$ as the level of risk chosen by agent. When agent $i$ invests $x_i$ in technology $A$, her revenue is given by $(1 - x_i + x_i Y^A)$. Denote by $X = (x_1, x_2, \cdots, x_n)$ denote a profile of individual risks. Note that in risk management or moral hazard issues, agents would incur a personal cost of effort, while sharing benefits with others. A consequence is that the existence of transfers would reduce incentives to produce effort. In contrast, in our model the cost of risk-taking (a higher variance) is shared with neighbors through transfers.

The individual utility of agent $i$ can be written as

$$U_i(X; \Lambda) = \sum_j \lambda_{ij} E(Y_j) - \frac{\sigma^2}{2} \sum_j \sum_k \lambda_{ij} \lambda_{ik} \text{cov}(Y_j, Y_k)$$

with, letting symbol $\mathbf{1}$ stand for the indicator function, $\text{cov}(Y_j, Y_k) = \sigma^2 \cdot x_j x_k (\mathbf{1}_{j=k} + \sigma^2 \cdot x_j x_k (\mathbf{1}_{j=k} +$

$^9$Note that if agents had a binary choice between $A$ and $B$, $x_i \in [0,1]$ could be interpreted as a probability, and we would solve a Bayesian equilibrium.
\( \rho \cdot 1_{\{j \neq k\}} \), with \( 0 < \rho < 1 \). That is,

\[
U_i(X; \Lambda) = 1 + (\mu - 1) \sum_j \lambda_{ij} x_j - \frac{\kappa \sigma^2}{2} \sum_j \lambda_{ij}^2 x_j^2 - \frac{\kappa \rho \sigma^2}{2} \sum_{j \neq k} \lambda_{ij} \lambda_{ik} x_j x_k
\]  

(1)

Assuming that both investment decisions and individual realizations are observable by neighbors, we analyze Nash equilibria. Formally, a profile \( X^* \) is a (pure) Nash equilibrium if it satisfies that, for all \( i \), for all \( x_i \in [0, 1] \), \( U_i(x_i^*, x_{-i}^*; \Lambda) \geq U_i(x_i, x_{-i}^*; \Lambda) \).

Since we impose \( x_i \in [0, 1] \), we need two assumptions. The first assumption relates the Sharp ratio to the product of the correlation parameter and the risk-aversion coefficient:

**Assumption 1** \( \frac{\mu - 1}{\sigma^2} \leq \kappa \rho \)

Denoting \( h = \frac{\mu - 1}{\sigma^2} \) for convenience, assumption 1 writes \( \frac{h}{\rho} \leq 1 \). The second assumption imposes that all own shares exceed a threshold related to the correlation parameter:

**Assumption 2** For all \( i \), \( \lambda_{ii} \in \left[ \frac{\rho}{1+h}, 1 \right] \)

Observe that every own share exceeds one half under assumption 2.

### 3 Equilibrium analysis

In this section, we characterize equilibrium risk levels as a function of the matrix of exchanges. More precisely, we will relate risk levels to the position of agents on the network of transfers. We will first focus on the case of homogenous own shares, and then pursue with the general case.

Before proceeding, optimizing agents’ utilities, we obtain the following system of first order equations:

\[
\lambda_{ii} x_i^* + \rho \sum_{j \neq i} \lambda_{ij} \cdot x_j^* = h \forall i
\]  

(2)

Equation (2) shows that strategic interaction emerges as a consequence of correlations between projects. Moreover, since \( \rho > 0 \), individual actions are strategic substitutes.

Define vector \( D \) such that \( D_i = \frac{1}{\lambda_{ii}} \). The linear system of first order condition can be inverted in general, i.e. a solution \( X^* = h(I + \rho \Gamma)^{-1} D \) exists and is unique. However,
the system is not always invertible. Since the determinant of the matrix \((I + \rho \Gamma)\) is a polynomial of order \(n\) in parameter \(\rho\), there exists a set of at most \(n\) values of parameter \(\rho\) such that the system is not invertible. For the rest of the paper, we will assume that \(\rho\) does not belong to this set. Note that this does not guarantee that equilibrium risk levels belong to \([0, 1]\). We will give conditions thereafter that guarantee existence and uniqueness of interior solutions.

3.1 Revenue sharing with homogenous own shares

By homogenous own shares, we mean that \(\lambda_{ii} = \lambda_0\) for all \(i\). In that case, the linear system is easily solved. We obtain:

**Proposition 1** Under assumption 1, in societies with homogenous own shares, the equilibrium level of risk is homogenous and given by

\[
x^{HOS}(\lambda_0) = \frac{h}{\lambda_0 + \rho(1 - \lambda_0)}
\]  

In particular, it is independent of the distribution of off-diagonal elements of the matrix of exchanges.

