A demand-based mechanism driving the income-pollution relation*

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March 22, 2013

Abstract

The purpose of this paper is twofold. First, we state the general conditions on preferences and pollution technology supporting a non-monotonic relationship between income and environmental quality. In particular, the emergence of a non-monotonic relationship between income and pollution requires either the pollution technology or the preferences on consumption and environmental quality to be non-homothetic. In addition we show that a non-monotonic relationship between pollution and income may emerge quite easily with non-homothetic preferences. We complete our analysis by incorporating this demand-based mechanism into a dynamic general equilibrium model to study the dynamic evolution of the income-pollution relation along the development process.

JEL classification codes: Q2; D62; H23
Keywords: Environmental Kuznets Curve; Pollution; Abatement; Homotheticity; Economic Growth

*Financial support from the Spanish Ministry of Economy and Competitiveness through grant ECO2011-23959; the Xunta de Galicia through grant 10PXIB300177PR; and the Generalitat of Catalonia through grant SGR2009-1051 is gratefully acknowledged. The paper has benefited from comments by participants in the 5th Workshop on Energy and Environmental Economics (A Toxa).

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1. Introduction

A vast literature has studied in recent period the relationship between environmental degradation and economic growth. The address of this issue is very relevant for predicting the long-run effects of economic growth on social and individual welfare. Following this motivation a large number of theoretical and empirical papers has investigates whether or not pollution is an inescapable consequence of income growth. These studies assume that pollution is a by-product of either production or consumption and that private agents can still devote resources to abate the levels of pollution. At the empirical ground there is not a consensus about the nature of the relationship between income and pollution, whereas the theoretical studies have focused on different mechanisms that may be behind this relation. In this paper we theoretically show that the relationship between pollution and income may follow a non-monotonic path if preferences on consumption and environmental quality are non-homothetic. To this end, we characterize the dynamics of pollution in a general equilibrium model that incorporates this kind of preferences.

The aforementioned literature has mostly considered the so-called Environmental Kuznets Curve (EKC, henceforth) as an empirical hypothesis: the relationship between pollution and income would exhibit an inverted U-shaped path. More precisely, as Stern (2004) states, "during the early stages of economic growth, degradation and pollution increases, but beyond some level of income per capita the trend reverses." The eventual existence of an EKC would not only lead economic growth to be compatible with the preservation of environmental quality, but would even convert economic growth into a tool to improve that quality. This conclusion stresses the importance of empirically testing the existence of this EKC and of theoretically exploring its potential determinants.

There is a large empirical literature that tries to estimate regularities in the behavior of pollution along the development process. The results from these empirical studies are inconclusive about the relationship between pollution and income. In particular, the empirical evidence is not robust to changes in the econometric specification and in data. Some studies support the existence of an inverted U-shaped, and even an N-shaped, relationship between income per capita and the emissions of some pollutants. However, one cannot derive the existence of empirical regularities between pollution and income per capita at the aggregate level.

The empirical debate should lead to looking for the existence of possible economic foundations that may drive the relationship between pollution and income along the development process. Following the terminology popularized by Grossman and Krueger (1991), Brock and Taylor (1995) characterize the links between economic growth and environmental quality through the interaction of three channels: scale, composition and technique effects. Economic growth creates a scale effect as increasing output tends to raise emissions. When income grows composition effect also emerges as the sectoral structure changes firstly from agriculture to industry leading pollution to increases, and subsequently the sectoral structure moves from industry to services provoking a decreasing in pollution. Finally, there is a technical effect as technological progress replaces dirty technologies and improves the productivity of pollution abating environmental effort. Dinda (2004) or Kijima et al. (2011) provide an overview
to the theoretical literature that tries to understand and disentangle the underlying mechanisms that may drive the relationship between income and pollution.

Our purpose in this paper is twofold. We first derive the general conditions on preference and technology supporting a non-monotonic relationship between income and pollution. We prove that the dependence of pollution on income is fully determined by the relationship between two types of elasticities: (i) the income-elasticities of consumers’ expenditures on consumption and abatement; and (ii) the elasticities of pollution with respect to emissions and abatement. In particular, the emergence of a non-monotonic relationship between income and pollution requires either the pollution technology or the preferences on consumption and environmental quality to be non-homothetic. By using this general result, we also clarify the existing theoretical contributions on the relationship between income and pollution. In particular, we show that many of the theoretical models obtaining a negative relationship between income and pollution are based on one of these non-homotheticities though they are disguised under seemingly different assumptions.

In addition we show that a non-monotonic relationship between pollution and income may emerge quite easily with non-homothetic preferences. In particular, we propose a demand-based mechanism that are sufficiently flexible to allow any shape for the income-pollution path (monotonic, inverted U-shaped or N-shaped) by imposing plausible constraints on the parameter configuration. The key ingredient of our mechanism is the existence a minimum consumption requirement that leads income-elasticities of consumption and abatement to be different and to vary with income. We pretend to show that the income-pollution relation is a consequence of the process of development. It is well accepted that the first steps of this process are driven by the existence of a minimum level of subsistence that conditions the emergence of the different stages of development. With these non-homothetic preferences, changes in income alter the composition of consumers’ expenditure in favor of abatement. This change in expenditure composition may generate a negative relationship between pollution and income even when the scale of abatement in pollution is smaller than the scale of emissions. This result is in stark contrast with the literature that states that the existence of this negative relationship requires abatement to have a return to scale in the production of pollution larger than the one of emissions (see, for instance, Andreoni and Levinson, 2001; or Plassmann and Khanna, 2006a).

