Corporate Cash and Employment*

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Abstract

In the aftermath of the U.S. financial crisis, both a sharp drop in employment and a surge in corporate cash have been observed. In this paper, based on U.S. data, we document that the negative relationship between the corporate cash ratio and employment is systematic, both over time and across firms. We develop a dynamic general equilibrium model where heterogenous firms need cash in their production process. We analyze the dynamic impact of aggregate shocks and the cross-firm impact of idiosyncratic shocks. We show that liquidity and productivity shocks tend to generate a negative comovement between the cash ratio and employment. In contrast, standard credit shocks produce a positive relationship. A calibrated version of the model yields a negative comovement that is close to the data.

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1 Introduction

In the aftermath of the U.S. financial crisis, both a sharp decline in employment and an accumulation of cash held by firms have been observed. While both variables are part of firms’ decisions, they are typically not considered jointly in the literature. To what extent are these two features related? Holding liquid assets facilitates the firm’s ability to pay for the wage bill. But employment and cash decisions also react to changes in firms environment, e.g., changes in credit conditions. Therefore, examining these two variables jointly sheds light on the role of financial frictions on employment, especially during the crisis. It also provides crucial information on firms’ behavior and on their response to shocks. The contribution of this paper is twofold. First, it provides stylized facts on the relationship between the corporate cash position and employment. Second, it delivers an explanation to the empirical evidence by building a tractable dynamic general equilibrium framework, including both cash and employment decisions.

We first document a robust negative comovement between the corporate cash ratio and employment on U.S. data, which is not specific to the recent financial crisis. Using Flow-of-Funds data over the period 1980-2011, the correlation between HP-filtered employment and the share of liquid assets in total assets is $-0.52$. Moreover, using firm-level data from Compustat, the annual cross-firm correlation between employment and the cash ratio is on average $-0.29$ over the same period. Section 2 provides a detailed description of this data analysis.

To understand the optimal cash and employment decisions, we consider an infinite-horizon general equilibrium model with heterogeneous firms that need liquid funds in their production process. Liquidity is closely related to labor because firms have liquidity needs in order to finance the wage bill, which is part of working capital. We adopt a structure similar to Christiano and Eichenbaum (1995), who divide periods in two subperiods. In the first subperiod, firms use credit to install capital, while they need liquid funds to pay workers in the second subperiod.1 In contrast to the literature introducing working capital in macroeconomic models (see Christiano et al. 2011, for a survey), we assume that firms do not have full access to external liquidity and cannot borrow all their short-term needs. This generates a demand for cash. Liquidity that is external to the firm may take several

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1This assumption is consistent with the evidence that firms with a higher labor share have a higher cash ratio (see Section 2).
forms, such as credit lines, trade credits, trade receivables to customers, or late wage payments. Liquidity shocks are changes in the availability of external liquidity. We assume that firms may be hit by technology shocks, by changes in their ability to obtain long-term credit (i.e., standard credit shocks) and by liquidity shocks. These shocks can be at the aggregate or at the idiosyncratic level.

The model is designed to be tractable so that several results can be derived analytically. It suggests that both aggregate technology and liquidity shocks can explain the negative comovement between employment and the corporate cash ratio. For example, a reduction in external liquidity generates two effects. On the one hand, lower liquidity reduces the financial opportunities of firms and depresses labor demand. On the other hand, the reduction in external liquidity makes the production process more intensive in cash to ensure that wages are fully financed. Firms assets are then tilted towards cash. Combining these two effects implies that the cash ratio increases while employment declines. On the contrary, a standard credit shock generates a positive comovement between the two variables. This analysis points out the crucial role played by the tightening of liquidity conditions in the aftermath of the Lehman crisis. While no initial sharp reduction in credit supply was observed during the recent financial crisis, firms experienced a significant deterioration in their expected liquidity conditions. For example, Gilchrist and Zakrajsek (2012) argue that banks cut the existing corporate lines of credits during the crisis. Also, short-term loans to business firms decreased by 9% between 2008 and 2009 (using Survey of Terms of Business Lending, maturity of less than 30 days) while the liquidity ratio sharply increased from 3.9% to 5%.

Since our analysis focuses on portfolio decisions between liquid and less liquid assets, it is natural to consider the cash ratio rather than the level of cash. The cash ratio is mainly driven by the availability of external liquid funds. Therefore, the liquidity shock mentioned above drives the comovement between the cash ratio and employment in an unambiguous way, unlike the level of cash. Indeed, the reduction in labor demand depresses the demand for cash per se through a “size effect”: since cash is used to finance the wage bill, a lower labor demand implies

\[2\] Ivashina and Sharfstein (2010) show that firms initially drew heavily on their credit lines, but that subsequently credit conditions tightened. Campello et al. (2011) show that some firms had their credit lines canceled and that other firms had to renegotiate their credit lines with a higher cost. More generally, credit line agreements may contain restrictive covenants that may limit the ability of borrowers to draw on their lines. See also Chari et al. (2008) or Kahle and Stulz (2013).
a lower demand for cash. This negative size effect then interacts with the positive “portfolio effect” described above.

Idiosyncratic shocks generate heterogeneity among firms regarding their cash holding and employment decisions. The model is parametrized using moments distribution from firm-level data. Despite its simplicity, the model performs relatively well quantitatively to reproduce the negative cross-firm correlation between the cash ratio and employment. Our benchmark calibration gives a correlation of −0.18, while it is −0.29 in the data.

The optimal choice of corporate liquidity is rarely introduced in macroeconomic models, even in models with financial frictions. When it is, the focus is on investment, not labor. Liquid assets are usually held by households, typically in the form of money, to finance their consumption. However, firms also have liquidity needs. Papers incorporating firms’ liquidity are typically in the spirit of Holmstrom and Tirole (2011) and Woodford (1990); they include Aghion et al. (2010), Kyiotaki and Moore (2012) or Bacchetta and Benhima (2013). However, these papers do not analyze employment fluctuations.

While the link between liquidity and employment has not received much attention so far, our analysis is related to several strands of the literature. First, there is a growing literature that incorporates firms’ financial frictions in a macroeconomic context. For instance, Covas and den Haan (2011) and Jermann and Quadrini (2012) analyze corporate external finance decisions over the business cycle, such as debt and equity. However, these papers do not introduce cash. For example, in their theoretical model, Jermann and Quadrini (2012) have working capital that is fully financed by an intra-period loan. Other papers focus more closely on the relationship between financial factors and the labor market. This literature stresses the role of financial frictions influencing labor demand. Most of these papers provide a more detailed analysis of the labor market than we do, but they do not consider cash holdings. Our analysis focuses on the impact of liquidity conditions on labor demand.

Our paper is also related to a vast theoretical literature in corporate finance on firms’ cash holdings and corporate saving. Our approach shares features with

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3There are obviously some exceptions. For example, Stockman (1981) considers a cash-in-advance constraint both for consumption and capital.

several recent papers that provide analyses at the firm level or in environments with heterogeneous firms. Some papers are particularly close to our approach as they focus on the role of financing conditions.\(^5\)\(^6\) Our paper differs from this literature by focusing on employment which plays a key role in the working capital management. In addition, we provide a general equilibrium analysis which is important in the context of employment as this is an input that is not generated by the firm (in contrast to capital). As a result, market-clearing wage fluctuations can potentially offset partial equilibrium effects. This is particularly relevant in the context of liquidity management as the wage bill affects firms’ liquidity needs. Another difference is that we make a clear distinction between liquid and less liquid assets. The recent dynamic models in the corporate finance literature consider cash a negative debt or as a residual between cash flow and investment.\(^7\)

Finally, our approach is consistent with the findings of the empirical literature on the determinants of corporate cash.\(^8\) This literature stresses in particular the precautionary motive to save cash and shows that this motive increases with cash flow uncertainty or with more uncertain access to capital markets (see for instance Almeida et al., 2004). Some papers have also analyzed the use of short-term credits, like credit lines, and their interaction with corporate cash holdings. They tend to show that cash is a substitute to credit lines, as suggested by our analysis. For instance, Campello et al. (2011) find a negative correlation between cash and credit lines.\(^9\)

5 For example, Eisfeldt and Muir (2013) develop a partial equilibrium model, examining both the aggregate corporate sector and the distribution of heterogeneous firms, to analyze the positive relationship between external finance and cash accumulation (in contrast to cash holding). They argue that changes in the cost of external finance drives internal and external funds hoarding in the same direction. Bolton et al. (2013) and Hugonnier et al. (2013) show that worsening external funding conditions increase cash holdings and decrease investment. Falato et al. (2013) explain this increase by a reduction of tangible capital that can be used as collateral for firms’ borrowing. Finally, the explanation given by Gao (2013) relies on the increased use of just-in-time production techniques, which increase the need for liquid funding.

6 Some papers consider other determinants of firms’ cash holdings. Boileau and Moyen (2012) analyze the effect of funding risk on corporate liquidity by making the distinction between credit lines usage and cash. Armenter and Hnatkovska (2011) develop a model to explain the gradual increase in corporate net saving. They show that firms have been relying more on internal funds for precautionary motives and because of tax reforms.