Equation (3) shows that individual risk level is decreasing in the value of own share. One the one hand, lowering \(\lambda_0\) reduces exposure to own project (first term in the denominator), which pushes agents to take more risk. One the other hand, lowering \(\lambda_0\) increases exchanges, thus enhances strategic interaction, which in total reduces incentives to take risk (second term in the denominator). Equation (3) the first effect dominates. Hence, for societies with homogenous own shares, more revenue sharing enhances risk taking.

Some polar cases deserve attention. The first one is autarky. The second one is a society in which agents share all their their income equally. The third one is a society in which agents keep for themselves a same own share and share equally the rest of income.

*Autarky.* If \(\lambda_0 = 1\), the optimal level of risk of any agent is \(x^e = h\). Note that \(h \in ]0, 1[\) by assumption 1.
**Full equal sharing.** This corresponds to the matrix of exchanges $\Lambda^{FES}$ with $\lambda_{ij}^{FES} = \frac{1}{n}$ for all $i, j$. The risk level is written $x^{FES} = \frac{nh}{1+(n-1)p}$.

**Partial equal sharing.** This sharing-rule can be represented by the matrix of exchanges $\Lambda^{PES}(\lambda_0)$ such that $\lambda_{ii}^{PES} = \lambda_0$, for all $i$, and $\lambda_{ij}^{PES} = \frac{1-\lambda_0}{n-1}$, for all $i, j \neq i$. To illustrate, consider, that a fixed proportion, say $\tau_0$, of incomes is collected and equally redistributed. Then, agent $i$ receives $(1 - \frac{(n-1)}{n} \tau_0) y_i + \frac{m}{n} \sum_{j \neq i} y_j$. Denoting $\lambda_0 = 1 - \frac{(n-1)}{n} \tau_0$, the equilibrium level of risk is increasing in the taxation rate $\tau_0$ (less however than in the absence of strategic interaction).

While societies with homogenous own shares generate homogenous risk levels, individual utilities depend on the whole distribution of transfers. Let $\Lambda_i$ denote the line $i$ vector of the matrix of exchanges. We say that the profile $\Lambda_i$ is more diversified than the profile $\Lambda'_i$ whenever $\sum_j \lambda_{ij}^2 < \sum_j \lambda'_{ij}^2$. It is easily shown that, when own shares are homogenous, more diversification is always beneficial to agent $i$’s equilibrium utility. Furthermore:

**Proposition 2** The matrix of full equal sharing guarantees the highest equilibrium utility for each agent.

This usual result expresses that if agents were able to coordinate in order to collectively implement full equal sharing, every agent would be better off.

### 3.2 Revenue sharing with heterogenous own shares

Considering now heterogenous own shares, we will relate risk taking to a Bonacich measure of the network of transfers. This network will be represented by the $n \times n$ matrix\(^{10}\) $\Gamma = [\gamma_{ij}]$, with $\gamma_{ii} = 0$ for all $i$, and $\gamma_{ij} = \frac{\lambda_{ij}}{\lambda_i}$ for all $i, j \neq i$. The element $\gamma_{ij}$ is equal to the ratio of the share that agent $j$ gives to agent $i$ over agent $i$’s own share.

We define now the Bonacich measure that will shape risk levels. Consider a $n \times n$ matrix $M$ with nonnegative elements $m_{ij}$ and let $J$ denote the column vector of ones.

\(^{10}\)Technically, the transformation of our game echoes the one introduced in Ballester, Calvo and Zénou (2006), remark 2 pp. 1409.
Consider also a scalar $\alpha \in ]-1,1[$. If $|\alpha|$ times the greatest modulus of eigenvalues of matrix $M$ is less than 1, the matrix $(I - \alpha M)$ is invertible. Further, defining $B(M; \alpha) = (I - \alpha M)^{-1}J$, the solution can be written

$$B(M; \alpha) = \sum_{k=0}^{\infty} \alpha^k M^k J$$

(4)

Note that when $\alpha < 0$, the contribution of the network to the measure is ambiguous: odd paths contribute negatively to the measure, even paths positively. To avoid confusion, we shall speak about Bonacich measure, without reference to centrality.

The next theorem relates risk levels to the Bonacich measure associated with the network represented by matrix $\Gamma$:

Theorem 1 Suppose that both assumptions 1 and 2 hold. An equilibrium risk profile $X^* = h(I + \rho\Gamma)^{-1}D$ exists and is unique. It can be written

$$x^*_i = \frac{h}{\rho} \left( 1 - (1 - \rho)B_i(\Gamma; -\rho) \right)$$

(5)

for all $i \in N$, where $B_i(\Gamma; -\rho) \in [0,1]$ and is given by equation (4). Hence, $x^*_i \in [h, \frac{h}{\rho}[$.