Many studies analyzing the relationship between income and pollution are based on models without feedback from the environment to economic growth. However, as Arrow et al. (1995) has pointed out, the economic activity is also affected by the evolution of environmental quality. Understanding the links between environment and economic growth then requires to follow a dynamic general equilibrium framework. In the final part of the paper we insert our demand-based mechanism in a general equilibrium model with capital accumulation and exogenous technical progress to characterize the dynamics of the income-pollution relation. In this framework there are two forces driving the relationship between pollution and income: (i) our demand-based mechanism; and (ii) the exogenous trend imposed by the technical progress on pollution. These two forces operate in the opposite direction when the scale of abatement in the production of pollution is smaller than the scale of emissions. We show that the balance between the forces depends on the level of income, which may generate non-monotonic
relation between pollution and income. In particular, we show that the trend effect dominates for sufficiently large levels of income. Hence, if the scale of abatement in the production of pollution is smaller than the scale of emissions, long-run economic growth will inevitably lead to an environmental degradation. However, during the transition the demand-based mechanism may dominate, so that a negative relationship between pollution and income may emerge along the middle stages of development process.

The paper is organized as follows. Section 2 derives the general conditions on preferences and pollution technology supporting a negative relationship between pollution and income. Section 3 presents a partial equilibrium model to illustrate how the demand-based mechanism based on non-homothetic preferences operates in driving the income-pollution relationship. Section 4 incorporates that demand-based mechanism in a general equilibrium model with capital accumulation to characterize the evolution of the relationship between pollution and income along the development process. Section 5 presents some conclusions and final remarks.

2. Revisiting the conditions for EKC

The purpose of this section is to derive general conditions on preferences and technologies that may generate a non monotonic relationship between pollution and income, as a necessary step to obtain an environmental Kuznets curve. Our main contribution at this point would be to clarify the current knowledge on this issue, that is, to find the common foundations among the existing contributions on this issue. To this end, we consider an economy with a representative individual, who derives utility from consumption and disutility from pollution associated to consumption. This representative individual then has a utility function \( u(c, p) \), where \( c \) represents consumption and \( p \) is net pollution. Utility is increasing and concave in \( c \) and decreasing and convex in \( p \). Net pollution is given by a function \( P(c, e) \), where \( e \) is the level of income that the representative individual spend on abatement. Net pollution is an increasing function in consumption and decreasing in abatement effort. Finally, the representative consumer faces to the following budget constraint \( R = c + e \), where \( R \) is disposable income.

Let us denote the income elasticities of consumption and abatement effort as \( \varepsilon_c^R \) and \( \varepsilon_e^R \), respectively. We assume that both consumption and environmental quality are normal goods, so that \( \varepsilon_c^R > 0 \) and \( \varepsilon_e^R > 0 \). We also define the elasticity of net pollution with respect to consumption and abatement effort as \( \varepsilon_P^R \) and \( \varepsilon_p^R \), respectively. Given the properties of the pollution function, we note that \( \varepsilon_P^R > 0 \) and \( \varepsilon_p^R < 0 \).

In this framework, the following result characterizes the pollution-income relationship in terms of the properties of preferences and pollution technology:

**Proposition 2.1.** \( \frac{\partial p}{\partial R} \leq 0 \) if and only if

\[
\frac{\varepsilon_c^R}{\varepsilon_c^R} + \frac{\varepsilon_P^R}{\varepsilon_p^R} \leq 0. \tag{2.1}
\]

**Proof.** By totally differentiating \( P(c, e) \) and dividing the result by \( \partial R \), we obtain

\[
\frac{\partial p}{\partial R} = P_c \left( \frac{\partial c}{\partial R} \right) + P_e \left( \frac{\partial e}{\partial R} \right), \tag{2.2}
\]
where $P_e$ and $P_c$ denotes the partial derivative of $P(c, e)$ with respect to $c$ and $e$, respectively. By using the definitions of $\varepsilon_c^R$, $\varepsilon_e^R$, $\varepsilon_p$, and $\varepsilon_p^R$, Equation (2.2) can be written as

$$\frac{\partial p}{\partial R} = \left( \varepsilon_p \varepsilon_c^R + \varepsilon_p^R \varepsilon_e \right) \left( \frac{p}{R} \right).$$

(2.3)

The proposition directly follows from (2.3). ■

From this general condition (2.1) we can derive the specific mechanisms that can drive the pollution-income path and, therefore, the existence of an environmental Kuznets curve. We identify two complementary mechanisms that may generate a non monotonic relationship between income and pollution: (i) a supply-side mechanism based on a non homothetic technology for pollution $P(c, e)$; and (ii) a demand-side mechanism based on a non homothetic utility function $u(c, p)$.

2.1. Conditions on pollution technology

From Condition (2.1) we derive two conclusions about the role that the pollution technology plays in driving the income-pollution relation. We first observe that a negative relationship between pollution and income may not require the scale of abatement in pollution to be larger than the scale of consumption (i.e., emissions). This condition is only necessary when the income elasticity of the expenditure on consumption are equal to the income elasticity of the expenditure on abatement (i.e., when the utility function is homothetic).

In addition, we conclude that an alternative to generate a non monotonic relation between pollution and income is to consider a non homothetic technology for net pollution $P(c, e)$. This property leads the elasticities of pollution with respect to its determinants to depend on income. Therefore, contrary to Lemma 1 in Plassman and Khanna (2006b), we show that non homogeneity of $P(c, e)$ is not a necessary condition for the EKC provided that the utility $u(c, p)$ is homothetic. What is necessary for a monotonic income-pollution relationship is that the slope of the isoquant curves of $P(c, e)$ are constant along a ray emanating from the origin. In this case, the tangency point between the production possibility frontier in $c$ and $e$ and the relative price moves along a ray emanating from the origin when this frontier expands with income. In order words, the marginal rate of transformation between $c$ and $e$ does not depend on income. This only requires the technology to be homothetic. For instance the function $P(c, e) = \theta \log(c) - \pi \log(e)$, is not homogenous but is homothetic and, therefore, the ratio $\frac{\varepsilon_p}{\varepsilon_p}$ is constant in income (i.e. constant in $\xi$). The technology introduced by Andreoni and Levinson (2001) $P(c, e) = c - e^\alpha e^\beta$ generates a non monotonic relationship between pollution and income because it is non homothetic if $\alpha + \beta \neq 1$.