7 This contrasts with an older corporate finance literature, see Holmstrom and Tirole (2011).

8 See, for example, Bates et al. (2009) and Almeida et al. (2013) for surveys.

9 Similarly, Sufi (2009) and Lins et al. (2010) show that internal cash is used more in bad times while firms are more likely to use credit lines in good times. Acharya et al. (2013) build a model to show that firms would rather use credit lines instead of cash reserve when they face a low aggregate risk.
The rest of the paper is organized as follows. Section 2 describes the negative comovement between corporate cash and employment. Section 3 presents the model and shows the basic mechanism that can lead to this negative relationship. In Section 4, we calibrate the model to analyze the dynamic impact of aggregate shocks. In Section 5, we examine the impact of idiosyncratic shocks on cross-firm correlations. Section 6 discusses various extensions and Section 7 concludes. Several results are derived in the Appendices.

2 Stylized Facts

In this section, we document the negative correlation in the U.S. between the corporate cash ratio and employment, both in aggregate terms and at the firm level. We first illustrate the aggregate correlation between corporate cash and employment over the business cycle. We use annual data in the non-farm non-financial corporate sector. The cash ratio, defined as the share of corporate liquidity to total assets, is built from the Flow-of-Funds of the United States. We define cash as the sum of private foreign deposits, checkable deposits and currency, total time and savings deposits and money market mutual fund shares. Corporate employment in logarithm is drawn from the Bureau of Labor Statistics. The upper panel in Figure 1 displays the HP-filtered component of employment and the cash ratio over the sample 1980-2011.

[ insert Figure 1 here ]

We clearly observe a negative comovement between the two variables. This negative relationship is particularly striking during the Great Recession since the corporate liquidity ratio experienced a large boom between 2007 and 2009 while employment has been strongly depressed. Taking the 1980-2011 sample, the contemporaneous correlation between employment and the cash ratio is strongly negative ($-0.52$) and significant at 1%.\textsuperscript{10} We show below that this is consistent with productivity and liquidity shocks in our model.

While our analysis focuses on the cash ratio and on portfolio effects, it is useful

\textsuperscript{10}In order to avoid any spurious correlation, we also compute the correlation when cash is divided by the one-year lagged value of total assets instead of its current value. The correlation is still negative ($-0.33$) and significant. The correlation on quarterly data and by excluding the Great Recession is lower ($-0.20$) but still significant at 5%. Robustness exercises are provided in the online appendix.
to examine the evolution of the level of cash. The lower panel in Figure 1 displays the cyclical component of employment and the inflation-adjusted level of cash over the same period. Interestingly, while the correlation between the cash *ratio* and employment is significantly negative, the correlation between the cash *level* and employment is low (-0.09) and insignificant. This difference is consistent with our model. While the dynamics of the cash ratio are driven by a pure portfolio effect, the cash level results from the combination of a portfolio effect and a size effect, making its dynamics more ambiguous. Namely, when the production process is more intensive in cash (portfolio effect), employment and thus the production scale typically decrease (size effect), which also means that less cash is needed. The implications for the level of cash are therefore ambiguous. We argue below that this zero correlation is consistent with a combination of technology and liquidity shocks which affect the level of cash in opposite ways.

The aggregate correlations that have been documented are driven by macroeconomic shocks common to all firms. In order to capture the heterogeneity among firms, we assess the correlation between the corporate cash ratio and employment using disaggregated firm-level data from Compustat. The sample contains US non-financial firms from 1980 to 2011. We focus only on firms that are active during the whole period, which allows us to have a homogeneous panel. In addition, we drop the 10% largest firms. This is a standard procedure (e.g., see Covas and den Haan, 2011) as the largest firms may have a specific behavior. For example, the cash holding of multinational companies might be driven by foreign tax incentives (see Foley et al., 2007). We also exclude financial and utilities firms, firms which are not incorporated in the US market and those engaged in major mergers.\(^\text{11}\) This is justified by the fact that part of the stock of cash holding is affected by acquisition. We also drop all firms with negative or missing values for: total assets, sales, cash and employees.\(^\text{12}\) We use the number of employees per firm (Compustat data item #29) as our measure of employment. The corporate cash ratio is defined as the ratio between cash and short term investment (Compustat data item #1) and the book value of assets (Compustat data item #6). A firm-specific linear trend is removed from both employment and the cash ratio. Figure 2 plots the year-

\(^{11}\) Using Compustat data items, we remove firms when 6000<SIC<6999, 4900<SIC<4949, curc<\$ and sale_fn = AB. The online appendix shows that the correlation is −0.28\% when we do not exclude the 10\% largest firms.

\(^{12}\) The sample is reduced to 14 658 firm-year observations. Data description and descriptive statistics are provided in the online appendix.
by-year cross-firm correlation coefficients between these two variables with their significance level.

[ insert Figure 2 here ]

Over the period, the cross-section correlation between detrended employment and cash ratio is $-0.29$ on average and it is significant at 1%. The negative correlation is significant in all periods and is lower than average during the Great Recession. While we only present unconditional correlations, the negative relationship between employment and cash holding is robust when we use OLS with firms-fixed effects, years-fixed effects, and standard control variables (see online appendix). In particular, this relationship is not driven only by macroeconomic shocks or by systematic differences across firms. Our model also accounts for this idiosyncratic correlation.

An important assumption of our model is that cash holding decisions are determined by the financing of the wage bill. This explanation suggests that there should be a relationship between the level of cash held by firms and their labor share. Figure 3 confirms this view by displaying the aggregate cash ratio of firms classified into two groups: those with a labor share below the median and those with a labor share above the median.\footnote{The labor share of firms is industry-specific. We use the NBER-CES Manufacturing Industry Database. For each industry, we compute the labor share by year, defined as the ratio between payroll and production. Then, using our Compustat dataset, we attribute to each manufacturing firm the value of the labor share associated to its industry. Firms with a labor share below (above) the median are “low (high) labor-share firms”. We compute the average value of the cash ratio by class and by year.}

[ insert Figure 3 here ]

In this paper, we argue that the demand for cash and employment are driven by future prospects about the availability of external liquidity and productivity. There are alternative potential explanations of the negative correlation between the cash ratio and employment. First, the demand for cash can be driven by the cyclicality in the cost of cash. For example, during the crisis, the flight to liquidity can be explained by the drop in interest rates which decreased the opportunity cost of cash. However, the negative correlation is robust to the inclusion of years-fixed effects, which indicates that it is not driven exclusively by business cycle effects like the cost of cash. A second alternative explanation emphasizes the role of unexpected shocks. For example, following a negative unexpected productivity shock, firms
lay off workers, which generates more cash flow. However, using our panel of firms from Compustat, we show that the correlation coefficient remains negative and significant when we control for cash flows (see online appendix). Moreover, the correlation coefficient is still negative and significant when we use the lagged cash ratio and when we control for the size of the firm (see online appendix). These two pieces of evidence suggest that the correlation between employment and the cash ratio is not driven solely by unexpected shocks.

3 A Dynamic Model of Corporate Cash Holdings

The single-good economy is inhabited by infinitely-lived heterogeneous entrepreneurs and identical households. Entrepreneurs produce, hire labor, invest, borrow, and hold cash. Households work, consume, lend to entrepreneurs and also hold cash. We abstract from financial intermediaries. Liquidity is modeled by dividing each period in two subperiods, which we refer to as beginning-of-period and end-of-period. The market for less liquid assets, called bonds, only opens at the beginning-of-period. Firms have a liquidity need at the end-of-period as they have to pay for the wage bill.\(^\text{14}\) This liquidity need can be covered either by external liquidity or by cash holdings. Therefore, the need for cash is affected by changes in the availability of external liquidity. We first describe the problem of entrepreneurs and then turn to their optimal behavior, focusing on optimal labor demand and cash. We characterize analytically the properties of the model in this partial equilibrium. Finally, we describe the general equilibrium model by introducing households.

3.1 Entrepreneurs

There is a continuum of entrepreneurs of length 1. Entrepreneur \(i \in [0,1]\) maximizes:

$$E_t \sum_{s=0}^{\infty} \beta^s u(c_{H+s})$$

\(^{14}\)For convenience we only consider labor as end-of-period input. In a related context, Gao (2013) considers raw material instead of labor.
where $c_{it+s}$ is the consumption of entrepreneur $i$ in period $t+s$. Entrepreneur $i$ produces $Y_{it}$ out of capital $K_{it}$ and labor $l_{it}$ through the production function

$$Y_{it} = F(K_{it}, A_{it})$$

where $F$ is a standard constant-return-to-scale production function and $A_{it}$ is total factor productivity (TFP). In this section, we assume full capital stock depreciation within a period. TFP is composed of an aggregate component and an idiosyncratic one:

$$A_{it} = A_t + c^A_{it}$$

(2)

where $A_t$ follows an AR(1) process and $c^A_{it}$ follows a Markov process, with $E(A_t) = A$ and $\int_0^1 c^A_{it} di = 0$.