Assumption 2 guarantees existence and uniqueness, while assumption 1 therefore guarantees that solutions belong to $[0,1]$. Theorem 1 shows that risk levels are differentiated when own shares are heterogenous, and related to the structure of transfers. Moreover, risk levels are bounded below by $h$ and above by $\frac{h}{\rho}$, which are the same bounds than in the case of homogenous own shares.

Comparative statics with respect to the volume of transfers. We generalize the idea of ‘more revenue sharing’ as follows. Starting from any society, revenue sharing

---

11 For $\alpha > 0$, if the greatest modulus of eigenvalues of $M$ is smaller than $\frac{1}{\rho}$, $B(M; \alpha)$ can be interpreted as a vector of Bonacich centrality measure defined over the network with link $m_{ij}$ representing the intensity of the connection from agent $i$ to agent $j$. The quantity $B_i(M; \alpha)$ measures the sum of the values of paths from agent $i$ to others through the network, where the value of a path is the product of link intensities on that path. This measure (actually, a slightly modified version) was introduced in Bonacich (1987).
increases when own shares are decreased and other shares are increased, in a way that preserves symmetry. Formally:

**Definition** [more revenue sharing] Consider one matrix of exchanges $\Lambda$, and let $\Lambda' = \Lambda + \Theta$ with $\theta_{ii} = - \sum_{j \neq i} \theta_{ij}$ for all $i$, and $\theta_{ij} = \theta_{ji}$ for all $i, j$. There is more revenue sharing in $\Lambda'$ than in $\Lambda$ if for all $i$, $\theta_{ii} \leq 0$ and for all $i, j \neq i$, $\theta_{ij} \geq 0$. (with at least one strict inequality)

We obtain:

**Theorem 2** Under assumptions 1 and 2, more revenue sharing increase risk taking on average.

Recall that one the one hand, lowering own shares reduces exposure to own project, inciting agents to take more risk; one the other hand, lowering own shares increases strategic interaction, which in total reduces incentives to take risk. Similarly to proposition 1, theorem 2 confirms that when the intensity of interaction is sufficiently low, the first effect prevails on average. As a direct application of theorem 2, and denoting $\bar{\lambda} = \min_i \lambda_{ii}$, $\bar{\lambda} = \max_i \lambda_{ii}$, $\bar{x} = x^*(\bar{\lambda})$ and $\bar{x} = x^*(\bar{\lambda})$, the average level of risk belongs to the interval $[\bar{x}, x]$.

However, more revenue sharing does not guarantee an increase of all individual risk levels. To illustrate, let $\Lambda' = \Lambda + \Theta$ be such that there is more revenue sharing in $\Lambda'$ than in $\Lambda$ and such that some agent, say agent 1, is unaffected by the modification $\Theta (\theta_{1j} = \theta_{j1} = 0$ for all $j$). Then, there exists one agent, say $i_0$, eventually distinct from agent 1, such that $x_{i_0}^* < x_{i_0}^*$ (proof omitted).

**Remark.** If assumption 2 does not hold, strategic interaction is high, inducing that risk levels may not be restricted to the interval $[h, \frac{h}{\rho}]$ (violating theorem 1). Further, more revenue sharing may reduce risk taking (violating theorem 2). The next example illustrates the point. Suppose $\rho = .9$ and consider the following matrices:

$$\Lambda = \begin{pmatrix} .64 & .09 & .27 \\ .09 & .48 & .43 \\ .27 & .43 & .30 \end{pmatrix}, \quad \Lambda' = \begin{pmatrix} .68 & .08 & .24 \\ .08 & .5 & .42 \\ .24 & .42 & .34 \end{pmatrix}.$$
We find $\frac{x_1^1}{h} \simeq 1.13$, $\frac{x_2^1}{h} \simeq 1.34$, $\frac{x_3^1}{h} \simeq .67$, entailing $\frac{1}{3h} \sum_i x_i^* \simeq 1.05$; and $\frac{x_1^2}{h} \simeq .68$, $\frac{x_2^2}{h} \simeq .04$, $\frac{x_3^2}{h} \simeq 2.45$, implying $\frac{1}{3h} \sum_i x_i'^* \simeq 1.06$. First, no risk level associated with matrix $\Lambda$ lies in the interval $[h, \frac{h}{\rho}]$. Second, there is more revenue sharing in $\Lambda$ than in $\Lambda'$, but less risk on average.