2.2. Conditions on preferences

Other way of obtaining a non-monotonic pollution-income relation is by means of a non homothetic utility function $u(c, p)$. This forces the marginal rate of substitution between $c$ and $p$ to depend on income. In terms of condition (2.1), this means that the income elasticities of expenditures on consumption and abatement are different and depends on income.
Non homothetic preferences arise in quite plausible scenarios. A natural source of this property on preferences is when individuals have aspirations in consumption: like, for instance, minimum level of subsistence; or endogenous preferences caused by habit formation or consumption externalities. In the next sections, we will analyze the income-pollution relation in presence of aspirations in consumption. Preferences are also non homothetic when the utility function is additively separable between $c$ and $p$, with the two parts with different degrees of homogeneity. An example of these preferences is given by the utility given by Stokey (1998):

$$u(c, p) = \frac{e^{\gamma - \sigma}}{1 - \sigma} - B \frac{p^{1 - \gamma}}{1 - \gamma},$$

with $\sigma \neq \gamma$. Other possible source of non homothetic preferences is the case where utility depends on consumption and environmental quality, which is given by the difference of an endowment of quality and net pollution. In other words, preferences are given by $u(c, q - p)$, where $q$ is the endowment of environmental quality. In this case, the marginal rate of substitution between consumption and abatement may depend on income if pollution is not a homogenous function of degree zero.

By considering the aforementioned conditions on pollution technology and preferences, we should clarify the results from Plassman and Khanna (2006b). As these authors shown, by plugging $P(c, e)$ in the utility function $u(c, p)$ we obtain a utility function on consumption and effort $v(c, e)$. If this last utility function is not homothetic we can obtain that the relation between the two arguments $c$ and $e$ is not monotone in income. Technically, this means that the slope of the indifference curves of $v(c, e)$ are not constant along a ray emanating from the origin. Observe that $v(c, e)$ is not homothetic because at least one of the following conditions holds: (i) $P(c, e)$ is no homothetic; or (ii) $u(c, p)$ is no homothetic.

3. A model with non-homothetic preferences

In the remainder of paper we will characterize the pollution-income path when preferences defined on consumption and pollution are non-homothetic. As was shown in the previous section, a non-monotonic relationship between pollution and income can easily be generated with these preferences because the marginal rate of substitution between consumption and environmental quality depends on income in this case. Our purpose in this section, is to show how this demand-based mechanism operates. To this end, we consider a partial equilibrium model where income is exogenously given. First, we assume that aggregate consumption is subject to a minimum level of subsistence. Hence, we consider that the representative consumer derive utility of consumption and environmental quality. In particular, the utility function is given by

$$u(c, p) = \gamma \log (c - \tau) - \lambda \log (p),$$

with $\gamma$ and $\lambda$ belonging to $(0, 1)$, and where $c$ is consumption, $\tau$ is a minimum consumption requirement and $p$ is the effective level of pollution. The utility function (3.1) is non homothetic. In particular, the income elasticity of demand for consumption is smaller than one provided that $\tau$ is strictly positive. Obviously, this means that the
income elasticity of demand for environmental quality is larger than one. The demand of environmental quality then grows faster than the demand of consumption when income increases. However, since the minimum consumption \( \bar{c} \) is constant, the income elasticities of demand for consumption and environmental quality are asymptotically equal to one. Hence, we will see that for a sufficiently large level of income the consumption to environmental quality ratio tends to be constant.

Secondly, we consider that pollution \( p \) is a function of gross emissions, that depend on consumption, and the amount \( e \) of income that the individual spends on abatement. As in Selden and Song (1995), the key assumption is that

\[
\lim_{e \to 0} \left( \frac{\partial P}{\partial e} \right) > -\infty. \tag{3.2}
\]

In particular, we assume that a minimum constant level of abatement \( \varphi \) exists even when the individual does not spend any fraction of income on this activity. There are some economic interpretations for \( \varphi \). We can consider that \( \varphi \) is the home production of abatement: any kind of recycling or energy-saving activities that reduce the environmental damage of the economic activity. Alternatively, we can assume that \( \varphi \) is the public expenditure on abatement that the government finance by means of a lump-sum tax. In any case, Assumption (3.2) means that pollution \( p \) is a non-homothetic function. Finally, for the sake of simplicity, and without loss of generality, we assume that emissions is an increasing function of cross-economy average of consumption \( e^a \).

With this assumption, individuals take the level of gross emissions as given when they decide the optimal plan of consumption and abatement effort. With this assumption we obtain explicit solutions that makes clear how our mechanism driving the income-pollution relation works.\(^1\) Following, among many others, Smulders and Gradus (1996), we consider the pollution is given by the ratio between emissions and abatement. This technological specification for pollution is quite flexible for allowing preferences to drive the income-pollution relation. More precisely, we consider

\[
P(c, e) = (e^a)^\theta (\varphi + e)^{-\pi}, \tag{3.3}
\]

with \( \theta > 0 \) and \( \pi > 0 \). Observe that the elasticities of pollution with respect to consumption and abatement are respectively given by

\[
\varepsilon_P^c = \theta, \tag{3.4}
\]

and

\[
\varepsilon_P^e = -\pi \left( \frac{e}{\varphi + e} \right). \tag{3.5}
\]

While the elasticity of pollution with respect to consumption is constant, the elasticity of pollution with respect of abatement effort depends on effort devoted to this activity. We will see that this implies that the later elasticity will be an increasing function of income at the equilibrium because \( e \) is a normal good. Observe that the elasticities are also driven by the scale of emissions and of abatement in pollution, i.e., by \( \theta \) and \( \pi \), respectively.

\(^1\)In the next section we show that this assumption is not relevant for the qualitative conclusions of our analysis.
In our economy, the representative consumer faces to the budget constraint
\[ R = c + e, \]
(3.6)
where \( R \) represents the disposable income. We should impose that \( R > \bar{\sigma} \) in order to

guarantee a positive effective level of consumption \( c - \bar{\sigma} \). Otherwise the utility function
is not well defined for all possible values of income.