Entrepreneurs enter beginning-of-period $t$ with initial income $\Omega_{it}$ and can borrow in illiquid bonds $D_{it}$ to pay for their consumption, their capital, and cash $M_{it}$. Bonds $D_{it}$ are illiquid in the sense that they can only be traded at beginning-of-period. They yield a gross interest rate $r_t$, while cash bears no interest. Their beginning-of-period budget constraint is:

$$\Omega_{it} + D_{it} \geq c_{it} + K_{it} + M_{it}$$

(3)

The cash ratio $m_{it}$ is defined as the proportion of cash to total assets, i.e., $m_{it} \equiv M_{it}/(K_{it} + M_{it})$. As $D_{it}$ is never negative in equilibrium, it is never part of gross assets.\(^{15}\) Initial income is made of output and unused cash minus the gross interest rate payment on bonds and the cost associated with external liquidity used in the previous subperiod:

$$\Omega_{it} = Y_{it-1} + \tilde{M}_{it-1} - r_{t-1}D_{it-1} - \psi L_{it-1}$$

(4)

where $\tilde{M}_{it-1}$ is unused cash, $L_{it-1}$ is external liquidity obtained in the previous end-of-period and $\psi \geq 1$ is the cost associated with it.

Liquidity shocks affect the magnitude of external liquidity $L_{it}$ available to firms. At end-of-period $t$, firms need to pay for wages out of their cash or any liquid funds.

\(^{15}\) $D_{it}$ is not negative, because all firms are always constrained and because we abstract from equity issuance. If some firms were unconstrained, they could choose a negative $D_{it}$, and thus hold both bonds and cash.
they obtain in that subperiod. They face the following liquidity constraint:

\[ M_{it} + L_{it} \geq w_t l_{it} \]  \hspace{1cm} (5)

where \( w_t \) is the wage rate. Unused cash is simply defined as \( \tilde{M}_{it} = M_{it} - L_{it} - w_t l_{it} \). It will be equal to zero in most of our analysis. Liquid funds \( L_{it} \) are assumed to be limited to a proportion \( \kappa_{it} \) of current output, i.e.,

\[ L_{it} \leq \kappa_{it} Y_{it}. \]  \hspace{1cm} (6)

If \( r > \psi \), then typically \( L_{it} = \kappa_{it} Y_{it} \), which we will assume. Shocks to \( \kappa_{it} \) are therefore liquidity shocks, i.e., shocks that affect the amount of external liquidity. External liquidity can take several forms. If it is made only of short-term borrowing, total liquidity needs are equal to the wage bill \( w_t l_{it} \) and \( L_{it} = \kappa_{it} Y_{it} \) represents a short-term credit limit justified by standard moral hazard mechanisms. A shock to \( \kappa_{it} \) corresponds to a change in short-term credit conditions and the cost \( \psi \geq 1 \) then represents the gross interest rate on short-term loans. Alternatively, shocks to \( \kappa_{it} \) could also represent shocks to liquidity provided by customers. Assume that \( L_{it} \) represent early sales: the firm could sell a proportion \( \kappa_{it} \) of its output at end-of-period \( t \), while the rest is sold at beginning-of-period \( t + 1 \). This could correspond to the trade receivables made by the firm to the customers. In that case, the liquidity need is \( w_t l_{it} - \kappa_{it} Y_{it} \). Sales in the next subperiod are then \( (1 - \kappa_{it}) Y_{it} \) so that \( \psi = 1 \).\(^{16}\) External liquidity could also vary with the proportion of wages that have to be paid at end-of-period. Therefore, \( \kappa_{it} \) captures different forms of external liquidity that will affect in the same way the demand for cash holdings.

The liquidity shock \( \kappa_{it} \) is assumed to be composed of an aggregate component and an idiosyncratic one:

\[ \kappa_{it} = \kappa_t + \epsilon_{it}^\kappa \]  \hspace{1cm} (7)

where \( \kappa_t \) follows an AR(1) process and \( \epsilon_{it}^\kappa \) follows a Markov process, with \( E(\kappa_t) = \kappa \) and \( \int_0^1 \epsilon_{it}^\kappa \, d\xi = 0 \). In our benchmark analysis, we simply assume that \( \kappa_{it} \) is known at beginning-of-period \( t \). Assuming that the liquidity shock is anticipated is a convenient way of capturing the perceived availability of liquidity. More generally, we can think of expected changes in the distribution of \( \kappa_{it} \). In Section 6, we show

\(^{16}\)However, selling at a later period may imply storage costs or consumer search costs so that \( \psi > 1 \).
that anticipated changes in the variance of $\kappa_{it}$ can have the same effect.

Finally, we assume that the entrepreneur faces a standard credit constraint at beginning-of-period $t$. Due to standard moral hazard arguments, a fraction $0 \leq \phi_{it} \leq 1$ of production has to be used as collateral for bond repayments:

$$r_tD_{it} \leq \phi_{it}Y_{it}$$

The parameter $\phi_{it}$ is composed of an aggregate component and a firm-specific one:

$$\phi_{it} = \phi_t + \epsilon_i$$

where $\phi_t$ follows an AR(1) process with $E(\phi_t) = \phi$ and $\int_0^1 \epsilon_i^\phi d\epsilon_i = 0$. In this paper, we make the distinction between a standard credit shock, $\phi_{it}$, and a liquidity shock, $\kappa_{it}$. The former can be viewed as a standard disturbance on the banking sector since it affects the long-term credit. The latter corresponds to an exogenous change in the availability of external liquid funds, which may come from different sources.

### 3.2 Optimal Cash Holding and Employment

Entrepreneurs maximize (1) subject to (3), (5) and (8). The optimization of the entrepreneur is described in details in Appendix A. We assume that shocks are anticipated so the random variables $A_{it}$, $\kappa_{it}$ and $\phi_{it}$ are known at beginning-of-period $t$. As cash does not yield any interest, one can also verify that (5) is always binding so that $fM_{it} = 0$.

It is convenient to express production as a function of the capital-labor ratio $k_{it} = K_{it}/l_{it}$. We have $F(K_{it}, A_{it}l_{it}) = A_{it}l_{it}f(k_{it}/A_{it})$ where $f(k) = F(k, 1)$. In the Cobb-Douglas case where $F(K, Al) = K^\alpha(Al)^{1-\alpha}$, the optimality conditions with respect to $l_{it}$ and $K_{it}$ imply a constant capital-labor ratio across firms such that (see Appendix A):

$$k_{it} = k_t = k(w_t)$$

where $k(w_t) = \alpha w_t/(1 - \alpha)$. More generally, we would have $k_{it} = A_{it}k(w_t/A_{it})$ (see Appendix A). In this section, we consider the Cobb-Douglas case. The more

\[\text{17}\text{The presence of credit constraints at the beginning-of-period is not crucial to the main mechanisms we analyze, but it allows to study the impact of credit market shocks. Moreover, it is a convenient assumption with heterogenous firms, as it puts a limit to the size of the most productive firms.}\]
general CES case is treated in Section 6.

The cash ratio, which is a key variable in our analysis because it reflects the cash-intensity of production, can be derived from the above results. Using (5), (10), and \( L_{it} = \kappa_{it}Y_{it} \), we find:

\[
\frac{M_{it}}{K_{it}} = \frac{1}{k_t} \left[ w_t - \kappa_{it}A_{it}f(k_t) \right]
\]

The demand for cash per unit of capital is equal to the demand of cash per unit of labor, divided by the capital-labor ratio. The demand for cash per unit of labor is itself simply equal to the liquidity need per unit of labor \( (w_t) \), minus external liquidity per unit of labor \( (\kappa_{it}A_{it}f(k_t)) \). Equation (11) implies that the cash ratio, which depends solely on \( M_{it}/K_{it} \), decreases with \( \kappa_{it} \) and \( A_{it} \). The reason is that both higher \( \kappa_{it} \) and \( A_{it} \) imply smaller liquidity needs at end-of-period \( t \).

To analyze labor demand, we will focus on cases where entrepreneurs are credit-constrained and have log utility. Appendix A shows that the credit constraint is binding whenever the wage paid by firms, \( w_t \), is lower than the marginal return of labor, denoted \( w_t^* \). With a Cobb-Douglas production function, we find that \( w_t^* = A_{it}[\kappa_{it} + (1 - \psi\kappa_{it})/r_t](1 - \alpha)f(k_t) \). Moreover, with log utility Appendix A shows that optimal consumption is \( c_{it} = (1 - \beta)\Omega_{it} \).

In that case, it is useful to rewrite the constraint (3) using (5), (8), and \( L_{it} = \kappa_{it}Y_{it} \). This gives:

\[
\beta\Omega_{it} + \frac{\phi_{it}Y_{it}}{r} + \kappa_{it}Y_{it} = K_{it} + w_t l_{it}
\]

Equation (12) gives the budget constraint aggregated over the two subperiods. Total financing of firms, on the left-hand side, pays for inputs, on the right-hand side. Both the long-term and short-term financing conditions, represented respectively by \( \phi_{it} \) and \( \kappa_{it} \), affect the capacity of firms to finance labor \( l_{it} \). Using (12), the optimal behavior of entrepreneurs is described in the following proposition.