Note the crucial role played by the heterogeneity of own shares. If own shares are homogenous, risks levels are always decreasing with the value of own share and increasing with parameter $\rho$. If own shares are heterogenous, assumption 2 is crucial to keep these results valid on average.

**Comparative analysis with respect to parameter $\rho$.** Parameter $\rho$ contributes to the intensity of strategic interaction. Noticing that the risk profile is positive, we deduce from equation (2) that, for each $\rho \in [0, 1]$, the equilibrium risk profile is lower than the one with no strategic interaction (i.e. $\rho = 0$). We find the following inuitive result:

**Proposition 3** Under assumption 2, average risk taking is decreasing in parameter $\rho$.

**Participation constraint.** Although the history of social norms may have built the matrix of exchange, irrespective of contemporaneous individual incentives, it is of interest to examine in which circumstances individual utilities are higher under transfers than under autarky. The issue is non trivial. When risk levels are high, variances are high and utilities are possibly low. Does the benefit from sharing revenue out-weights the negative externalities that agents’ choices may generate to others?

For any matrix of exchanges $\Lambda$, the individual participation constraint is satisfied if individual utility is greater than utility under autarky. The next proposition gives a sufficient condition:

**Proposition 4** The participation constraint is satisfied if and only if

$$\lambda_{ii} x_i^2 + \rho \sum_{j\neq i} \lambda_{ij} x_j^2 \leq h^2$$

(6)

In particular, it is satisfied if for all $j$, $\max_{i \neq j} \lambda_{ij} \leq \lambda_{jj}$.

Under Proposition 4, individual participation constraint is satisfied if each own share exceeds each share she gives to neighbors. This condition does not fully recover assumption 2. Note that, if every own share exceeds one half, both individual participation constraint and assumption 2 are valid.
Remark. In the above counter example, the participation constraint is satisfied for both matrices Λ and Λ′.

4 Efficient allocation of risks

In this model, the sign of externalities is endogenous to the risk chosen by agents. Indeed, an increase of the level of risk of an agent induces both higher expected return and higher variance for her neighbors. In consequence, whether agents over or under invest in the risky technology is ambiguous. In what follows, we will first characterize the efficient allocation, and second we will give sufficient conditions under which the efficient and equilibrium risk profiles can be compared on average.

The efficient allocation. We consider as efficient a risk profile that maximizes the sum of utilities in the society, that we denote \( W(X; \Lambda) = \sum_i U_i(X; \Lambda) \). Let \( \Psi \) be the \( n \times n \) matrix with \( \psi_{ij} = \sum_k \lambda_{ki} \lambda_{kj} \) for all \( i, j \). Define the \( n \times n \) matrix \( \Phi = [\phi_{ij}] \) with \( \phi_{ij} = 0 \) for all \( i \), \( \phi_{ij} = \frac{\psi_{ij}}{\psi_{ii}} \) for all \( i, j \neq i \). Finally, define the vector \( E \) such that \( E_i = \frac{1}{\psi_{ii}} \).

Remark that, being symmetric and column-stochastic, matrix \( \Psi \) can be interpreted as a matrix of exchanges of a transformed game. We obtain a characterization of the efficient risk profile as follows:

**Proposition 5** If matrix \( \Psi \) satisfies assumptions 1 and 2, an efficient risk profile \( \hat{X} = h(I + \rho \Phi)^{-1}E \) exists and is unique. It can be written

\[
\hat{x}_i = \frac{h}{\rho} \left( 1 - (1 - \rho)B_i(\Phi; -\rho) \right)
\]

(7)

for all \( i \in N \), where \( B_i(\Phi; -\rho) \) is given by equation (4), with \( \hat{x}_i \in [h, \frac{h}{\rho}] \).

Proposition 5 states that the efficient risk allocation is given by the Bonacich measure of an appropriate interaction matrix\(^{12}\). However, matrix \( \Psi \) must satisfy assumptions 1 and 2, which is not guaranteed for every matrix of exchanges \( \Lambda \) (some applications are given thereafter).

Over/Under investment with regard to the efficient allocation. Since the

\(^{12}\)Again, except for at most \( n \) values of parameter \( \rho \), the system is invertible.
efficient risk profile corresponds to the matrix of exchanges of a modified game, we can apply theorem 2. Corollary 1 sums up the point formally:

**Corollary 1** The average equilibrium level of risk is lower (resp. higher) than the average efficient level of risk if those three conditions apply simultaneously: (i) both matrices Λ and Ψ satisfy assumption 2; (ii) \( \lambda_{ii} \geq \psi_{ii} \) (resp. \( \lambda_{ii} \leq \psi_{ii} \)) for all \( i \); (iii) \( \lambda_{ij} \leq \psi_{ij} \) (resp. \( \lambda_{ij} \geq \psi_{ij} \)) for all \( i, j \neq i \).