Taking as given the average level of consumption in the economy, this consumer
then maximizes (3.1) subject to (3.6) and (3.3), and the non-negative constraint in \( c \)
and \( e \). We characterize the symmetric equilibrium \( c = c^* \). Observe that the minimum
level of abatement \( \varphi \) (i.e., Assumption (3.2)) implies that the consumer does not want
to spend positive values of income on abatement for values of \( R \) sufficiently close to
\( \bar{\sigma} \). By solving the consumer problem, we obtain the following interior demands for
consumption and abatement effort, respectively:
\[ c = \frac{\gamma (R + \varphi) + \lambda \pi \bar{\sigma}}{\gamma + \lambda \pi}, \]
(3.7)
and
\[ e = \frac{\lambda \pi (R - \bar{\sigma}) - \gamma \varphi}{\gamma + \lambda \pi}. \]
(3.8)
Given the values of \( \bar{\sigma} \) and \( \varphi \), the value of \( e \) given by (3.8) is positive if and only if \( R \) is
larger than
\[ R^L \equiv \bar{\sigma} + \frac{\gamma \varphi}{\lambda \pi}. \]
(3.9)
If \( R < R^L \), then a corner solution emerges with \( e = 0 \) and \( c = R \). In this case, the net
pollution is given by
\[ P = R^\theta \varphi^{-\pi}. \]
(3.10)
Therefore, the net pollution \( P \) is an increasing function of income when \( R < R^L \).

Provided that \( R > R^L \), the consumer follows the interior plan given by (3.7) and
(3.8). Observe that along this interior solution the income elasticities of \( c \) and \( e \) are
respectively given by:
\[ \varepsilon^c_c = \frac{\gamma R}{\gamma (R + \varphi) + \lambda \pi \bar{\sigma}}, \]
(3.11)
and
\[ \varepsilon^e_c = \frac{\lambda \pi R}{\lambda \pi (R - \bar{\sigma}) - \gamma \varphi}. \]
(3.12)
Note that \( \varepsilon^c_c < 1 \) and \( \varepsilon^e_c > 1 \) at an interior equilibrium (i.e., as \( R > R^L \)). Moreover,
both elasticities converge to unity as income tends to infinity because the minimum
levels of consumption \( \bar{\sigma} \) and abatement effort \( \varphi \) tend to be negligible as \( R \) grows.
This behavior of income elasticities clearly determines the responds of consumption
and abatement effort to income changes. Hence, these values of income elasticities
are crucial to understand the relation between income and pollution along the interior
equilibrium. By plugging (3.7) and (3.8) in (3.3), we obtain
\[ \frac{\partial p}{\partial R} = \frac{\lambda \pi p}{(\gamma + \lambda \pi)^2 c (\varphi + e)} \left[ \gamma (\theta - \pi) (R + \varphi) - (\gamma \theta + \lambda \pi \bar{\sigma}) \bar{\sigma} \right]. \]
(3.13)
Observe that the pollution decreases with income at an interior equilibrium if the scale of abatement in pollution is larger than the scale of emissions (i.e., $\pi > \theta$). However, even when $\theta > \pi$ there exists a threshold level of income given by

$$R^U = \left[ \frac{\gamma \theta + \lambda \pi^2}{\gamma (\theta - \pi)} \right] \bar{c} - \varphi, \quad (3.14)$$

such that the pollution-income relation is negative (positive) for $R < (>) R^U$. At this point, we should compare the two thresholds $R^L$ and $R^U$. By comparing (3.9) and (3.14), we obtain that $R^L < R^U$ if and only if

$$\varphi < \varphi^* \equiv \frac{\lambda \pi^2 \bar{c}}{\gamma (\theta - \pi)}. \quad (3.15)$$

The following proposition summarizes the income-pollution relationship that arise in our economy:

**Proposition 3.1.** Assume that $R > \bar{c}$. Hence:

(i) If $\pi > \theta$, then $\frac{\partial \varphi}{\partial R} > 0$ for $R < R^L$ and $\frac{\partial \varphi}{\partial R} < 0$ for $R > R^L$.

(ii) If $\theta > \pi$ and $\varphi < \varphi^*$, then $\frac{\partial \varphi}{\partial R} < 0$ when $R \in (R^L, R^U)$, whereas $\frac{\partial \varphi}{\partial R} > 0$ when $R \notin (R^L, R^U)$.

(iii) If $\theta > \pi$ and $\varphi > \varphi^*$, then $\frac{\partial \varphi}{\partial R} > 0$.

**Proof.** The result directly follows from (3.10) and (3.13). ■

From the previous proposition we can characterize the path of the relationship between income and pollution.

**Corollary 3.2.** Assume that $R > \bar{c}$. Hence, the income-pollution relation is given by:

(i) A $N$-shaped curve if and only if $\bar{c} > 0$, $\varphi \in (0, \varphi^*)$ and $\theta > \pi$.

(ii) A inverted $U$-shaped curve if and only if $\varphi > 0$ and $\pi > \theta$.

(iii) A $U$-shaped curve if and only if $\varphi = 0$, $\bar{c} > 0$ and $\theta > \pi$.

(iv) A monotonically increasing curve if and only if $\theta > \pi$ and either $\bar{c} = 0$ or $\varphi > \varphi^*$.

(v) A monotonically decreasing curve if and only if $\varphi = 0$ and $\pi > \theta$.

The relevant contribution of our analysis is that a negative relation between income and pollution can arise even when the scale of abatement in pollution is smaller than the scale of emissions (i.e., when $\theta > \pi$.) This is in stark contrast with the previous literature that theoretically derived a non-monotonic relationship between income and pollution (see, e.g., Andreoni and Levinson, 2001; Plassmann and Khanna, 2006a; or Egli and Steger, 2007.) As Colorally 3.2 shows the presence of a minimum consumption requirement allows a non monotonic relationship between pollution and income when
\( \theta > \pi \). In particular, in this case we can obtain a N-shaped relation between income and pollution as some empirical studies derive from data.\(^2\)

The economic intuition of our results are quite simple. In the first levels of development, where income is too close the minimum level of subsistence, individuals are not willing to make any effort on abatement. They devote all income to consumption, so that gross emissions reach the maximum level given by \( R^p \), whereas the abatement is constant and equal to \( \varphi^\pi \). Hence, in this first step of development with low level of income, the net pollution is an increasing function of income.