**Proposition 1 (Individual policy functions)** Suppose that \( u(c_{it}) = \ln(c_{it}) \) and 
\( F(K, A_l) = K^{\alpha}(A_l)^{1-\alpha} \). If \( r_t > \psi > 1 \) and \( A_{it}f(k_t) < w_t < A_{it}[\kappa_{it} + (1 - \psi\kappa_{it})/r_t](1 - \alpha)f(k_t) \), where \( k_t \) is given by (10), then the liquidity constraint (5) and the credit constraints (6) and (8) are binding and the policy functions for \( K_{it}, M_{it}, l_{it}, D_{it}, \) and \( \Omega_{it+1} \) satisfy:

\[
l_{it} = Z_{it}\Omega_{it}
\]

\[
K_{it} = k_tZ_{it}\Omega_{it}
\]
\[ M_{it} = (w_t - \kappa_{it} A_{it} f(k_t)) Z_{it} \Omega_{it} \]  
\[ D_{it} = \phi_{it} A_{it} f(k_t) Z_{it} \Omega_{it} / r_t \]  
\[ \Omega_{it+1} = [(1 - \psi_t \kappa_{it}) - \phi_{it}] A_{it} f(k_t) Z_{it} \Omega_{it} \]  
where \[ Z_{it} = \frac{\beta r_{it}}{r_t[k_t + w_t] - (\kappa_{it} r_t + \phi_{it}) A_{it} f(k_t)} \].

**Proof.** See Appendix A. ■

We call \( Z_{it} \) the financial multiplier. It measures the impact of a change in income on labor demand. Notice that a decline in the financing conditions \( \phi_{it} \) or \( \kappa_{it} \) implies a smaller \( Z_{it} \). This is actually the only channel through which \( \phi_{it} \) affects the policy functions of assets so that \( M_{it} \) and \( K_{it} \) move proportionately in the same direction and the cash ratio remains unaffected. In contrast, a change in \( \kappa_{it} \) affects both the financial multiplier and the relative need for cash, thereby leading to a negative relation between the cash ratio and employment. These effects give the main insight from the results in Proposition 1 and are given in the following corollary.

**Corollary 1** *Ceteris paribus, firms with lower liquidity \( \kappa_{it} \) or lower productivity \( A_{it} \) have lower employment \( l_{it} \) and a higher cash ratio \( m_{it} \). Moreover, \( \phi_{it} \) affects negatively employment \( l_{it} \) but has no effect on the cash ratio \( m_{it} \).*

**Proof.** From (13)-(15) it is easy to see that \( M_{it} / K_{it} \) is decreasing in \( \kappa_{it} \) and \( A_{it} \), while \( l_{it} \) is increasing in \( \kappa_{it} \) and \( A_{it} \). Similarly, \( M_{it} / K_{it} \) is invariant in \( \phi_{it} \) while the financial accelerator \( Z_{it} \), hence \( l_{it} \), is increasing in \( \phi_{it} \). ■

Corollary 1 illustrates the main mechanism in the model. An expected decrease in \( \kappa_{it} \) implies a smaller amount of available liquid funds at end-of-period \( t \). As a response, firms naturally increase the proportion of cash in their portfolio, as seen in (11). At the same time, they reduce their labor demand and their production, as outside funding decreases. This can be easily seen from (12). The same occurs with a decline in productivity \( A_{it} \), since lower productivity also implies lower liquid funds at the end-of-period. In contrast, changes in the credit constraint \( \phi_{it} \) do not have a direct impact on \( m_{it} \) since they only affect long-term credit. Indeed, \( \phi_{it} \) affects the scale of production through the financial multiplier \( Z_{it} \) but does not affect the structure of the portfolio between liquid and illiquid assets. Therefore, \( \phi_{it} \) could not
explain the negative relationship between the cash ratio and employment. However, the degree of the credit constraint affects the response of $l_{it}$ to shocks.

The results can be even starker when we consider the level of cash rather than the cash ratio, as shown in the following corollary.

**Corollary 2** If $r_t k_t > \phi_{it} A_{it} f(k_t)$, then, ceteris paribus, firms with lower liquidity $\kappa_{it}$ or lower productivity $A_{it}$ have higher cash holdings $M_{it}$, while firms with lower $\phi_{it}$ have lower cash holdings.

**Proof.** From (13)-(15) it is easy to see that $M_{it}$ is decreasing in $\kappa_{it}$ and $A_{it}$, as long as $r_t k_t > \phi_{it} A_{it} f(k_t)$. Similarly, since $Z_{it}$ is decreasing in $\phi_{it}$, then $l_{it}$ is decreasing in $\phi_{it}$. ■

A lower $\phi_{it}$ leaves the cash ratio unchanged, but reduces the scale of production through the financial accelerator $Z_{it}$. Since cash is used in the production process, this reduces the level of cash. This is a “size” effect. Lower $\kappa_{it}$ and $A_{it}$ also reduce the scale of production through the financial multiplier, but they also make the production process more intensive in cash through the cash ratio. This is a “portfolio” effect. If $r_t k_t > \phi_{it} A_{it} f(k_t)$, the portfolio effect dominates and the level of cash increases.

The next two sections verify numerically the ceteris paribus result from Corollaries 1 and 2 in a dynamic model where the income level $\Omega_{it}$ is endogenous and the wage rate $w_t$ is determined in the labor market. Section 4 focuses on aggregate shocks and the time-series dimension, while Section 5 focuses on the cross-firm dimension.

### 3.3 Closing the Model

The model is closed by introducing households. Since the emphasis is on firms, households are modeled in a simple way and the full description is left for Appendix B. Identical households provide an infinitely elastic supply funds $D_t$ to firms at interest rate $r = 1/\beta_h$, where $\beta_h$ is the households’ discount factor. This is justified by a utility function linear in consumption and the absence of financial frictions. We assume that $\beta_h \geq \beta$. Similarly, we assume that households’ utility is linear in cash so that their supply of cash is infinitely elastic at rate 1.

Households have a labor supply $l^h(w_t)$ that depends positively on the wage rate. In our specification, we have $l^h(w_t) = (w_t/\bar{w})^\eta$ where $\eta > 0$ is the Frisch elasticity of
labor supply and $\bar{w}$ is a positive constant (see Appendix B). The wage rate is then determined endogenously so that $l^s(w_t) = \int_0^1 l_{it} di$ where $l_{it}$ is the labor demand by firm $i$ in period $t$. According to Proposition 1, $l_{it} = l(w_t, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it})$, so the equilibrium wage is defined by

$$l^s(w_t) = \int_0^1 l(w_t, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it}) di,$$  \hspace{1cm} (19)

At end-of-period $t$ households also supply liquid funds $L_t$ at rate $\psi$, where $\psi - 1$ is a sunk cost incurred by households when providing external liquidity. They always have sufficient cash since they receive their wages at end-of-period $t$ while they consume at beginning-of-period $t+1$. As mentioned above, these liquid funds can take different forms such as early purchases or short term credit.

4 Aggregate Shocks

In this section, we focus on the time-series dimension, as described in Figure 1, of the relationship between the cash ratio and employment. For this purpose, we assume that all entrepreneurs are identical and only face aggregate shocks, so $\epsilon_i^A = \epsilon_i^C = \epsilon_i^\phi = 0$. We also assume that entrepreneurs are always constrained by setting $\beta < \beta^h$. In this context, we calibrate the model to analyze the dynamic impact of three relevant shocks: liquidity shocks $\kappa_i$, productivity shocks $A_t$, and standard credit shocks $\phi_i$. We show that our model generates a negative comovement between cash and labor in the presence of liquidity and productivity shocks, but not with standard credit shocks.

4.1 Equilibrium

In the absence of idiosyncratic shocks, the only potential source of heterogeneity between firms is their wealth. Since labor demand is linear in wealth, we can then write $\int_0^1 l(w_t, A_t, \kappa_t, \phi_t, \Omega_t) di = l(w_t, A_t, \kappa_t, \phi_t, \Omega_t)$ where $\Omega_t = \int_0^1 \Omega_{it} di$. We consider a constrained equilibrium in the Cobb-Douglas case defined as follows:

Definition 1 (Constrained equilibrium under aggr. shocks only, Cobb-Douglas case)

For a given aggregate wealth $\Omega_t$ and a given realization of $A_t$, $\kappa_t$, and $\phi_t$, a constrained period-$t$ equilibrium is a level of employment $l_t$, of capital $K_t$, of cash
$M_t$, of debt $D_t$, of financial multiplier $Z_t$ and of future wealth $\Omega_{t+1}$ satisfying Equations (13) to (18), where $r_t = 1/\beta_h$, the wage $w_t$ clears the labor market so that $l^*(w_t) = l(w_t, A_t, \kappa_t, \phi_t, \Omega_t)$ with $l^*(w_t) = (w_t/\bar{w})^u$ and $k_t$ is the corresponding capital-labor ratio given by Equation (10). Finally, the equilibrium wage must satisfy $w_t < w^*_t$.