If agents exchange a small share with the society, externalities are mainly mould by the return effect, i.e. they are positive. Symmetrically, if agents exchange too much with the society, the variance effect dominates the shaping of externalities.

In real world, agents do not exchange too much, and under investment might be often observed\(^{13}\). For instance, under partial equal sharing, the conditions hold as soon as \( \lambda_0 \geq \frac{1}{n} \). More generally, two simple conditions related to own shares guarantee under investment\(^{14}\):

\[
\begin{align*}
\frac{n\lambda_{ii}^2}{2} - 2\lambda_{ii} + \frac{1-\rho(n-2)}{1+\rho} & \geq 0 \\
\lambda_{ii} & \geq \frac{1}{2}
\end{align*}
\]

(8)

Note that the first inequality of system (8) is automatically satisfied when \( \rho \leq \frac{1}{n-1} \). Note also that under investment obtains for all parameter \( \rho \) if \( \lambda_{ii} \geq \frac{1}{\sqrt{n}} \) for all \( i \).

**Remark.** Under full equal sharing, the equilibrium risk profile coincides with the efficient one.

## 5 Conclusion

This paper analyzed a model of risk choice under revenue sharing, in presence of systematic risk. We considered agents with homogenous initial wealth and same risk

\(^{13}\)This result provides a possible explanation of the lack of investment in risky innovations in developing villages (Valente [1997]).

\(^{14}\)Indeed, condition (i) in corollary 1 is implied by \( \lambda_{ii}^2 + \frac{(1-\lambda_{ii})^2}{n-1} \geq \frac{1}{1+\rho} \), which can be written as the first inequality of system (8). Further, both conditions (ii) and (iii) are satisfied if \( \lambda_{ii} \geq \frac{1}{2} \) for all \( i \); indeed, condition (ii) is implied by \( \lambda_{ii} \geq \lambda_{ii}^2 + (1 - \lambda_{ii})^2 \), and condition (iii) is written \( 0 \leq (\lambda_{ii} + \lambda_{jj} - 1)\lambda_{ij} + \sum_{k \neq i,j} \lambda_{ki}\lambda_{kj} \).
aversion, and we examined the impact of the heterogeneity of transfers on individual risk taking.

Our first conclusion is that the joint presence of systematic risk and revenue sharing generates strategic interaction. In particular, positive correlation entails that risk choices are strategic substitutes, thus potentially reducing the positive impact of transfers on risk levels. Second, the study shows that heterogeneity in own shares is crucial to differentiate risk levels. The structure of transfers in a society shapes optimal risk levels through the Bonacich measure of a network representing a slight transformation of the matrix of exchanges. Further, we showed that more revenue sharing generates more risk taking on average, although heterogeneity may induce a reduction of the risk taken by some individuals. Last, the structure of transfers indicates whether agents over or under invest with regard to that maximizing the sum of utilities, and we found that societies with high own shares generally suffer from under investment in the risky technology.

This work may open at least two directions for future research. First, it would be interesting to test some empirical implications of the analysis. More precisely, capturing proxi variables for risk taking as well as for the network of transfers, theorems 1 and 2 might be tested. Second, our model examined the impact of heterogeneity of transfers risk-taking, but it disregarded the fact that transfers may be themselves sensitive to the level of risk in the society. Even if our analysis suggested that the participation constraint is satisfied for the most realistic cases, it would be challenging to propose a set up with endogenous sharing rules in the context of social networks.

APPENDIX

Definition 1 (Strict diagonal-dominance) A $n \times n$ matrix $M = [m_{ij}]$ is strictly diagonal-dominant if $|m_{ii}| > \sum_{j \neq i} |m_{ij}|$ for all $i$.

Definition 2 (row-stochasticity) A $n \times n$ matrix $M = [m_{ij}]$ is row-stochastic if $m_{ij} \in \mathbb{R}^+$ for all $i, j$ and $\sum_{j} m_{ij} = 1$ for all $i$.

