Long term care social insurance with two-sided altruism

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Abstract

The purpose of this paper is to look at the design of a social insurance for long term care when altruism is two-sided. The state of dependency has the implication that both the parent and the aiding child are made worse off relative to the state of autonomy. In a world of perfect information, the first best allocation would be implementable by inducing the imperfectly altruistic child to provide the right amount of assistance to his dependent parent and through either a transfer between the two states of nature or a state contingent inheritance tax to smooth marginal utilities across the two states of nature. If those instruments are not available, we design a scheme comprising a long term care benefit and a payroll tax on children’s earnings. This scheme is supposed to redistribute resources across individuals and between the state of nature and to induce caring children to provide the optimal amount of assistance to their parents.

Keywords: long term care, social insurance, two-sided altruism

JEL: H2, H5.
1 Introduction

Long-term care is becoming a big concern for policy makers. Following the rapid aging of our societies, the needs for LTC are expected to grow and yet there is a lot of uncertainty as to how to finance those needs. Family solidarity that has been the main provider of LTC is reaching a ceiling and the market is quite absent. Not surprisingly, one would expect that the state would take the relay. The purpose of this paper is to design an optimal social insurance scheme in a setting in which both parents and children are altruistic towards each other. Children’s altruism is however partial. First it is only triggered by the occurrence of dependency and second it is limited. Another feature of our model is that parents treat differently the aid from their children and the care they get from the market. In a pure market economy, the dependent parent has to devote his resources to purchase LTC but also possibly to leave some bequests to his child to induce him to help him more than he would do otherwise.

One cannot be but struck by two parallel evolutions: the soaring needs for LTC and the growing share of inherited wealth in overall capital accumulation. LTC is surely not a problem for the very wealthy households, but for most households long-term care can eat most of their assets and make it impossible to bequeath anything.

It is thus not surprising that some people have thought of using the proceeds of estate taxation to finance long term care. An example of this is the English green paper proposing a voluntary inheritance levy. Accordingly people in England and Wales could pay a one-off "inheritance levy" of up to £12,000 in return for free long-term care in their old age. The fee would either be deducted from the estates of older people when they die or paid on retirement. The aim is to avoid forcing many pensioners to sell their family homes to fund massive nursing home bills.

There is indeed a close link between bequests and long term care. In a world without good LTC insurance households are forced to oversave or to self-insure meaning that in case of good health they end up with an excess of saving that can lead to involuntary bequests. It is then clear that if it were possible to tax those bequests the proceeds could be used to finance long-term care. Actually if some type of insurance, private or public were available, one would end up with the same result, that is, a perfect smoothing of
consumption between the two states of the world, dependency or not. Brunner and Pech (2012) made that argument by showing that a tax on bequests to finance LTC causes a smaller deadweight loss than an income or consumption tax. This holds true whether or not the parents are altruistic. The problem is that in a world with heterogeneous agents it might be impossible to distinguish bequests that are made by dependent parents from those made by autonomous parents.

The matter gets more complicated when we look at the case where long term care is provided by formal services financed by saving and informal services coming from children. For the time being we assume that formal and informal care are complements. Dependent parents will have to exchange those informal services against the prospect of some bequests. If children are not altruistic, we will have a pure quid pro quo exchange; if they are altruistic they might help their parents even if these cannot bequeath anything. The amount of help will depend on the extent of filial altruism. In case of perfect altruism, children will provide the optimal amount of assistance to their parents.

In a world of identical individuals, the social insurance scheme will serve two purposes: it redistributes resources across the two states of nature and it induces the child to help his parent. If individuals differ in the level of their wage, social insurance must also redistribute resources across individuals.

2 Identical individuals

2.1 The model setup

Consider a population (which size is normalized to one) consisting of one parent (subscript ‘p’) and one child (subscript ‘c’) families. The parent is a pure altruist towards his child, while the child’s altruism towards the parent may be imperfect. The parent is retired and has accumulated wealth $y$. He faces the probability $\pi$ of becoming dependent and needing long-term care. The need of LTC requires expenditures of amount $L$. In case of dependency the parent would like to benefit from the aid of his child, who is ready to help his parent out of altruism but also with the expectation of some inheritance $b$. Additionally, the parent can buy LTC insurance coverage on the private
market at a price \( p \geq \pi \). For \( p = \pi \) LTC insurance is actuarially fair. The parent decides how much LTC insurance coverage \( I \) to buy and how much he wants to leave as a bequest to his child. In case of autonomy, the child inelastically supplies one unit of labor at a wage rate \( w \). In case the parent needs long-term care, the time spend on the labor market is reduced by the time spend for informal care provision and gross earnings amount to \( w(1 - a) \). Care provided by the child reduces the monetary loss from LTC by \( h(a) \leq L \) (with \( h' > 0, h'' < 0 \)) since then the parent requires less professional care services.