Since the aggregate labor demand depends on $A_t, \kappa_t, \phi_t$ and $\Omega_t$, the equilibrium wage also depends on those variables: $w_t = w(A_t, \kappa_t, \phi_t, \Omega_t)$. For an individual firm, we saw that the credit constraint is binding whenever $w_t < w^*_t$. At the aggregate level, we can show that there exists an increasing function $\Omega^*(A_t, \kappa_t, \phi_t)$ so that $w_t < w^*_t$ is equivalent to $\Omega_t < \Omega^*$. When the wage is low, firms want to use all their resources to produce. However, because firms’ resources are limited by the credit constraints, the aggregate labour demand is low when the aggregate wealth is low, which maintains the equilibrium wage at a low level and firms are constrained in equilibrium. In this section, we focus on cases where this condition is satisfied and we discuss the case where firms are unconstrained in Section 6. The following Proposition shows under which conditions the steady state is constrained:

**Proposition 2 (Constrained steady state under aggregate shocks only)** The steady state is constrained if and only if $\beta < \beta_h$.

**Proof.** See Appendix C. ■

Individual agents and the aggregate economy will fluctuate around a constrained steady state. Intuitively, on the one hand, a wage that is lower than the marginal productivity of labor makes the credit constraint binding, as stated in Proposition 1. On the other hand, the credit constraint makes the equilibrium wage dependent on aggregate wealth. When $\beta < \beta_h$, the net interest rate $1/\beta_h - 1$ is below the propensity to consume out of wealth $1/\beta - 1$, so firms never accumulate sufficient wealth to be able to provide an equilibrium wage equal to marginal productivity.

### 4.2 Calibration

Table 1 shows the calibration used for the parameters. The first five parameters are calibrated on standard values. Firms’ discount factor equals $\beta = 0.97$ while the one for households’ is set to $\beta_h = \beta/0.99$. Consequently, the steady-state annualized
The real interest rate is 2%. The Frisch parameter, $\eta$, is set to unity.\footnote{The online appendix shows that the dynamics are slightly affected by the calibration of this parameter.} We set the share of capital in production, $\alpha$, to 0.36. We assume that the cost of using liquidity, $\psi$, is lower than the gross interest rate, such that $\psi = 1.01$.\footnote{This assumption is needed for credit lines to be attractive. In reality, the rate on short-term credit lines is typically higher. But the borrowing period is shorter so that the actual borrowing cost is typically lower.} The other parameters are calibrated to match key empirical targets, using the Compustat database described in Section 2. The liquidity parameter $\kappa$ is set to 0.59 in order to match the mean of the cash ratio which equals 11\% over the sample.\footnote{The online appendix shows that our results are robust to the calibration of this parameter.} In addition, the proportion of output to be collateralized $\phi$ is set to $(0.32 \times r)$ to match the average debt-to-sales ratio of 0.32.\footnote{We define debt as the sum of long-term debt (Compustat data item #9) and debt in current liabilities (Compustat data item #34). Sales correspond to Compustat data item #117.} Finally, we normalize $A$ to unity. The autoregressive parameter of the AR(1) process for $\kappa_t$, $\phi_t$ and $A_t$ is set to 0.95.

4.3 Impulse Response Functions

We examine the impact of a 0.1 percent decrease in aggregate liquidity, technology and credit from their steady-state level. The impulse response functions are computed by determining the equilibrium wage, $w_t$, that clears the labor market and using in turn the policy functions (13) to (17).\footnote{We check that we do have $w_t < w_t^*$ every period.}

| insert Table 1 here |

Liquidity Shock Figure 4 shows the case of a negative liquidity shock, i.e., a decline in $\kappa_t$. This shock implies that firms have smaller external liquid funds to pay for wage bills at end-of-period. On the one hand, firms need to adjust their portfolio by increasing their cash ratio $m_t$ at beginning-of-period. On the other hand, as the financing conditions deteriorate, the firm has to reduce its size, implying that both employment $l_t$ and wages $w_t$ decline. However, this size effect is dominated by the portfolio effect, which implies that the level of cash $M_t$ increases. The need for cash decreases over time as $\kappa_t$ returns to its initial value. In addition,
less attractive production conditions also imply a decline in investment $K_t$ and in debt $D_t$. The equilibrium decline in wage causes an initial increase in firms’ income $\Omega_t$. Overall, the liquidity shock clearly generates a negative comovement between employment and the cash ratio.

**Productivity Shock**  Figure 5 shows the impact of a decline in productivity $A_t$. One of the effects of this decline is to decrease the amount of external liquid funds available at end-of-period. This effect dominates on impact and implies an increase in both the cash ratio and - to a lesser extent - the cash level. The other, more standard, effect is to decrease production through a tighter financial multiplier. This implies a decline in investment, labor demand, wages and debt. Lower inputs and lower productivity both generate a lower production. Lower production in turn implies a smaller need for cash in subsequent periods. This size effect dominates the portfolio effect, generating a drop in the level of cash, even though the cash ratio remains high.

As discussed above, the size and the portfolio effects affect the level of internal liquidity in opposite ways (see Equation 15). Therefore, the comovement between the level of cash and employment depends on the relative importance of these two effects. We observe that a liquidity shock generates a negative comovement between the two variables, while a technology shock generates a positive comovement (except on impact), because the portfolio effect dominates in the former case while the size effect dominates in the latter. Therefore, the combination of the two shocks induces an ambiguous effect on the relationship between the level of cash and employment. In contrast, the cash ratio is exclusively driven by the portfolio effect in response to these two shocks (see Equation 11), implying a negative comovement with employment. This result is in line with the empirical evidence stressed in Section 2 and illustrated in Figure 1.

**Credit Shock**  Figure 6 describes the response to a negative credit shock $\phi_t$. Lower borrowing at beginning-of-period implies lower investment and lower labor demand. Cash holdings decrease sharply, both because of the decrease in labor demand and of the lower need for liquidity per unit of labor due to a lower equilibrium wage. However, a lower debt increases internal funds so that investment and production can quickly recover. This leads to a quick recovery of labor demand and of cash. Overall, cash and employment are clearly positively correlated.
The results in Figures 4 and 5 appear consistent with the aggregate evidence shown in Figure 1. During the financial crisis, the negative comovement between cash and employment was particularly pronounced, while the main shock was emanating from the financial sector. If we assume that the main shock was a liquidity shock $\kappa_t$, the model indeed produces a strong negative relationship. Instead, if we assumed that the main shock was a standard credit shock $\phi_t$, we would observe a positive comovement. In other words, the model is inconsistent with a credit shock being the main shock during the crisis, but is consistent with a liquidity shock. If we consider the period before the crisis, the negative comovement between the cash ratio and employment can be explained from our model by the combination of both technology and liquidity shocks.

Interestingly, in all experiments, debt evolves in the same pattern as labor, which is in line with Covas and den Haan (2012) and Jermann and Quadrini (2012) who stress that debt is procyclical. As a result, the comovement between the cash ratio and debt exhibits the same pattern as the comovement between the cash ratio and labor. It is negative for liquidity and productivity shocks and positive for credit shocks. This is in line with the Flow-of-Funds data which suggest a negative correlation between the log of debt and the cash ratio (both HP-filtered).

5 Cross-firms Correlations

We now assess whether the calibrated model is able to explain the cross-firm evidence of a negative correlation between cash and employment. To examine this issue, we reintroduce heterogeneous firms that are hit by idiosyncratic productivity shocks $e^A_{it}$ and liquidity shocks $e^K_{it}$. Instead we assume for simplicity that the aggregate economy does not fluctuate by setting $A_t = A$, $\kappa_t = \kappa$. As a benchmark, we assume that credit constraints do not vary across firms and time and set $\phi_{it} = \phi$. We relax this assumption later by assuming that firms can have different levels of credit constraints.

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23 The online appendix displays the impulse response functions when the standard credit shock is correlated with the liquidity shock. Not surprisingly, the former generates a lower decrease in the cash ratio.
5.1 Equilibrium

As in the case with aggregate shocks only, we consider a constrained equilibrium defined as follows:

**Definition 2 (Constrained equilibrium under idios. shocks only, Cobb-Douglas case)**

For a given period-t distribution of wealth, productivity and liquidity \(\{\Omega_{it}, A_{it}, \kappa_{it}\}_{i \in [0,1]}\), a constrained period-t equilibrium is given by the firm-specific levels of employment \(l_{it}\), of capital \(K_{it}\), of cash \(M_{it}\), of debt \(D_{it}\), of \(Z_{it}\) and of future wealth \(\Omega_{it+1}\) satisfying Equations (13) to (18), where \(r_t = 1/\beta_h\), the wage \(w_t\) clears the labor market such that (19) is satisfied with \(l^*(w_t) = (w_t/\bar{w})^0\) and \(k_t\) is the corresponding capital-labor ratio given by Equation (10). Finally, the equilibrium wage must satisfy \(w_t < w_{it}{^*}\) for all \(i \in [0,1]\).

In our simulation exercise, we check ex post that we do have \(w_t < w_{it}{^*}\) for all \(i\).