After reaching a minimum level of income (i.e., \( R > R^U \) in the model), individuals become to be willing to spend income in abatement because the level of consumption is sufficiently larger than the minimum requirement \( \bar{\pi} \). Furthermore, given the income-elasticities of consumption and abatement, the response of the later to increases in income is relatively larger (see (3.11) and (3.12)). Hence, the net pollution will decrease with income even when the scale of emissions in pollution is larger than the scale of abatement. For levels of income lightly larger than \( R^L \) the ratio \( \varepsilon_c^R/\varepsilon_e^R \) are sufficiently smaller, such that Condition (2.1) determining a negative income-pollution path holds.

As the economy develops the income elasticities converge to one and, thus, the ratio \( \varepsilon_c^R/\varepsilon_e^R \) increases. At the same time, the absolute value of the elasticity of net pollution with respect to abatement \( \varepsilon_P^p \) increases with income as the effort in abatement increases. However, this may not be sufficient to change the sign of the relation between elasticities

\[
\frac{\varepsilon_c^R}{\varepsilon_e^R} + \frac{\varepsilon_P^p}{\varepsilon_P^p}
\]

such that the income-pollution path still maintains a negative slope. In particular, if the scale of emissions in pollution is smaller than the scale of abatement, then the net pollution monotonically converges to zero. Otherwise, the income-pollution path becomes positive when income reaches a sufficiently large level (i.e., \( R > R^U \) in the model.)

4. A dynamic general equilibrium model

In the previous section we consider a static partial equilibrium model, and we studied how exogenous changes of income affected the level of pollution. In the present section we extend the analysis of the income-pollution relationship to a dynamic general equilibrium model. This extension is necessary to complete our analysis for two reasons. Firstly, the income is now endogenously determined as an equilibrium result, so that there is a feedback between income and pollution along the equilibrium path. In addition, both income and pollution exhibit in this new framework a trend. In particular, the trend of pollution will be positive if the scale of emissions in pollution is larger in absolute terms than the scale of abatement. We will study whether or not a negative relationship between pollution and income still arises when both variables exhibit a positive trend.

\(^2\)For instance, Sengupta (1997) found that the emissions of \( CO_2 \) exhibit an N-shaped relation with income. However, as Kijima et al. (2010) point out, there is not theoretical studies explaining the possible fundamentals of this income-pollution path.
We extend the neoclassical growth model to study the dynamic path of pollution. In particular, the economy consists of competitive firms and an infinitely lived representative consumer. We assume that a single good is produced in each period by means of a constant-returns-to-scale technology, which uses labor and capital as inputs. For simplicity in the exposition, and without loss of generality, we consider a Cobb-Douglas production function. Hence, output per capita is given by
\[ y_t = k_t^\alpha \left( A t l_t \right)^{1-\alpha}, \]
with \( \alpha \in (0, 1) \), \( A_t = (1 + \eta)^t \), and where \( k_t \) and \( l_t \) are the stock of capital and the employment per capita, respectively. Furthermore, the production of this single good generates, as a by-product, emissions of pollutants to the atmosphere. We consider that gross pollution is an increasing function of output. However, the representative consumer can devote an amount \( e_t \) of his income to abate the emissions. Hence, the net or unabated pollution is given by
\[ p_t = \frac{\overline{y}_t^\theta}{(\varphi_t + e_t)^\pi}, \quad \text{(4.1)} \]
with \( \theta > 0 \) and \( \pi > 0 \), and where \( \overline{y}_t \) is the average output across the economy and \( \varphi_t \) is the exogenous, time-varying (and cost-free) abatement. Therefore, the single good can be consumed, invested or used for abating pollution.

The representative consumer is endowed with an initial stock of capital \( k_0 \) and with a unit of time in each period that inelastically supplies as labor, i.e., \( l_t = 1 \). The consumer’s preferences are represented by the utility function
\[ u(c_t, p_t) = \frac{\left( c_t - \overline{c}_t \right) ^\gamma p_t ^{-\lambda} \left[ \frac{1}{1-\sigma} - 1 \right]}{1-\sigma}, \quad \text{(4.2)} \]
with \( \gamma \) and \( \lambda \) belonging to \( (0, 1) \), and where \( c_t \) is per capita consumption, \( \overline{c}_t \) is a time-varying aspiration or minimum requirement in consumption, and \( \sigma > 0 \) is the inverse of the intertemporal elasticity of substitution of the composite good. Aspiration means that consumer takes an exogenous reference with respect which his consumption is compared to. We consider that both minimum consumption and exogenous level of abatement permanently grow at the stationary growth rate (exogenous rate of technical progress) to guarantee the existence of balanced growth path, i.e.,
\[ \overline{c}_t = (1 + \eta)^t \overline{c}_0, \]
and
\[ \varphi_t = (1 + \eta)^t \varphi_0, \]
with \( \overline{c}_0 > 0 \) and \( \varphi_0 > 0 \). The objective of consumer is to maximize
\[ \sum_{t=0}^{\infty} \beta^t u(c_t, p_t), \]
subject to (4.1) and the budget constraint
\[ (1 + r_t) k_t + w_t = k_{t+1} + c_t + e_t + \delta k_t, \quad \text{(4.3)} \]
where \( r_t \) and \( w_t \) are the rental rates of capital and labor, respectively; \( \beta \in (0,1) \) are the subjective discount rate; and \( \delta \in [0,1] \) is the depreciation rate of capital stock. In solving this maximization problem, consumer takes the emissions as given, so that production is a source of inefficiency in this economy.