In sum, the parent’s and child’s utility are given by

\[
U_p = \pi \left[ H(y + (1 - p)I + h(a) - L - b) + u(w(1 - a) + b) \right] + (1 - \pi) \left[ u(y - pI - \tilde{b}) + u(w + \tilde{b}) \right],
\]

\[
U_c = \pi \left[ u(w(1 - a) + b) + \beta H(y + (1 - p)I + h(a) - L - b) \right] + (1 - \pi) \left[ u(w + \tilde{b}) + \beta u(y - pI - \tilde{b}) \right],
\]

where \( \beta \in [0,1] \) reflects the child’s degree of altruism and \( \tilde{a} \) indicates the state of staying healthy. The utility functions satisfy \( u', H' > 0 \) and \( u'', H'' < 0 \).

The timing of the model is as follows: first the government announces its policy. Then, the parent and the child play the following two stage game. In stage 1, the parent decides the amount of LTC insurance coverage and the level of bequests. In stage 2, after the state of nature is revealed, the child decides how much informal care to provide in case the parent needs long-term care. To determine the subgame perfect Nash equilibrium we solve this game by backward induction. But, before we turn our attention to the laissez-faire we study the first-best allocation which provides a benchmark against which we can compare the laissez-faire allocation.

### 2.2 First-best solution

With \( ex \ ante \) identical families, we can define the optimal allocation as the one maximizing the expected utility of a representative dynasty. Assuming that both parents
and children receive equal social weights, the first-best problem can be written as

\[
\max_{m,c,\hat{m},\hat{c},a} W = \pi [H(m) + u(c)] + (1 - \pi) [u(\hat{m}) + u(\hat{c})]
\]

s.t. \( (1 - \pi) (\hat{m} + \hat{c} - L) + \pi (m + c) = y + (1 - \pi)w + \pi [w(1 - a) + h(a)] \) (4)

where the decision variables are informal care provision \( a \) and parent’s and child’s consumption in both states of nature. We denote the latter by \( m, \hat{m}, c \) and \( \hat{c} \) respectively. In the first-best all variables are directly set, assuming full information and disregarding the multi-stage structure of the game. The specification of the game will of course be relevant below, when we study the decentralization of the first-best optimum. Denoting the Lagrange multiplier associated with the resource constraint (4) by \( \mu \), the first order conditions (FOCs) characterizing the optimal solution can be written as follows

\[
H'(m) = u'(\hat{m}) = u'(c) = u'(\hat{c}) = \mu, \quad (5)
\]

\[
w = h'(a). \quad (6)
\]

Equation (5) states the equality of marginal utilities of incomes across generations and states of nature (full insurance) while equation (6) describes the efficient choice of informal care. It states that the opportunity costs of informal care \( w \) should be equal to its marginal benefit.

### 2.3 Laissez faire allocation

#### 2.3.1 Stage 2: choice of children

In case of autonomy, children do not have to make any decision. They consume their income and bequest enjoying a utility \( u(w + \hat{b}) \). When their parents are in need of long-term care, they solve the following problem

\[
\max_a u(w(1 - a) + b) + \beta H(y + (1 - p)I - L + h(a) - b).
\]

The first order condition of the above problem is given by

\[
-wu'(w(1 - a) + b) + \beta h'(\omega + (1 - p)I - L + h(a) - b) = 0.
\]
The above equation yields \( a^* = a(b, I) \). We have the following comparative static effects

\[
\frac{\partial a^*}{\partial I} = \frac{-\beta H''(m)h'(a)(1 - p)}{SOC^a} < 0, \tag{9}
\]

\[
\frac{\partial a^*}{\partial b} = \frac{wu''(c) + \beta H'(m)h'(a)}{SOC^a} > 0, \tag{10}
\]

as the second order condition (SOC) is negative

\[
SOC^a = w^2 u''(c) + \beta H'(m)h''(a) + \beta H''(m)h'^2 < 0. \tag{11}
\]