5.2 Calibration

Beside the parameter values described in the previous section, we aim at calibrating a range for \(\kappa_{it} = \kappa + \epsilon_{it}^\kappa\) and \(A_{it} = A + \epsilon_{it}^A\). We assume that these shocks can take 10 equidistant possible realizations. The two shocks are assumed to follow an independent first-order Markov process with transition probability of \(\frac{0.25}{9}\). More precisely, each firm has a probability of 75% to stay in the same state for \(\kappa\) (\(A\)) and a probability of 25% to switch to one of the 9 other states, with an identical probability for each of these states. We calibrate the range for \(\kappa_{it}\) and \(A_{it}\) (namely, we set the minimum and maximum values) to match some distribution moments observed at the firm level. Table 1 provides the interquartile values to match, computed from the Compustat database described in Section 2. The range of the idiosyncratic liquidity and productivity shocks \(\kappa_{it}\) and \(A_{it}\) are set to reproduce: (i) the interquartile range for the empirical cash ratio and (ii) the interquartile ratio for sales. This implies \(\kappa_{it} \in [0.55; 0.635]\) and \(A_{it} \in [0.988; 1]\). All the other parameters are calibrated as described in Section 4.2. The numerical method to obtain the steady-state wage and distribution of firms is described in Appendix D.

5.3 Results

Table 2 displays firm-level moments computed from the stationary distribution. Interestingly, we observe that our calibrated model is able to reproduce the mean
of the cash ratio observed in the data. In addition, the model does a good job in replicating the interquartile ratio of employment. Turning to the negative cross-firm correlation between the cash ratio and employment the model provides a negative correlation of $-0.18$ under our benchmark calibration. This number is somewhat smaller than the precise number found in the data ($-0.29$).

To understand this result, Figure 7 shows the impact of an idiosyncratic innovation of $\kappa_{it}$ and $A_{it}$ on the value of the labor normalized by wealth ($l_{it}/\Omega_{it}$) and the cash ratio ($m_{it}$), both weighted by the distribution probability.

This figure shows that, as $\kappa_{it}$ decreases, the cash ratio is higher and labor is lower for a given $\Omega_{it}$. Similarly, firms facing a negative productivity shock adjust their labor downward. At the same time, they enjoy lower liquidity flows from their sales, which forces them to increase their cash ratio. Therefore, it appears that the cash ratio increases when $A_{it}$ decreases while labor decreases. Consequently, this figure shows a negative relationship between the cash ratio and labor. Firms facing a negative liquidity shock are able to finance less labor with the same amount of cash. To accommodate for this shock, they both accumulate more cash in order to pay for the wage bill and diminish their level of labor to limit the wage bill.

However, while the normalized labor ($l_{it}/\Omega_{it}$) is independent of $\Omega_{it}$ according to Proposition 1, the level of labor $l_{it}$ is driven by the size of the firm $\Omega_{it}$, which depends on the history of shocks. As a consequence, the correlation between the cash ratio and labor is driven not only by $A_{it}$ and $\kappa_{it}$ as suggested by Figure 7, but also by $\Omega_{it}$. Table 3 complements the previous figure by showing the weighted value of these variables by class of firms.

While firms with a level of wealth below median have on average a substantially lower level of employment than firms with a level of wealth above median, their cash ratio is about the same on average. On the one hand, idiosyncratic innovations on liquidity ($\kappa$) and technology ($A$) have a direct effect on the cash ratio and
labor, as shown in Figure 7. On the other hand, they also affect firms’ wealth and therefore employment for a given level of cash. This heterogeneity of wealth generates noise that dampens the correlation. These two elements can explain why the unconditional correlation of cash and labor is negative, but low.

We now show that the credit constraint affects the correlation between the cash ratio and employment through the size effect. To do so, we consider two groups of firms differing with their value of $\phi_i$, namely firms with strong financial constraints and firms with milder financial constraints. In order to be consistent with the calibration strategy described above, we set the two values of $\phi_i$ in order to match moments of the debt-to-sales ratio. More precisely, we match the value of this ratio for the bottom and top 25% of the distribution. This strategy implies that financially-constrained firms are those with $\phi_i = (0.06 \times r)$ while less constrained firms have $\phi_i = (0.31 \times r)$. The lower panel in Table 2 displays the results. In the data, we find that firms with the lowest debt-to-sales ratio exhibit a less negative correlation between the cash ratio and labor.\textsuperscript{24} Our model is able to reproduce this fact. Precisely, we find that the correlation between the cash ratio and labor is $-0.08$ for firms with a low value of $\phi_i$ while it is $-0.20$ for firms with a large value of $\phi_i$. Therefore, the simulation results reveal that the correlation between cash and labor is stronger for less financially-constrained firms. Those firms have a larger financial multiplier since they have more resources through their level of borrowing. Consequently, their labor is less sensitive to productivity and liquidity shocks, while their cash ratio is not affected by the level of $\phi_i$. This implies that the correlation between cash and labor is larger for a large $\phi_i$.

6 Extensions

The benchmark model has abstracted from several elements that could be relevant to the analysis. In this section we describe several extensions. First, we examine how results are affected when capital depreciates gradually and the production function has a more general CES form. Second, we analyze the case where firms are not credit constrained. Third, we discuss the impact of liquidity uncertainty with unanticipated liquidity shocks. Fourth, we discuss the impact of unexpected productivity shocks that provide an alternative explanation for the negative co-

\textsuperscript{24}Applying Fisher’s classical Z-transformation to the coefficients, we can conclude that the correlations coefficients of the two groups of firms are significantly different.
movement between cash and employment.

6.1 Partial Capital Depreciation and CES Production Function

In the baseline analysis, we assumed that capital depreciated completely from one period to the other and we adopted a Cobb-Douglas production function. Here we relax these two assumptions by allowing capital to depreciate only partially and the production function to follow a more general CES specification where $$\begin{align*}
F(K, AL) &= \left[ \alpha(bK)^{\sigma} + (1 - \alpha)(AL)^{\sigma} \right]^{1/\sigma}.
\end{align*}$$ Introducing partial depreciation affects the beginning-of-period budget constraint, which now becomes:

$$\begin{align*}
\Omega_{it} &= Y_{it-1} + (1 - \delta)K_{t-1} + \bar{M}_{it-1} - r_{t-1}D_{it-1} - \psi L_{it-1}
\end{align*}$$

(20)

where $$0 < \delta < 1$$ is the depreciation rate of capital. This changes the firms’ optimization and the computation of the equilibrium wage (see Appendix E).

We consider the baseline calibration discussed in Section 4.2 except that $$\delta = 0.1$$, implying an annual capital depreciation rate of 10%, $$b$$ is normalized to unity and $$\sigma = -0.25$$, meaning that the elasticity of substitution between capital and labor is set to 0.8 (instead of 1 for the Cobb-Douglas specification). Figure 8 shows that our results are robust to the partial depreciation and the CES production function.

[ insert Figure 8 here ]

Specifically, liquidity and technology shocks generate a negative comovement between labor and the cash ratio while the standard credit shock does not. It is worth noticing that imperfect substitutability between capital and labor helps to generate the negative comovement in response to a technology shock since it reinforces the drop in capital. Using our benchmark calibration, we find that the cross-section correlation is $$-0.10^{25}$$.

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25In order to ensure that the average cash ratio is consistent with the data, we reduced the range of values for the idiosyncratic liquidity shock, such that $$\kappa_{it} \in [0.55; 0.59]$$. We still consider 10 equidistant realizations of this shock.
6.2 Unconstrained Firms

We assumed so far that \( r < 1/\beta \), so that firms are always credit-constrained. This has two advantages: it enables us to examine the effect of a standard credit shock and it helps sustain an equilibrium with heterogeneous firms. It is however important to examine how this assumption affects the response of the economy to liquidity and productivity shocks. We show that in the absence of credit constraints, a liquidity shock affects essentially the cash ratio while a productivity shock affects essentially labor. Cash and labor are thus more disconnected than in the benchmark constrained case.

In order to simulate the unconstrained case, we set \( r \) equal to \( 1/\beta \) and assume that \( \phi \) is sufficiently high so that firms never hit their credit limit. We otherwise use the same calibration as in the benchmark model. Since \( r = 1/\beta \), the level of wealth is undetermined in the steady state. For comparison purposes, we set the initial level of \( \Omega \) to the same level as in the benchmark steady state. Figure 9 shows the simulation results.

Following a negative liquidity shock, the economy experiences a decrease in employment and an increase in the cash ratio as in the benchmark. Indeed, on the one hand, firms need more cash to produce. On the other hand, as cash is costly, labor becomes less productive, so the demand for labor and the equilibrium wage decrease. Notice, however, that the effect on employment and the wage is much milder when firms are unconstrained as compared to the benchmark, where firms are constrained. Indeed, as long as the cost of liquidity \( \psi - 1 \) is not too high, the liquidity shock barely affects labor productivity. Therefore, in the absence of constraint, firms do not change their labor demand dramatically. In the presence of credit constraints, the demand for labor and hence the equilibrium wage depend on firms’ resources. Since fewer external resources are available, firms have to cut on labor hiring, generating a stronger reaction of labor demand.

Consider now the effect of a negative productivity shock. While employment decreases as in the benchmark, the cash ratio remains constant. Indeed, the productivity shock has a direct negative effect on the availability of external liquidity, but it has also a negative indirect, general equilibrium effect on the wage and hence on liquidity needs. In the absence of credit constraints, the equilibrium wage is more sensitive to productivity as compared to the case with credit constraints,
where labor demand and the wage depend on wealth. Since, in the latter case, the response of aggregate wealth is sluggish, then so are the responses of labor and the wage. Finally, since the wage, and hence liquidity needs, decrease more when firms are unconstrained, the increase in the cash ratio is mitigated as compared to the benchmark. Actually, the decrease in liquidity needs perfectly compensates for the decrease in external liquidity, leaving the cash ratio unchanged.