Preliminary result 1 If a $n \times n$ matrix $M$ is strictly diagonal-dominant, the equation $MZ = J$ admits a unique solution.
Lemma 1 Consider three parameters $\delta \in ]0,1[$, $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$, such that $\alpha \frac{1-\delta}{\delta} \neq \beta$. Consider also a $n \times n$ row-stochastic matrix $A = [a_{ij}]$ such that $a_{ii} > \frac{\delta}{1+\delta}$ for all $i$. Define $E = [e_{ij}]$, with $e_{ii} = 0$ for all $i$ and $e_{ij} = \frac{a_{ij}}{a_{ii}}$ for all $i, j \neq i$. The system of equations such that, for all $i$,

$$a_{ii}x_i + \delta \sum_{j \neq i} a_{ij}x_j = \alpha + \beta a_{ii}$$

admits a unique solution

$$x_i = \frac{\alpha}{\delta} - \left( \alpha \frac{1-\delta}{\delta} - \beta \right) B_i(E; -\delta)$$

with $B_i(E; -\delta) \in ]0,1[$.

Proof of lemma 1. We consider the following transformation:

$$v_i = \left( \frac{1}{\alpha \frac{1-\delta}{\delta} - \beta} \right) \left( \frac{\alpha}{\delta} - x_i \right)$$

Equation (9) becomes:

$$a_{ii} \left( \frac{\alpha}{\delta} - \left( \alpha \frac{1-\delta}{\delta} - \beta \right) v_i \right) + \delta \sum_{j \neq i} a_{ij} \left( \frac{\alpha}{\delta} - \left( \alpha \frac{1-\delta}{\delta} - \beta \right) v_j \right) = \alpha + \beta a_{ii}$$

Dividing all terms by $a_{ii}$, and taking account of $\sum_{j \neq i} a_{ij} = 1 - a_{ii}$, one obtains:

$$v_i + \delta \sum_{j \neq i} e_{ij} v_j = 1$$

or in matrix form $(I + \delta E)V = J$. Since $a_{ii} > \frac{\delta}{1+\delta}$ for all $i$, the matrix $I + \delta E$ is strictly diagonal-dominant. The preliminary result 1 applies with $M = I + \delta E$.

Inverting the system, the solution writes as a Bonacich measure $v_i = B_i(E; -\delta)$ with

$$B(E; -\delta) = \sum_{k=0}^{\infty} (-\delta)^k E^k J$$. Rearranging,

$$B(E; -\delta) = I(I - \delta E)J + (\delta E)^2(I - \delta E)J + (\delta E)^4(I - \delta E)J + \cdots$$

Factorizing, one obtains:

$$B(E; -\delta) = \left( \sum_{k=0}^{\infty} \delta^{2k} E^{2k} \right) \cdot (I - \delta E)J$$
Notice that $\delta^{2k}[E^{2k}]_{ij} > 0$ for all $k, i, j$. Further, if $a_{ii} > \frac{\delta}{1+\delta}$ for all $i$, the vector $(I - \delta E)J > 0$. Hence, $B(E; -\delta) > 0$. More, note that a solution of $(I + \delta E)Z = J$ also writes $Z = J - \delta E Z$. That is, if $Z > 0$, clearly $Z < J$. □

**Proof of proposition 2.** We examine the allocation $x = (x_1, x_2, \ldots, x_n)$ that maximizes individual utility, and we will see that the maximized utility is lower than the equilibrium utility under full equal sharing. We conclude that the equilibrium utility, which is lower than the maximized utility over all allocations, is also smaller than the equilibrium utility under full equal sharing.

We first notice that

$$U^{FES}_i = 1 + x^{FES}_i \left[ \mu - 1 - \frac{\rho \kappa \sigma^2}{2} x^{FES}_i \right] - \frac{(1 - \rho) \kappa \sigma^2 (x^{FES}_i)^2}{2 n}$$

(16)

Second, in general,

$$\frac{\partial U_i}{\partial x_j} = \lambda_{ij} \left( \mu - 1 - \rho \kappa \sigma^2 \sum_k \lambda_{ik} x_k \right) - \frac{\lambda_{ij} \rho \kappa \sigma^2}{2} \sum_k \lambda_{ik} x_k - (1 - \rho) \kappa \sigma^2 \lambda_{ij}^2 x_j$$

(17)

That is, imposing $\frac{\partial U_i}{\partial x_j} = 0$ and rearranging, we find:

$$\lambda_{ij} x_j + \rho \sum_{k \neq i} \lambda_{ik} x_k = h$$

(18)

Equivalently,

$$(1 - \rho) \lambda_{ij} x_j + \rho \sum_k \lambda_{ik} x_k = h$$

(19)

Summing over all agents, we obtain

$$\sum_k \lambda_{ik} x_k = x^{FES}$$

(20)

Hence, the individual utility $U^{max}_i$, as maximized over all possible allocations, is written:

$$U^{max}_i = 1 + x^{FES}_i \left[ \mu - 1 - \frac{\rho \kappa \sigma^2}{2} x^{FES}_i \right] - \frac{(1 - \rho) \kappa \sigma^2 (x^{FES}_i)^2}{2 n}$$