### 2.3.2 Stage 1: choice of parents

The problem of the parents is to choose the LTC insurance coverage and the level of bequests to their children. The latter will depend on the state of nature. In case of dependency parents have more needs but at the same time they might leave a higher bequest to induce more assistance from their children. Their problem is the following

\[
\max_{I, b} U_p = \pi [H(y + (1 - p)I - L + b(a^*) - b) + u(w(1 - a^*) + b)]
+ (1 - \pi) [u(y - pI - \hat{b}) + u(w + \hat{b})]. \tag{12}
\]

Taking into consideration equation 8 the FOCs with respect to \( I, b \), and \( \hat{b} \) can be written as

\[
\pi H'(m) \left[ (1 - p) + h''(a) \right] \left[ (1 - \beta) \right] - (1 - \pi) pu'(\hat{m}) \leq 0, \tag{13}
\]

\[
-H'(m) \left[ 1 - h'(a) \right] \frac{\partial a^*}{\partial b} (1 - \beta) + u'(c) = 0, \tag{14}
\]

\[-u'(\hat{m}) + u'(\hat{c}) = 0. \tag{15}\]

It can be easily verified that for \( \beta = 1 \) and actuarial fair insurance \( p = \pi \), the parent buys the efficient amount of insurance coverage, that is he is fully insured \( H'(m) = u'(\hat{m}) \). Additionally, he leaves the efficient amount of bequests so that \( H'(m) = u'(c) \). The latter in turn goes hand in hand with an efficient amount of aid provided by the child \( h'(a) = w \); see equation (8). This result is not surprising as for perfect altruism on both the parent’s and the child’s side, the optimization problem of both family members coincides with the one of the social planner. Whenever \( \beta < 0 \) this is no longer the case and the child puts a too low value on the parent’s benefits of his informal care provision.
2.4 Decentralization of the first-best allocation

Assume for the time being that there is no asymmetry of information so that all relevant variables including informal aid are publicly observable. In the following we show that the FB allocation within our multi-stage setting can be decentralized by a lump-sum transfer from the healthy to the dependent elderly ($D, D$) supplemented by a tax on labor income $\tau_a$ and bequests $\tau_b$ for those children whose parents are dependent. Transfers must be chosen such that the government’s budget constraint, i.e.

$$\pi D = (1 - \pi)\hat{D} + \pi [\tau_a w(1 - a^*) + \tau_b b^*]$$  

is satisfied.

2.4.1 Family’s problem reconsidered

In case of autonomy, children do not have to make any decision. They consume their income and bequest enjoying a utility $u(w + \hat{b})$. When their parents are in need of long-term care, they now solve the following problem

$$\max_a u((1 - \tau_a)w(1 - a) + (1 - \tau_b)b) + \beta H(y + (1 - p)I + D - L + h(a) - b)$$  

The first order condition of the above problem is given by

$$-(1 - \tau_a)wu'(c) + \beta H'(m)h'(a) = 0.$$  

The above equation yields $a^* = a(b, I, \tau_a, \tau_b, D)$. To implement the first-best we need a tax on child’s income, i.e. $\tau_a = 1 - \beta$. The comparative static effects are\(^1\)

\[
\begin{align*}
\frac{\partial a^*}{\partial I} &= \frac{-\beta H'(m)h'(a)(1 - p)}{SOC^a} < 0, \\
\frac{\partial a^*}{\partial \tau_a} &= \frac{(1 - \tau_a)wu''(c)(1 - \tau_b) + \beta H'(m)h'(a)}{SOC^a} > 0, \\
\frac{\partial a^*}{\partial \tau_b} &= \frac{-wu'(c) - (1 - \tau_a)w^2(1 - a)u''(c)}{SOC^a} \leq 0, \\
\frac{\partial a^*}{\partial D} &= \frac{-\beta H'(m)h'(a)}{SOC^a} < 0.
\end{align*}
\]

\(^1\)The second order condition is $SOC^a = ((1 - \tau_a)w)^2u''(c) + \beta H'(m)h''(a) + \beta H''(m)h'^2 < 0$. 