6.3 Liquidity Uncertainty

In our analysis, firms know perfectly the amount of external liquidity they can get at the end-of-period, i.e., $\kappa_{it}$ is known at beginning-of-period $t$. If instead we assume that only the distribution of $\kappa_{it}$ is known, we can analyze the impact of an increase in uncertainty in $\kappa_{it}$. Not surprisingly, an increase in liquidity uncertainty increases the demand for cash and decreases employment on average.\textsuperscript{26} In particular, if we assume that labor is set at the beginning of period, then an increase in uncertainty has the same effect as an anticipated negative liquidity shock.

To understand this result, consider the simple case where there are two possible states for $\kappa_{it}$: $\kappa_{it}^L = \kappa - \varpi_t$ and $\kappa_{it}^H = \kappa + \varpi_t$, with $\varpi_t > 0$. The magnitude of $\varpi_t$, and thus the variance of $\kappa_{it}$, is known at the beginning of period but $\kappa_{it}$ is revealed only at the end of period. When $\varpi_t$ increases, the firm increases its cash holdings. When labor is predetermined, firms actually hold just enough cash to be able to finance the wage bill in the worst case where $\kappa_{it} = \kappa_{it}^L$. The reason is that insufficient cash would leave the firm with no revenues ($\Omega_{it+1} = 0$).\textsuperscript{27} This prospect deters firms from putting themselves in such a situation, as the utility is logarithmic and $\log(0) = -\infty$. In the event where $\kappa_{it} = \kappa_{it}^H$, firms do not draw down on the whole line of credit as it is costly ($\psi > 1$), and they set $L_{it} = \kappa_{it}^L Y_{it}$. Thus, cash holdings move proportionately to $\varpi_t$ and firms behave exactly as if their anticipated liquidity shock was $\kappa_{it}^L$.

\textsuperscript{26}This hoarding behavior is reminiscent of the literature on precautionary savings initiated by Bewley (1986) and Aiyagari (1994).

\textsuperscript{27}This implicitly assumes that the punishment the firms face for not honoring the contract entails both that households do not work and that money holdings are seized, leaving the firms. This also supposes that money is a perfectly pledgable asset and that households are credible enough to implement that punishment.
6.4 Unanticipated Productivity Shocks

In this paper we focus on active liquidity management by firms, i.e., the optimal choice of cash holdings $M_t$. However, a proportion of cash holding may come from unexpected unused cash $\tilde{M}_t$, which has been equal to zero so far in our analysis. This may give an alternative explanation to the negative comovement between cash and employment. Assume that productivity shocks are not known at beginning-of-period $t$ and that firms can adjust their employment within the end-of-period (i.e., employment is not predetermined as in 6.2). In that case, unused cash $\tilde{M}_t$ is no longer necessarily equal to zero. For example, an unexpected decline in $A_t$ implies a lower need for liquidity and thus higher $\tilde{M}_t$. Thus, we would have a negative comovement between unexpected cash holding $\tilde{M}_t$ and labor demand. However, if the productivity shock is persistent (e.g., as in (2)) the path of productivity in subsequent periods is anticipated as in our benchmark analysis. Overall, except for the effect on impact, the dynamic effect of an unanticipated productivity shock is similar to an anticipated productivity shock.

The model therefore predicts a temporary increase in relative cash holdings. After an initial negative shock, the cash ratio is reduced to adjust for lower expected productivity. In contrast, it is sometimes argued, especially in the wake of the financial crisis, that firms keep holding cash because of low investment opportunities. For this argument to hold in our model, we should assume repeated negative productivity shocks. Alternatively, we would need to add some adjustment costs for reducing money holdings or assume that firms’ liquidity management is totally passive, i.e., firms would not choose their optimal level of $M_t$.

7 Conclusion

This paper has documented a negative comovement between the corporate cash ratio and employment. Even though such a relationship may appear surprising at first sight, we show that it can be explained both by liquidity and productivity shocks. Precisely, these two shocks might make production less attractive or more difficult to finance, while they also generate a need for liquidity necessary to pay wage bills, which can be satisfied by holding more cash. Moreover, we argue that our analysis is useful in understanding the motives for firms’ cash holdings and in shedding light on the dominant shocks during the financial crisis.
Besides explaining an interesting stylized fact, the simple model developed in this paper could be extended to analyze the role of corporate liquidity in a macroeconomic environment. Several extensions could be of interest. First, instead of focusing on the business cycle frequency, the model could be used to examine longer term developments. The model would actually be consistent with the documented gradual increase in cash holdings if we assume changes in the production process that imply more end-of-period payments (e.g., with more extensive use of just-in-time technologies as reported in Gao, 2013, or with an increase in production outsourcing). A second extension, that would lead to a richer analysis, is to introduce financial intermediaries. Third, for a better analysis of the financial crisis, it would be of interest to introduce demand shocks. Finally, the role of policy intervention would be a natural extension. The last two extensions would be related to the existing DSGE literature incorporating working capital to study monetary policy.
Appendix

A The Entrepreneur’s Problem

Entrepreneurs maximize (1) subject to (3), (5) and (8). They also take into account (4) and the production function \( Y_{it} = F(K_{it}, A_{it}l_{it}) \). The production function has constant returns to scale so we can write \( Y_{it} = A_{it}l_{it}f(k_{it}/A_{it}) \), with \( f(k) = F(k, 1) \) and with \( k \) the capital-labor ratio \( K/l \). Let \( \lambda_{it} \) denote the multiplier associated with the credit constraint (8). Let \( \gamma_{it} \) and \( \eta_{it} \) denote the multipliers associated to the budget constraints (3) and (5) respectively. We make the guess that there exists a function \( k(\cdot) \) such that \( k_{it} = k(w_{it}, A_{it}) \).\footnote{We show later that this guess is satisfied and we will specify \( k(\cdot) \).} If we assume that \( w_{it} > A_{it}f[k(w_{it}, A_{it})] \) and that \( r_t > \psi > 1 \), then \( \tilde{M}_{it} = 0 \), \( M_t > 0 \) and \( L_{it} = \kappa_{it}Y_{it} \).\footnote{This can be shown analytically but is not done here for parsimony.} The Lagrangian problem is then

\[
\mathcal{L}_{it} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \{ u(c_{is}) + \gamma_{is} [Y_{is-1} - r_{s-1}D_{is-1} - \psi_{s-1}k_{is}Y_{is-1} + D_{is} - c_{is} - K_{is} - M_{is}] + \eta_{is} [M_{is} + k_{is}Y_{is} - w_{it}l_{is}] + \lambda_{is} [\phi_{it}Y_{is} - r_tD_{is}] \}.
\]

The entrepreneur’s program yields the following first-order conditions with respect to \( c_{it}, D_{it} \) and \( M_{it} \):

\[
u'(c_{it}) = \gamma_{it} \tag{21}\
\gamma_{it} = \beta r_t \mathbb{E}_t \gamma_{it+1} + r_t \lambda_{it} \tag{22}\
\gamma_{it} = \eta_{it} \tag{23}
\]

Combining (21) and (22) give the standard Euler equation:

\[
u'(c_{it}) = \beta r_t \mathbb{E}_t \nu'(c_{it+1}) + r_t \lambda_{it} \tag{24}
\]
The optimality conditions with respect to \( l_{it} \) and \( K_{it} \) are:

\[
w_t \eta_{it} = F_{it} \left[ \kappa_{it} \eta_{it} + \phi_{it} \lambda_{it} + \beta (1 - \psi_{it} \kappa_{it}) E_t \gamma_{it+1} \right]
\]

(25)

\[
\gamma_{it} = F_{Kit} \left[ \kappa_{it} \eta_{it} + \phi_{it} \lambda_{it} + \beta (1 - \psi_{it} \kappa_{it}) E_t \gamma_{it+1} \right]
\]

(26)

Replacing \( \gamma_{it} = \eta_{it} \) in (25) and combing with (26), we obtain:

\[
w_t = \frac{F_{it}}{F_{Kit}}
\]

(27)

\( F \) has constant returns to scale so we can write: \( F(K, Al) = A f(K/Al) \). Therefore, \( F_K(K, Al) = f'(K/Al) \) and \( F_l(K, Al) = A [f(K/Al) - K f'(K/Al)/Al] \). As a consequence, \( w_t/A_{it} = \tilde{w}(k_{it}/A_{it}) \), with \( k_{it} = K_{it}/l_{it} \) and \( \tilde{w}(k) = [f(k) - k f'(k)]/f'(k) \). Since \( F \) is concave in both arguments, we have \( f'' < 0 \), which implies that \( \tilde{w}(k) \) is strictly increasing in \( k \). If there exists a solution \( k(w_t, A_{it}) \) to that equation, then this solution is unique and satisfies \( k(w_t, A_{it}) = A_{it} \tilde{w}^{-1}(w_t/A_{it}) \). As a result, we have:

\[
K_{it} = k(w_t, A_{it}) l_{it}
\]

(27)

where \( k(w, A) = A \tilde{w}^{-1}(w/A) \).