(21)

That is, $U^{max}_i < U^{FES}_i$ if and only if

$$\sum_j \lambda_{ij}^2 x_j^2 > \frac{(x^{FES}_i)^2}{n}$$

(22)
But, from equations (19) and (20), we deduce that:

\[ \lambda^2_{ij} x^2_j = \left( \frac{h - \rho FES}{1 - \rho} \right)^2 \]  

(23)

Hence, inequality (22) is satisfied if and only if

\[ n \left( \frac{h - \rho FES}{1 - \rho} \right)^2 > \frac{(FES)^2}{n} \]  

(24)

Recalling that \( x^{FES} = \frac{nh}{1 + (n-1)\rho} \), inequality (24) is written

\[ (1 + (n-1)\rho)^2 - 2n\rho(1 + (n-1)\rho) + n^2 > (1 - \rho)^2 \]  

(25)

That is,

\[ (n-1)^2(1 - \rho)^2 + 2n(1 - \rho)(1 + (n-1)\rho) > (1 - \rho)^2 \]  

(26)

which is equivalent to

\[ (1 - \rho)(n - 2) + 2(1 + (n-1)\rho) > 0 \]  

(27)

which is true, and we are done. ■

**Proof of theorem 1.** The system of first order conditions is linear. Under assumption 2, the matrix \( I + \rho\Gamma \) is diagonal-dominant, which guarantees that the system represented by equation (2) is invertible. Further, the solution is unique by linearity of the system of first order conditions, and more, Bonacich measures are well defined.

Noticing that the matrix of exchanges is row-stochastic (it is both symmetric and column-diagonal), we apply lemma 1 with \( \delta = \rho, \alpha = h, \beta = 0, A = \Lambda \). ■

**Proof of theorem 2.** We use the following lemma:

**Lemma 2 (adapted from Farkas’s lemma)** Let \( Q \) be an \( n \times n \) matrix. If the equation \( Q^T x = J \) admits a positive solution, then for all \( y \in \mathbb{R}^n \) such that \( Qy \geq 0 \), we have \( \sum y_i \geq 0 \).

We let matrix \( \Lambda_\rho \) denote the matrix with diagonal element \( \lambda_{ii} \) and off-diagonal elements \( \rho \lambda_{ij} \). We will see that the conditions of the lemma apply if we fix \( Q = \Lambda_\rho \) and \( y = x^* - x^* \).
First, we prove that there exists a positive solution to \((\Lambda_\rho)^T x = J\):

The condition writes:

\[
\lambda_i x_i + \rho \sum_{j \neq i} \lambda_{ij} x_j = 1
\]  

(28)

Since the matrix \(\Lambda\) is column-stochastic, the matrix \(\Lambda^T\) is row-stochastic. We can therefore apply lemma 1 with \(\delta = \rho\), \(\alpha = 1\), \(\beta = 0\), \(A = \Lambda^T\), and we conclude that there is a positive solution to the system \((\Lambda_\rho)^T x = J\).

Second, we see that \(\Lambda_\rho (x' - x) \geq 0\):

Indeed, we observe that \([\Lambda_\rho x']^*_i = h\), and \(\Lambda_\rho x' = \Lambda_\rho x' - \Theta_\rho x\). Then, \([\Lambda_\rho x']^*_i = h - [\Theta_\rho x']^*_i\). Hence, recalling that \(\Lambda_\rho x' = h\),

\[
[\Lambda_\rho (x' - x)]^*_i = -(\theta_i x'^*_i + \rho \sum_{j \neq i} \theta_{ij} x'^*_j)
\]  

(29)

Since matrix \(\Lambda'\) satisfies assumption 2, theorem 1 implies that \(h < x'^*_i < \frac{h}{\rho}\) for all \(i\). Given that all \(\theta_{ij} > 0\), \(x'^*_i + \rho \sum_{j \neq i} \frac{\theta_{ij}}{\theta_{ii}} x'^*_j > h + h \sum_{j \neq i} \frac{\theta_{ij}}{\theta_{ii}}\). Since \(\sum_{j \neq i} \frac{\theta_{ij}}{\theta_{ii}} = -1\), \(x'^*_i + \rho \sum_{j \neq i} \frac{\theta_{ij}}{\theta_{ii}} x'^*_j > 0\) and we are done.