6
The problem for the parents is now the following

\[
\max_{I,b} U_p = \pi [H(y + (1 - p)I + D - L + h(a^*) - b) + u(1 - \tau_a)w(1 - a^*) + (1 - \tau_b)b] \\
+ (1 - \pi) [u(y - pI - \hat{D} - \hat{b}) + u(w + \hat{b})]
\]  \tag{24}

Again taking into consideration equation (??) the FOCs with respect to \(I, b, \) and \(\hat{b}\) are

\[
\pi H'(m) \left[ (1 - p) + h'(a^*) \frac{\partial a^*}{\partial I} (1 - \beta) \right] - (1 - \pi)pu'(\hat{m}) \leq 0, \tag{25}
\]

\[
-H'(m) \left[ 1 - h'(a^*) \frac{\partial a^*}{\partial b} (1 - \beta) \right] + (1 - \tau_b)u'(c) = 0, \tag{26}
\]

\[-u'(\hat{m}) + u'(\hat{c}) = 0. \tag{27}\]

Equations (25)–(27) yield \(I^*(D, \hat{D}, \tau_a, \tau_b), b^* = b(D, \hat{D}, \tau_a, \tau_b)\) and \(\hat{b}^* = \hat{b}(D, \hat{D}, \tau_a, \tau_b)\).

When social insurance is chosen such that the parent (and thus also the child) is fully insured implying \(H'(m) = u'(\hat{m})\) equation (25) holds as inequality and \(I^* = 0\). The tax on bequests must be chosen such that its effect on informal care is offset, i.e.

\[
\tau_b = h'(a^*) \frac{\partial a^*}{\partial b} (1 - \beta).
\]

Note that \(\partial a^*/\partial b\) is not independent of \(\tau_b\).

There are different ways to decentralize this first best optimum; basically we need two types of tools: a distortive tax that fosters child’s assistance except in case of pure ascending altruism \((\beta = 1)\), and a redistributive device between the two states of nature.

In the above the set of tools \(D, \hat{D}, \tau_a, \tau_b\) does the job.

### 2.5 Second best

We now turn to the second best. We assume that the government can only use uniform taxes on bequest \(\tau_b\) and on labor earnings \(\tau_a\) to finance a LTC benefit \(g\). We use the notation \(g\) rather then \(D\) to avoid confusion with the first-best implementation. We assume away private insurance for conveniency but also because in most countries this market is extremely thin. In the simple framework of this model it should be possible to distinguish the two types of bequests and hence to tax them differently. However, as
soon as heterogeneity is introduced as in the next section it is not realistic to assume that bequests can be distinguished.

With our instruments we write the revenue constraint as:

\[
\pi g = \tau_a (1 - \pi a)w + \tau_b (\pi b + (1 - \pi)\hat{b})
\]

2.5.1 The child’s problem

A child solves

\[
\max_a u[(1 - \tau_a)w(1 - a) + (1 - \tau_b)\hat{b}] + \beta H [y - L - b + g - h(a)],
\]

which apart from the change in notation is equivalent, to (17) so that the FOC continues to be given by (18) and can be written as

\[
(1 - \tau_a)wu'(c) = \beta H(m)h'(a).
\]

The solution is given by

\[
a = a^*(b, \tau_a, \tau_b, g).
\]

The comparative statics properties of this function are described by equations (20)—(23). Using subscripts to denote partial derivatives, we thus have

\[
a_b > 0, a_{\tau_b} < 0, a_{\tau_a} \gtrless 0, a_g < 0.
\]

2.5.2 Parent’s problem

Turning to the parent’s problem, it is given by

\[
\max_{b, \hat{b}} \pi \left[ H[y - L - b + g + h(a)] + u[(1 - \tau_a)w(1 - a) + (1 - \tau_b)\hat{b}] \right] + (1 - \pi) \left[ u(y - \hat{b}) + u[(1 - \tau_a)w + (1 - \tau_b)\hat{b}] \right].
\]

The two FOC’s are:

\[
u'(\hat{m}) = u'(\hat{c})(1 - \tau_b),
\]

\[
H'(m) \left[ 1 - h'(a)a_b \right] = u'(c)[(1 - \tau_b) - (1 - \tau_a)wa_b].
\]

Using (28) equation (30) can be rewritten as

\[
H'(m) \left[ 1 - h'(a)(1 - \beta)a_b \right] = u'(c)(1 - \tau_b).
\]
These conditions define the bequests supply functions as: \( b = b(\tau_a, \tau_b, g) \) and \( \hat{b} = b(\tau_a, \tau_b) \). Substituting for \( b \) into \( a^c \), yields the level of aid as a function of the government’s instruments \( a = a^c[b(\tau_a, \tau_b, g), \tau_a, \tau_b, g] = \tilde{a}(\tau_a, \tau_b, g) \).