Given the initial stocks of capital \( k_0 \) and efficient units of labor \( A_0 \), and the initial levels of aspirations \( \bar{c}_0 \) and exogenous abatement \( \varphi_0 \), a competitive equilibrium is defined as a set of path for prices \( \{ r_t, w_t \} \), allocations \( \{ c_t, e_t \} \) and capital stock \( \{ k_t \} \), such that: (i) the path \( \{ c_t, e_t \} \) solves the representative consumer’s problem; (ii) the path \( \{ k_t \} \) maximizes the firms’ profits; and (iii) the market clearing condition for goods, labor and capital holds, i.e.,

\[
y_t = k_{t+1} + c_t + e_t - (1 - \delta) k_t. \tag{4.4}
\]

In equilibrium, competition among profit-maximizing firms ensures that both production factors are paid their marginal products. Hence, the profit maximization conditions are

\[
r_t = \alpha k_t^{\alpha - 1} A_t^{1 - \alpha}, \tag{4.5}
\]

and

\[
w_t = (1 - \alpha) k_t^\alpha A_t^{-\alpha}. \tag{4.6}
\]

The representative consumer’s problem involves two margins. First, total income must be allocated between total expenditure and investment. In addition, total expenditure must be distributed between consumption and abatement. These two trade-offs are characterized by the first order conditions of the aforementioned problem. By standard procedure, we find these first order conditions, then substitute in the consistency equilibrium condition \( y_t = y_t \), and finally rearrange the expressions to summarize the necessary and sufficient maximization conditions by means of the following system of equations:

\[
u_c(c_t, p_t) = -u_p(c_t, p_t) \left( \frac{\partial p_t}{\partial e_t} \right), \tag{4.7}
\]

\[
u_c(c_t, p_t) = \beta u_c(c_{t+1}, p_{t+1}) (1 + r_{t+1} - \delta), \tag{4.8}
\]

together with the resource budget constraint (4.3), the pollution function (4.1) and the following transversality condition

\[
\lim_{t \to \infty} u_c(c_t, p_t) k_t = 0,
\]

where \( u_c() \) and \( u_p() \) represent the marginal utilities of consumption and pollution, respectively. Observe that Equation (4.7) says that the marginal rate of substitution between consumption and abatement must be equal to unit (i.e., the relative price of abatement in terms of consumption) at the equilibrium. Equation (4.8) is the usual Euler condition stating that the marginal rate of substitution between present and future consumption must be equal to the future net rate of return on present investment.

Our economy exhibits a balanced growth path (BGP, henceforth) equilibrium, along which output, the stock of capital, consumption and abatement grow at the constant rate \( \eta \). In order to proceed with our analysis we now normalize the variables to remove
the consequences of the long-run growth. In particular, we introduce the following normalized variables:

\[ \hat{k}_t = k_t (1 + \eta)^{-t}, \quad \hat{c}_t = c_t (1 + \eta)^{-t}, \quad \hat{e}_t = e_t (1 + \eta)^{-t} \text{ and } \hat{y}_t = y_t (1 + \eta)^{-t}. \]

Note that the normalized variables remain constant along the BGP. Observe also that pollution \( p_t \) growth at the constant rate \((1 + \eta)^{\theta - \pi}\) along the BGP. Based on this conclusion, the next proposition characterize the asymptotic behavior of pollution.

**Proposition 4.1.** If \( \pi > \theta \) then \( \lim_{t \to \infty} P_t = 0 \), whereas \( \lim_{t \to \infty} P_t = \infty \) when \( \theta > \pi \).

In a BGP the relationship between pollution and income is always monotone. Furthermore, this relationship is negative only when the scale of emissions in pollution is smaller in absolute terms than the scale of abatement, i.e., \( \theta < \pi \). In any case, along the dynamic adjustment to the BGP there may exist a level of capital stock under which pollution decreases with income, even when \( \theta > \pi \). Unfortunately, the income-pollution relation along the dynamic transition cannot be analytically characterized. We will next calibrate our economy to simulate the dynamics of that relationship.

### 4.1. Numerical simulations

The model has to be solved and simulated in order to characterize the dynamic correlation between pollution and GDP. To this end, we choose the parameter values to replicate some facts observed in U.S. data. A period in the model corresponds to a quarter in actual data. The parameterization for the production function, the process governing the accumulation of capital, the discount factor and the intertemporal elasticity of substitution are those commonly used in the RBC literature (see, e.g., Cooley and Prescott, 1995). In particular, we set the values of those parameters to force the BGP equilibrium of our economy to match the share of capital income on GDP, the consumption to GDP ratio, the investment to GDP ratio, the capital stock to GDP ratio and the average growth rate observed in the data. However, it is more difficult to find macroeconomic empirical evidence to set the values of the other parameters, basically the environmental ones. We will follow the previous literature to set these parameters.

Table 1 describes the benchmark values of the parameters that we use in our numerical simulations. With these values, we first obtain that the steady-state ratios of consumption, investment and abatement to GDP are 0.63, 0.27 and 0.1, respectively. As mentioned before, there is no precise evidence about the share of aggregate abatement expenditure on GDP. The existing evidence reduces to the market expenditure on some particular pollutants.\(^3\) In light of these evidence, we might conclude that we obtain an excessive share of abatement expenditure on GDP. However, note that the abatement in the model should match the actual aggregate expenditure on abatement, which includes all domestic and market activities that are costly used to reduce the level of emissions. In any case, the aforementioned figures implies that the shares of consumption and investment on the GDP net of expenditure on abatement are 70% and

---

\(^3\)For instance, Hackett (2011) concludes that the US expenditure on abatement accounts for 2% of GDP.
30%, respectively, which are very close to those observed in the actual data. Secondly, following Kelly (2003) we set the same shares for consumption and pollution on the composite good, i.e., $\gamma = \lambda$. Finally, following Rubio et al. (2009) and Fernandez et al. (2011), we assume that pollution is a concave function of emissions. Furthermore, we assume that the scale of emissions in pollution is larger in absolute terms than the scale of abatement, i.e., $\theta > \pi$. We adopt this assumption because this is the worst scenario to obtain a negative relation between pollution and income. Hence, this assumption seems to be convenient to show the quantitative importance of our demand-based mechanism for the pollution-income relation.