Third, we apply lemma 2 and conclude that

\[
\sum_i (x'^*_i - x^*_i) \geq 0
\]  

(30)

which proves the theorem. ■

Proof of proposition 3. Consider one solution \(x\) associated with parameter \(\rho\) and one solution \(x'\) associated with \(\rho' > \rho\). For all \(i\), we have:

\[
\lambda_i x_i + \rho \sum_{j \neq i} \lambda_{ij} x_j = h
\]

and

\[
\lambda_i x'_i + \rho' \sum_{j \neq i} \lambda_{ij} x'_j = h
\]

Basically,

\[
\Lambda_\rho (x - x')_i = h - (\lambda_i x'_i + \rho \sum_{j \neq i} \lambda_{ij} x'_j)
\]
Hence, as $\rho' > \rho$,
\[
\Lambda_{\rho}(x - x')_i > h - \left(\lambda_{ii}x'_i + \rho' \sum_{j \neq i} \lambda_{ij}x'_j\right)
\]
That is, for all $i$,
\[
\Lambda_{\rho}(x - x')_i > 0
\]
Since matrix $\Lambda$ satisfies assumption 2, the solution of $\Lambda_{\rho}^Tz = J$ is positive. Thus, lemma 2 applies (see the proof of theorem 2), and we are done. ■

Proof of the proposition 4. In general, utility is written:
\[
U_i(X; \Lambda) = \sum_j \lambda_{ij}(\mu x_j + 1 - x_j) - \frac{\kappa \sigma^2}{2} \sum_j \lambda_{ij}^2 x_j^2 - \frac{\kappa \rho \sigma^2}{2} \left[\left(\sum_j \lambda_{ij} x_j\right)^2 - \sum_j \lambda_{ij}^2 x_j^2\right]
\]
That is,
\[
U_i(X; \Lambda) = 1 + \left(\sum_j \lambda_{ij} x_j\right)\left[\mu - 1 - \frac{\kappa \rho \sigma^2}{2} \sum_j \lambda_{ij} x_j\right] - \frac{\kappa (1 - \rho) \sigma^2}{2} \sum_j \lambda_{ij}^2 x_j^2
\]
The FOC can be written:
\[
\sum_j \lambda_{ij} x_j = 1 \cdot \left(h - (1 - \rho)\lambda_{ii}x_i\right)
\]
while equilibrium utility in isolation writes $1 + \frac{h}{2}(\mu - 1)$. Thus, little calculation indicates that agent $i$’s utility exceeds profit in isolation iff:
\[
\lambda_{ii}^2 x_i^2 + \rho \sum_{j \neq i} \lambda_{ij}^2 x_j^2 \leq h^2
\]
Or equivalently
\[
\left(\frac{\lambda_{ii}x_i}{h}\right)^2 + \rho \sum_{j \neq i} \left(\frac{\lambda_{ij}x_j}{h}\right)^2 \leq 1
\]
The FOC implies that $\lambda_{ii}x_i < h$. Hence, the inequality (35) is implied by:
\[
\frac{\lambda_{ii}x_i}{h} + \rho \sum_{j \neq i} \frac{\lambda_{ij}x_j}{h} \leq 1
\]
Notice that when assumption 2 applies, equilibrium risk levels are positive. Hence, assuming that $\max_{i \neq j} \lambda_{ij} \leq \lambda_{jj}$ for all $j$, we obtain that $\frac{\lambda_{ii}x_i}{h} < 1$, and we are done. ■
Proof of proposition 5. Simple computation entails:

\[
\frac{1}{\kappa} \frac{\partial W}{\partial x_i} = \frac{\mu}{\kappa} - \rho \sigma^2 \sum_j \lambda_{ji} \sum_{k \neq i} \lambda_{jk} x_k
\]

\[-\sigma^2 x_i \sum_j \lambda_{ji}^2 \]  

(37)

That is, \( \frac{\partial W}{\partial x_i} = 0 \) if and only if

\[\psi_{ii} \hat{x}_i + \rho \sum_{j \neq i} \psi_{ij} \hat{x}_j = h\]  

(38)

Since the matrix \( \Lambda \) is bi-stochastic, \( \sum_{j \neq i} \psi_{ij} = 1 - \psi_{ii} \); that is, the matrix \( \Psi \) is row-stochastic. We can therefore apply lemma 1 with \( \delta = \rho, \alpha = h, \beta = 0, A = \Psi \) (and thus \( E = \Phi \)). In particular, \( \psi_{ii} > \frac{\rho}{1+\rho} \) for all \( i \) implies that the matrix \( I + \rho \Phi \) is strictly diagonal-dominant. ■

REFERENCES
Ghiglino C. and S. Goyal, 2008, Keeping up with the neighbours: social interaction in a market economy, mimeo.
Kanbur, S., 1981, Risk taking and taxation: an alternative perspective, Journal of