2.5.3 The government’s problem

The second-best problem can now be expressed by the following Lagrangean:

\[
L = \pi [H [y - L - b + g + h(a)] + u[(1 - \tau_a)w(1 - a) + (1 - \tau_b)b]] +
(1 - \pi) [u(y - \hat{b}) + u[(1 - \tau_a)w + (1 - \tau_b)\hat{b}]]
- \mu \left[ \pi g - \tau_a(1 - \pi a)w - \tau_b(\pi b + (1 - \pi)\hat{b}) \right]
\]

Using the envelope theorem, we write the FOC’s of this problem as:

\[
\frac{\partial L}{\partial g} = \tau_a w \pi - \mu \pi g - \tau_a \pi a w \pi + (1 - \pi) \pi b + (1 - \pi) \pi b_{\tau_a} = 0
\]

\[
\frac{\partial L}{\partial \tau_a} = \left[ \pi u' (c) (1 - a) + (1 - \pi) u'(\tilde{c}) \right] w +
\pi u'(c) w(1 - \pi a) - \mu \pi \tau_a b a \tau_a + \tau_b \left[ \pi b_{\tau_a} + (1 - \pi) \hat{b}_{\tau_a} \right] = 0
\]

\[
\frac{\partial L}{\partial \tau_b} = \left[ \pi u'(c) b(1 - \pi) u'(\tilde{c}) \hat{b} \right] +
\mu \left[ \pi b + (1 - \pi) \hat{b} - \pi \tau_a w \pi a \tau_b + \tau_b \left[ \pi b_{\tau_b} + (1 - \pi) \hat{b}_{\tau_b} \right] \right] = 0
\]

Those FOC’s can be rearranged using the compensated form with

\[
\frac{dg}{d\tau_b} = \frac{\pi b + (1 - \pi) \hat{b}}{\pi}, \quad \text{and} \quad \frac{dg}{d\tau_a} = \frac{(1 - \pi a) w}{\pi}.
\]

In other words, we keep tax rates as sole decision variables, while accounting of course
for their impact on $g$, via the government’s budget constraint. This yields:

$$
\frac{\partial L^c}{\partial \tau_b} = \frac{\partial L}{\partial \tau_b} + \frac{\partial L}{\partial g} \frac{dg}{d\tau_b} = \pi (1 - \beta) H'(m) h'(a) \tilde{a}_{\tau_b} - \\
[\pi u'(c) b + (1 - \pi) u'(\bar{c}) \hat{b}] + H'(m) (\pi b + (1 - \pi) \hat{b}) - \\
\mu \left[ \tau_a w \tilde{a}_{\tau_a} - \tau_b \left[ \pi b_{\tau_b} + (1 - \pi) \beta \hat{c}_{\tau_a} \right] \right] = 0
$$

$$
\frac{\partial L^c}{\partial \tau_a} = \frac{\partial L}{\partial \tau_a} + \frac{\partial L}{\partial g} \frac{dg}{d\tau_a} = \pi (1 - \beta) H'(m) h'(a) \tilde{a}_{\tau_a} - \\
[\pi u'(c) (1 - a) + (1 - \pi) u'(\bar{c})] w + H'(m) (1 - \pi a) w - \\
\mu \left[ \tau_a w \tilde{a}_{\tau_a} - \tau_b \left[ \pi b_{\tau_a} + (1 - \pi) \beta \hat{c}_{\tau_a} \right] \right] = 0
$$

Using a shorter notation, we write

$$
- \left[ \pi u'(c) b + (1 - \pi) u'(\bar{c}) \hat{b} \right] + H'(m) (\pi b + (1 - \pi) \hat{b}) = \\
- \pi \left[ u'(c) - H'(m) \right] b - (1 - \pi) \left[ u'(\bar{c}) - H'(m) \right] \hat{b}
$$

and

$$
- \left[ \pi u'(c) (1 - a) + (1 - \pi) u'(\bar{c}) \right] w + H'(m) (1 - \pi a) w = \\
- \pi \left[ u'(c) - H'(m) \right] (1 - a) w - (1 - \pi) \left[ u'(\bar{c}) - H'(m) \right] w
$$