[Table 1]

We simulate the equilibrium dynamics when the initial stock of capital $k_0$ is such that their normalized value $\hat{k}_0$ accounts to the 20% of the stationary value of normalized stock $\hat{k}^*$. We will focus on the dynamic behavior of net pollution along the equilibrium path. Figure 1 graphs the simulated relationship between the logarithmic values of pollution and income. The left-hand side panel shows the behavior of the normalized variables (i.e., the detrended variables), whereas the right-hand side panel gives the dynamic behavior of the trended variables. Note that in this economy the dynamics are driven by two forces: (i) the dynamic adjustment derived from the imbalances in the capital stock with respect the BGP; and (ii) the trend derived from the exogenous technical progress. In studying the dynamic behavior of pollution, we should first focus in the effect of the first of these forces. This force includes the dynamic adjustment in the composition of consumers’ expenditure, which is the basis of our mechanism explaining a potential negative relationship between pollution and output. Therefore, we should decompose the effect of the aforementioned two forces to clarify the relationship between pollution and income. To this end, we first focus in the dynamic behavior of the normalized or detrended variables to illustrate how the demand-based mechanism works in our dynamic general equilibrium model.

[Figure 1]

Figure 1 illustrates that the relation between the detrended values of pollution and output follows an inverted U-shape path in our benchmark economy. As was explained in the previous section this dynamics are crucially determined by the non homotheticity of preferences, which leads the income-elasticities of consumption and abatement to depend on output. First, one should note that the behavior of pollution for sufficiently small levels of income (or, equivalently, capital) are driven by the absence of Inada condition of the abatement technology, which allows consumers to allocate all the income to consumption expenditure. However, we are especially interested on explaining the negative relationship between pollution and income that arises when a minimum level of income is reached. The presence of an aspiration in consumption $\tau_0$ generates that the income-elasticity of consumption and abatement are smaller and larger than unity, respectively. Hence, during the dynamic adjustment to the BGP, there is a meaningful change in the composition of final expenditure in favor of abatement. Figure 2 shows that the ratio from consumption to abatement is decreasing in income in our benchmark economy, whereas this ratio is constant in the model with
homothetic preferences (i.e., when $r_0 = \varphi_0 = 0$). Therefore, the detrended pollution decreases along the transition in the benchmark economy even when the scale of emissions in pollution is larger in absolute terms than the scale of abatement, i.e., $\theta > \pi$. The increasing fraction of income devoted to abatement more than compensates this relatively small returns on abatement exhibited by the net pollution.

[Figure 2]

Formally, the previous mechanism based on the adjustment of expenditure composition should be explained by using Proposition 2.1. From this result, and the analysis in Section 3, we can conclude that there exist two threshold for the detrended stock of capital $\hat{k}^L$ and $\hat{k}^U$, such that the dynamics of the detrended pollution $\hat{p}_t$ is determined by the position of the detrended stock of capital $\hat{k}$ with respect to these thresholds. First, the expenditure on abatement is zero, and then pollution is increasing if $\hat{k}_t < \hat{k}^L$. In the case of strict expenditure on abatement (i.e., $\hat{k}_t > \hat{k}^U$), the detrended pollution decreases with income when $\hat{k}_t < \hat{k}^U$, whereas if $\hat{k}_t > \hat{k}^U$ then the detrended pollution increases with income. In our simulations with $\hat{k}_0 = 0.2k^*$ we observe that $\hat{k}_t < \hat{k}^U$ along the entire dynamic adjustment. This means that the threshold $\hat{k}^U$ is in the right-hand side of the steady state $\hat{k}^*$. Figure 3 plots the dynamics of the detrended pollution as function of the deviations of the detrended capital with respect to its stationary value. If $\hat{k}^L < \hat{k}_0 < \hat{k}^*$ then the detrended pollution is monotonically decreasing, whereas this variable follows a non monotonically dynamic adjustment when $\hat{k}_0 > \hat{k}^*$.

[Figure 3]

We have just explained the behavior of the detrended pollution $\hat{p}_t$. However, we must also be interested whether or not the negative relationship between pollution and income still maintains when we incorporate the trends of those variables. The right-hand side of Figure 1 illustrates this relation in our benchmark economy. We observe that the negative relation still arises during part of the transition adjustment, although the relation becomes positive as the economy reaches a sufficiently large level of income. As was explained before the dynamic evolution of the non-detrended pollution depends on two forces: (i) the adjustment in the composition of final expenditure; and (ii) the exogenous technical progress. These two forces are of opposite sign in our benchmark economy with $\theta > \pi$. During the transition dynamics the income-elasticities of consumption and abatement monotonously approximate to unity, so that the difference between the two income-elasticities monotonously reduce during the transition.\(^5\) The changes in the composition of final expenditure then tends to disappear as the economy approaches to BGP. Therefore, the dynamics of pollution

\(^4\)Note that income monotonously increases when $\hat{k}_0 < \hat{k}^*$, whereas if $\hat{k}_0 > \hat{k}^*$ then this variable monotonously decreases. We have to take this into account to characterize the pollution-income relation.

\(^5\)For simplicity we have assumed that the $r_\tau$ and $\varphi_\tau$ grow at the stationary growth rate $\eta$. This implies that the expenditure adjustment only occurs during the transition. If this two fundamentals was constant, then preferences would be permanently non-homothetic and, therefore, the expenditure effect would also works in the long-run. However, this effect would asymptotically vanish, so that the trend effect would still dominate after some level of income.
tends to be only driven by the exogenous technical progress in the long-run. In our benchmark economy with $\theta > \pi$, the pollution monotonously increases in the long-run.\(^6\)

This dominance of the trend effect in the long-run would be larger, the larger the exogenous growth rate of productivity $\eta$. Obviously, the trend of pollution is a increasing function of $\eta$. In addition, the effect of the dynamic adjustment in the expenditure composition decreases with $\eta$. As was explained before, this effect tends to vanish as the economy approaches to the BGP. In a neoclassical growth model as ours, the speed of convergence is an increasing function of the exogenous rate of technical progress. The larger the growth rate of total factor productivity $\eta$, the smaller the dynamic adjustment to the BGP. Figure 4 shows that the trend effect almost dominates for any level of income when $\eta$ increases to 0.03. However, even in this case, the presence of our demand-based mechanism ensures that pollution for each level of income is smaller during the transition than in the case of an economy with homothetic preferences (i.e., when $c_0 = \varphi_0 = 0$).