We now rewrite the FOC’s as:

$$
\pi (1 - \beta) H'(m) h'(a) \tilde{a}_{\tau_b} - \pi \left[ u'(c) - H'(m) \right] b - (1 - \pi) \left[ u'(\bar{c}) - H'(m) \right] \hat{b} = \\
\mu \left[ \tau_a w \tilde{a}_{\tau_a} - \tau_b \left[ \pi b_{\tau_b} + (1 - \pi) \beta \hat{c}_{\tau_a} \right] \right]
$$

$$
\pi (1 - \beta) H'(m) h'(a) \tilde{a}_{\tau_a} - \pi \left[ u'(c) - H'(m) \right] (1 - a) w - (1 - \pi) \left[ u'(\bar{c}) - H'(m) \right] w = \\
\mu \left[ \tau_a w \tilde{a}_{\tau_a} - \tau_b \left[ \pi b_{\tau_a} + (1 - \pi) \beta \hat{c}_{\tau_a} \right] \right]
$$

To interpret these formulas, we take the standard approach that the cross-derivatives are negligible. Formally, $b_{\tau_a}^c = \hat{c}_{\tau_a}$. Then we have:

$$
\tau_b = \frac{-\pi \left[ u'(c) - H'(m) \right] b - (1 - \pi) \left[ u'(\bar{c}) - H'(m) \right] \hat{b}}{-\mu \left[ \pi b_{\tau_b} + (1 - \pi) \beta \hat{c}_{\tau_b} \right]} \\
\tau_a = \frac{\pi (1 - \beta) H'(m) h'(a) \tilde{a}_{\tau_a} - \pi \left[ u'(c) - H'(m) \right] y - (1 - \pi) \left[ u'(\bar{c}) - H'(m) \right] w}{\mu w \tilde{a}_{\tau_a}}
$$

(31)
The denominators in these two formulas are standard. They reflect the efficiency effect of taxes. Normally one expects \( a_c^\tau > 0 \) and

\[
\pi \frac{\partial \hat{b}_c}{\partial \tau_b} + (1 - \pi) \frac{\partial \hat{b}_c}{\partial \tau_b} < 0.
\]

To interpret equation (31), let us assume that in the laissez faire we have the following inequalities: \( u'(\hat{c}) = u'(\bar{m}) < u'(c) < u'(m) \). This is a reasonable assumption if \( L \) is large enough and \( y \) not too high compared to \( w \). In that case we have that \( \tau_b > 0 \). Both the tax and the public benefit contribute to reduce the gap between these marginal utilities. As shown above in the First-Best, they would be all equal to each other. Turning to equation (32), we immediately note that the first term in the denominator vanishes if \( \beta = 1 \). Otherwise it represents the need to foster child’s assistance. The second and third terms are the same as in the previous condition. They reflect the concern for insurance. They give the welfare gap between the young and the dependent elderly in the two states of nature. If the marginal utility of the dependent is high relative to that of the young, then there is a need for more tax to finance the social benefit.

Note that throughout this section we have assumed away private insurance. If private insurance were available, its role would depend on the existence and the size of loading costs. With zero loading costs, a first best could be achieved as mentioned above.

### 3 Heterogenous families

We now assume that families differ \textit{ex ante} in the children’s productivity level. The government levies a flat tax on earnings and on bequests to finance a uniform benefit \( g \). Thus, the social insurance scheme pursues several objectives: insurance, redistribution and inducement to child’s assistance.

The revenue constraint has to be modified such that:

\[
\pi g = \sum_i n_i \left[ \tau_a (1 - \pi a_i) w_i + \tau_b (\pi b_i + (1 - \pi) \hat{b}_i) \right]
\]

where \( n_i \) is the relative number of families with productivity \( w_i \). As in the previous section we successively analyze the optimizing behavior of children, parents and government.
3.1 Child’s problem

The problem of a child of type \( i \) is

\[
\max_{a_i} u[(1 - \tau_a)w_i(1 - a_i) + (1 - \tau_b)b_i] + \beta H [y - L - b_i + g - h(a_i)]
\]

From the FOC

\[
(1 - \tau_a)w_iu'(c_i) = \beta H(m_i)h'(a_i)
\]

we obtain \( a_i = a(b_i, \tau_a, \tau_b, g) \), where \( a_{ib} > 0, a_{i\tau_b} < 0, a_{i\tau_a} \gtrless 0, a_{ig} < 0 \).

3.2 Parent’s problem

The parent’s problem is also the same as the homogenous case:

\[
\max_{b_i, \hat{b}_i} \pi \left[ H [y - L - b_i + g - h(a_i)] + u[(1 - \tau_a)w_i(1 - a_i) + (1 - \tau_b)b_i] \right] + (1 - \pi) \left[ u(y - \hat{b}_i) + u[(1 - \tau_a)w_i + (1 - \tau_b)\hat{b}_i] \right].
\]

The two FOC’s are

\[
\begin{align*}
    u'(\hat{m}_i) &= u'(\hat{c}_i)(1 - \tau_b) \\
    H'(m_i) \left[ 1 - h'(a_i)a_{ib} \right] &= u'(c_i)[(1 - \tau_b) - (1 - \tau_a)w_i a_{ib}].
\end{align*}
\]

Using (33), (35) can be rewritten as

\[
H'(m_i) \left[ 1 - h'(a_i)(1 - \beta)a_{ib} \right] = u'(c_i)(1 - \tau_b),
\]

From these conditions one derives the bequest supply functions: \( b_i = b(\tau_a, \tau_b, g) \) and \( \hat{b}_i = b(\tau_a, \tau_b) \). This also leads to \( a_i = a(b_i(\tau_a, \tau_b, g), \tau_a, \tau_b, g) = \tilde{a}_i(\tau_a, \tau_b, g) \).

3.3 Government’s problem

The second best problem can be expressed by the following Lagrange expression:

\[
\mathcal{L} = \sum_i n_i \left\{ \pi H [y_i - L - b_i + p + h(a_i)] + u[(1 - \tau_a)w_i(1 - a_i) + (1 - \tau_b)b_i] + (1 - \pi) \left[ u(y - \hat{b}_i) + u[(1 - \tau_a)w_i + (1 - \tau_b)\hat{b}_i] \right] - \mu \left[ \pi p - \tau_a(1 - \pi a_i)w - \tau_b(\pi b_i + (1 - \pi)\hat{b}_i) \right] \right\}
\]
Maximizing this expression with respect to $g$, $\tau_b$ and $\tau_a$ yields the following FOC’s
\[
\frac{\partial L}{\partial g} = \sum_n n_i \{ [H'(m_i)h'(a_i) - u'(c_i)(1 - \tau_a)w_i] \dot{a}_{ig} + \pi H'(m_i) - \mu [\pi + \pi \tau_a w \dot{a}_{ig} - \tau_b \pi \dot{b}_g] \} = 0
\]
\[
\frac{\partial L}{\partial \tau_a} = \sum_n n_i \{ [H'(m_i)h'(a_i) - u'(c_i)(1 - \tau_a)w_i] \dot{a}_{ir_a} - \left[ \pi u'(c_i)(1 - a_i) + (1 - \pi)u'(\hat{c}_i) \right] w_i + \mu \left[ w_i(1 - \tau_a) - \pi \tau_a w_i \dot{a}_{ir_a} + \tau_b \left[ \pi b_{ir_a} + (1 - \pi)\dot{b}_{ir_a} \right] \right] \} = 0
\]
\[
\frac{\partial L}{\partial \tau_b} = \sum_n n_i \{ [H'(m_i)h'(a_i) - u'(c_i)(1 - \tau_a)w_i] \dot{a}_{ir_b} - \left[ \pi u'(c_i)b_i + (1 - \pi)u'(\hat{c}_i)b_i \right] + \mu \left[ \pi b_i + (1 - \pi)\dot{b}_i - \pi \tau_a w \dot{a}_{ir_b} + \tau_b \left[ \pi b_{ir_b} + (1 - \pi)\dot{b}_{ir_b} \right] \right] \} = 0
\]

As above we rearrange those FOC’s in terms of compensations with $dg/d\tau_b$ and $dg/d\tau_a$. Furthermore we use the operator $E = \sum_i n_i$.
\[
\frac{\partial L^c}{\partial \tau_b} = E \left\{ \pi (1 - \beta)H'(m)h'(a)\dot{a}_{rb} - \left[ \pi u'(c)b + (1 - \pi)u'(\hat{c})\dot{b} \right] + H'(m)(\pi b + (1 - \pi)\dot{b}) - \mu \left[ \tau_a w \dot{a}_{rb} - \tau_b \left[ \pi b_{\tau_a} + (1 - \pi)\dot{b}_{\tau_a} \right] \right] \} = 0
\]
\[
\frac{\partial L^c}{\partial \tau_a} = E \left\{ \pi (1 - \beta)H'(m)h'(a)\dot{a}_{ra} - \left[ \pi u'(c)(1 - a) + (1 - \pi)u'(\hat{c}) \right] w + H'(m)(1 - \pi a)w - \mu \left[ \tau_a w \dot{a}_{ra} - \tau_b \left[ \pi b_{\tau_a} + (1 - \pi)\dot{b}_{\tau_a} \right] \right] \} = 0
\]