[Figure 4]

\subsection{4.2. Social optimum}

Before closing our analysis, we will derive the social planned solution of our model in order to characterize the socially optimal relationship between pollution and GDP. This social planned solution is equivalent to a competitive equilibrium where consumers interiorize that their decisions on capital accumulation affect the flow of emissions. Hence, the difference between our competitive equilibrium and the socially planned solution is the consumer’s margin on the intertemporal allocation of consumption. This margin is characterized by the following social Euler condition:

$$u_c(c_t, p_t) = \beta \begin{cases} u_c(c_{t+1}, p_{t+1}) (1 + r_{t+1} - \delta) \\ + u_P(c_{t+1}, p_{t+1}) \left[ \left( \frac{\partial p_{t+1}}{\partial y_{t+1}} \right) \left( \frac{\partial y_{t+1}}{\partial k_{t+1}} \right) \right] \end{cases}.$$  \hspace{1cm} (4.9)

By comparing (4.8) and (4.9), we observe that they differ in the second term of the right-hand side of (4.9). In deciding the intertemporal allocation of consumption a benevolent social planner takes into account that the present investment will reduce future welfare since the corresponding increase in capital stock will drive emissions up. Therefore, one should expect that the social level of investment will be smaller than the level in the decentralized economy. We will show that point by simulating the social planned solution of our economy.

Figure 5 compares the relation between pollution and income in the competitive economy and in the socially planned solution. The shape of this relation are identical in the two solutions. However, the level of pollution is smaller in the socially planned economy than in the competitive one for any level of income. Hence, the relation between pollution and income does not qualitatively depend on whether or not consumers internalize the external effects derived from their decisions on consumption.

\(^6\)Obviously, the two aformentioned forces go in the same direction when $\theta < \pi$, such that the pollution converges to zero as capital tends to infinite.
and investment. By comparing the investment rate, the consumption rate and the effort on abatement in the two solutions, we can characterize the inefficiency. Figure 6 illustrates this comparison by computing path of these rates in the competitive economy and in the socially planned solution. Firstly, we observe that the investment rate is always larger in the competitive economy. Social planner then indirectly reduces the level of emissions by choosing a lower investment for a given level of income. Note that the reduction on investment has a permanent effect on the path of pollution because that reduction translates in a smaller stock of capital. This explains why the difference between the rate of investment between the competitive equilibrium and the socially planned solution increases along time. We also observe that the path of the investment rate has a hump shape in the two solutions. Obviously, this is a consequence of our demand-based mechanism based on the non homotheticity of preferences.7

Secondly, the comparison between the effort to abatement in the two solutions depends on the level of income. For income sufficiently small, the social planner devotes more effort to abatement than the consumers in the decentralized economy, whereas the inverse is true for large levels of income. The differences for small levels of income is due to the existence of a corner solution for abatement in the decentralized economy. On the contrary, the social planner devotes less effort to abatement for sufficiently large levels of income because the level of emissions is always smaller in the socially planned economy. Finally, we observe that the propensity to consume is always larger in the socially planned economy.

[Figures 5 and 6]

It is obvious that the socially planned solution can be decentralized by either taxing capital income or by subsidizing the expenditure allocated to abatement. The first of these policies reduces the level of investment and, therefore, the level of emissions. On the contrary, the later policy alters the intratemporal margin on final expenditure in favor of abatement.

5. Conclusion

We have analyzed the relationship between pollution and income along the process of economic growth. To this end, we have first derived the general conditions on preferences and pollution technology that ensures a negative income-pollution relation. In particular, we have proved that the emergence of a non-monotonic relationship between income and pollution requires either the pollution technology or the preferences on consumption and environmental quality to be non-homothetic. Based on this result, we have also shown that pollution may easily decrease with income when consumers are subject to a minimum consumption requirement that makes preferences on consumption and environmental quality non-homothetic. This result emerges even when the scale of emissions in the production of pollution is larger than the scale of pollution abating environmental effort. By incorporating this demand-based mechanism into a dynamic

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7By using a sample of 24 OCDE countries, Antras (2001) illustrates that there is clear evidence of a hump shape for the investment rate in the series. He also shows that assuming Stone-Geary preferences, the neoclassical growth model could explain this path of the investment rate.
general equilibrium model, we have showed that the relationship between pollution and income follows an N-shaped path.

The income-pollution path emerging in our economy has some interesting implication for the environmental-oriented fiscal policy and for the comparison across countries. Consider three countries with different levels of income, such that they are placed in different sections of the N-shaped relation between income and pollution. First of all, we observe that these countries may still exhibit the same level of net pollution. However, the future evolution of this pollution will dramatically different in each country. The trade-off between economic growth and environmental quality is transitory for low-income countries. The development in those countries will alter the composition of expenditure in favor of pollution abating effort, so that the positive relationship between pollution and income will be reversed at some point. Therefore, pollution is not a great problem for countries with a sufficiently small income, and the best policy to reduce pollution in those countries would be the development aid.

By the contrary, the trade-off between economic growth and environmental quality is permanent in high-income countries. The minimum consumption requirement is not meaningfully in those countries, so that the composition of expenditure is almost constant. Therefore, the evolution of pollution is only driven by the trend effect of economic growth. In this scenario, governments in the richest country should then set active environmental policies that reduce the impact of income on pollution by distorting consumers’ decisions on consumption and abatement. For instance, the government may either tax consumption or subsidize the effort devoted to abatement. Any of these policies reduces the consumption-abatement ratio, so that the pollution for any level of income is smaller in presence of these public interventions.
References


Table 1. Benchmark economy

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Figure 1. Income-pollution relation (in logs)
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Figure 6. Expenditures rates