We use a shorter notation to represent the gap between the child’s marginal utility of consumption and that of the dependent parent. Formally: $\Delta = u'(c) - H'(m)$ and $\hat{\Delta} = u'(\hat{c}) - H'(m)$. We now rewrite the FOC’s as:
\[
\pi (1 - \beta)E \left[ H'(m)h'(a)\dot{a}_{\tau_a} \right] - \pi \text{cov}(\Delta, w(1 - a)) - (1 - \pi) \text{cov} (\hat{\Delta}, w) - \pi E \Delta E w(1 - a) - (1 - \pi)E \hat{\Delta} E w
\]
\[
= \mu E \left[ \tau_a w \dot{a}_{\tau_a} - \tau_b \left[ \pi b_{\tau_a} + (1 - \pi)\dot{b}_{\tau_a} \right] \right]
\]

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\[ \pi (1 - \beta) E \left[ H'(m) h'(a) \hat{a}_r c \right] - \pi \text{cov}(\Delta, b) - (1 - \pi) \text{cov}(\hat{\Delta}, \hat{b}) \]
\[ - \pi E \Delta E b - (1 - \pi) E \Delta E \hat{b} \]
\[ = \mu E \left[ \tau_a w \hat{a}_r c - \tau_b \left[ \pi \hat{b}_c + (1 - \pi) \hat{b}_{rb} \right] \right] \]

To interpret these formulas, we proceed as in the previous section by assuming that the cross derivatives are nil. Then we have

\[ \tau_b = \frac{-\pi \text{cov}(\Delta, b) - (1 - \pi) \text{cov}(\hat{\Delta}, \hat{b})}{-\mu E \left[ \pi \frac{\partial \varphi}{\partial r_b} + (1 - \pi) \frac{\partial \varphi}{\partial r_b} \right]} + \frac{-\pi E \Delta E b - (1 - \pi) E \Delta E \hat{b}}{-\mu E \left[ \pi \frac{\partial \varphi}{\partial r_b} + (1 - \pi) \frac{\partial \varphi}{\partial r_b} \right]} \tag{36} \]

and

\[ \tau_a = \frac{\pi (1 - \beta) E \left[ H'(m) h'(a) \hat{a}_r c \right] - \pi \text{cov}(\Delta, w(1 - a)) - (1 - \pi) \text{cov}(\hat{\Delta}, \hat{w})}{\mu E w \hat{a}_r c} + \frac{-\pi E \Delta E w(1 - a) - (1 - \pi) E \hat{\Delta} E \hat{w}}{\mu E w \hat{a}_r c} \tag{37} \]

The interpretation of these two formulas is the same as in the case of identical individuals with one exception, namely the covariance terms that are expected to be negative. In other words, earnings inequality pushes for more taxation of either earnings or bequests. To see that we should realize that long term care, \( m \), comprises both \( y \) and \( g \) that are the same for all whereas both \( c \) and \( \hat{c} \) are closely linked to heterogenous wages.

4 Conclusion

The departure point of this paper is the concomitance of two trends: the increasing needs for LTC and unusual growth of inheritance. In a laissez faire economy with no insurance public or private, dependency would bite a big chunk of accumulated wealth with the consequence that bequests would greatly differ for kids with dependent parents and kids with healthy parents. Children’s care can only partially mitigate those
differences. Given that the market for LTC insurance is universally thin, one has to rely on public action to restore some balance between the two states of nature. In a first best world, a subsidy on children’s care, individualized lump sum taxes and a state contingent inheritance tax could achieve the optimum. If for obvious reasons differential inheritance tax and individualized lump sum taxes are not available, one has to rely on second best schemes. In this paper we have looked at two alternative ways of financing a flat rate LTC benefit: a wage tax and a uniform inheritance tax. The main conclusion is that under plausible conditions a tax on both inheritance and wage to finance public LTC flat rate benefit is desirable. The outcome would be different if heterogeneity would shift from children’s earnings to parents’ income.

References