Consider the CES production function \( F(K, Al) = [\alpha(bK)^\sigma + (1 - \alpha)(Al)^\sigma]^{1/\sigma} \).

In that case, the capital-labor ratio is given by:

\[
k_{it} = A_{it} \left( \frac{\alpha b^\sigma w_t}{(1 - \alpha) A_{it}} \right)^{\frac{1}{\sigma}}
\]

(28)

In the Leontief case where \( F(K, Al) = \min \{bK, Al\} \), we simply have: \( k_{it} = A_{it}/b \).

In the Cobb-Douglas case where \( F(K, Al) = K^\alpha (Al)^{1-\alpha} \), we have \( k_{it} = k_t = \alpha w_t/(1 - \alpha) \).

Denote \( \tilde{F}_l(w_t, A_{it}) = A_{it} f[k(w_t, A_{it})/A_{it}] - k(w_t, A_{it}) f'[k(w_t, A_{it})/A_{it}]/A_{it} \) and \( \tilde{F}_K(w_t, A_{it}) = f'[k(w_t, A_{it})/A_{it}] \). Combining (22), (25) and \( \eta_{it} = \gamma_{it} \), we get:

\[
\lambda_{it} = \frac{w^*(A_{it}, \kappa_{it}, r_t, w_t) - w_t}{r_t w_t - (\phi_{it} + \kappa_{it} r_t) A_{it} \tilde{F}_l(w_t, A_{it})} \beta r_t E_t \gamma_{it+1}
\]

where

\[
w^*(A_{it}, \kappa_{it}, r_t, w_t) = A_{it} [\kappa_{it} + (1 - \psi_{it} \kappa_{it})/r_t] \tilde{F}_l(w_t)
\]

(29)
is the return of one unit of hired labor. This means that the constraint is binding whenever \( w^* > w \). For the CES production function, we have: \( w^* = A_t[\kappa_t + (1 - \psi_t\kappa_t)/r_t](1 - \alpha)(f(k_t))^{1-\sigma} \). The Cobb-Douglas case is obtained simply by setting \( \sigma = 0 \).

**Log-utility**

**Lemma 1** When \( u(c_{it}) = \ln(c_{it}) \), we have \( c_{it} = (1 - \beta)\Omega_{it} \).

**Proof.** We make the educated guess that there exists \( \chi \) such that \( c_{it} = (1 - \chi)\Omega_{it} \). Given that shocks are known at the beginning-of-period, \( c_{it+1} = \chi\Omega_{it+1} \) is known at the beginning-of-period, so the Euler equation (22) can be written without the expectations operator

\[
\frac{1}{\Omega_{it}} = \beta r_t \frac{1}{\Omega_{it+1}} + r_t \lambda_{it}
\]

Combining our guess with (3), (11) and (10), we obtain

\[
\Omega_{it+1} = r_t \chi \Omega_{it} + \left[ (1 - \psi_t \kappa_t) + r_t \kappa_t \right] A_{it} f(k_{it}) l_{it} - r_t(w_t + k_{it}) l_{it}
\]

Similarly, combining (22), (25), (26) and \( \eta_{it} = \gamma_{it} \), we get:

\[
r_t \lambda_{it} = \frac{[\kappa_{it} + (1 - \psi_t \kappa_{it})/r_t] A_{it} f(k_{it}) - (w_t + k_{it})}{w_t + k_{it} - (\phi_{it}/r_t + \kappa_{it}) A_{it} f(k_{it})} \beta r_t \mathcal{E}_t \gamma_{it+1}
\]

If the constraint is not binding, then \( [\kappa_{it} + (1 - \psi_t \kappa_{it})/r_t] A_{it} f(k_{it}) = w_t + k_{it} \). Replacing in \( \Omega_{it+1} \), we obtain that \( \Omega_{it+1} = r_t \chi \Omega_{it} \). Replacing \( \Omega_{it+1} \) in the Euler equation where \( \Lambda_{t+1} = 0 \), we find \( \chi = \beta \).

If the constraint is binding, then \( D_{it} = \phi_{it} Y_{it} \), which implies that

\[
l_{it} = \frac{\chi \Omega_{it}}{w_t + k_{it} - (\phi_{it}/r_t + \kappa_{it}) A_{it} f(k_{it})}
\]

Replacing \( \Omega_{it+1} \) and \( \lambda_{it} \) in the Euler equation, and then replacing \( l_{it} \), we find again that \( \chi = \beta \). ■

Combining \( c_{it} = (1 - \beta)\Omega_{it} \) with the binding constraints (3), (5) and (8), we can easily derive equations (13)-(17) in Proposition 1. The Cobb-Douglas case is obtained by using Equation (10).
B The Household Problem

Identical households have a linear utility $U_t$ with a discount factor $\beta_h$, and no financial frictions:

$$E_t U_t = E_t \sum_{s=0}^{\infty} \beta_h^s \left[ c_{t+s}^h + (1 - \beta_h) M_{t+s}^h - \beta_h \bar{w} \frac{1^{1+1/\eta}}{1 + 1/\eta} \right]$$

(30)

with $\beta_h \geq \beta$. The households maximize this utility subject to their beginning-of-period and end-of-period budget constraints

$$r_{t-1} D_{t-1}^h + M_t^h + c_t^h + (\psi - 1) L_{t-1} = D_t^h + r_{t-1} M_{t-1}^h + r_{t-1}^L L_{t-1}$$

$$w_t l_t + M_t^h = L_t + \tilde{M}_t^h$$

where $c_t^h$ is households’ consumption, $D_t^h$ is household debt, $M_t^h$ are the household’s beginning-of-period money holdings, $\tilde{M}_t^h$ are the household’s end-of-period money holdings. $r_{t-1}^M$ is the return of 1 unit of cash. In the end-of-period, the households lend or spend (depending of the interpretation of the liquidity shock) part of their wage $w_t l_t$ to the firms. This lending/spending $L_t$ incurs costs $\psi - 1$ to the households and yields $r_{t-1}^M$. $\psi$ can be interpreted as a real sunk cost. $r_{t-1}^L$ can be interpreted as the equilibrium return on short-term lending or as an equilibrium price premium on early sales.

Consolidating the end-of-period $t - 1$ and the beginning-of-period $t$ budget constraints, we obtain:

$$r_{t-1} D_{t-1}^h + M_t^h + c_t^h = D_t^h + r_{t-1} M_{t-1}^h + (r_{t-1}^L - \psi) L_{t-1} + r_{t-1}^M w_{t-1} l_{t-1}$$

Households’ optimization then implies that, in equilibrium, $r_t = 1/\beta_h$, $r_{t-1}^M = 1$, $r_{t-1}^L = \psi$ and $l_t = (w_t/\bar{w})^\eta$.

C Equilibrium with aggregate shocks only

Before proving Proposition 2, we establish the following Lemma:

**Lemma 2** There exists an increasing function $\Omega^*$ so that $w_t < w^*(A_t, \kappa_t, \phi_t, w_t)$ is equivalent to $\Omega_t < \Omega^*(A_t, \kappa_t, \phi_t)$. If $\Omega_t < \Omega^*(A_t, \kappa_t, \phi_t)$, then the credit constraint
is binding and the dynamics of $K_t$, $M_t$, $D_t$, $l_t$ and $\Omega_{t+1}$ follow:

\begin{align*}
  l_t &= Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \\
  K_t &= k(w_t)Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \\
  M_t &= (w_t - \kappa_t f[k(w_t)]A_t)Z(w_t, A_t, \kappa_t, \phi_t)\Omega_t \\
  D_t &= \phi_t f[k(w_t)]A_tZ(w_t, A_t, \kappa_t, \phi_t)\Omega_t/r_t \\
  \Omega_{t+1} &= [(1 - \delta)(1 - \kappa_t) - \phi_t]f[k(w_t)]A_tZ(w_t, A_t, \kappa_t, \phi_t)\Omega_t
\end{align*}

where

\[ Z(w_t, A_t, \kappa_t) = \frac{\beta r_t}{r_t[k(w_t) + w_t] - (\kappa_t r_t + \phi_t)f[k(w_t)]A_t} \]

is the financial multiplier and

\[ w_t = w(A_t, \kappa_t, \phi_t, \Omega_t) \]

is the equilibrium wage so that $w(A_t, \kappa_t, \phi_t, \Omega_t)$ is the solution to $l^*(w_t) = Z(w_t, A_t, \kappa_t, \phi_t)\beta r_t\Omega_t$.

**Proof.** $w^*$ is given by Equation (29). Given that $\tilde{F}_t(.)$ and $k(.)$ are increasing functions, $w^*$ is increasing in $w_t$. Since we also have that the constrained equilibrium wage $w$ is increasing in $\Omega_t$, then there exists an increasing function $\Omega^*$ so that $w_t < w^*(A_t, \kappa_t, \phi_t, w_t)$ is equivalent to $\Omega_t < \Omega^*(A_t, \kappa_t, \phi_t)$. The rest of the Lemma derives from Proposition 1.

Using this Lemma, we can study the steady state. From Equation (35), we have that the steady-state wage must satisfy:

\[ w + k(w) - (\kappa + \beta_h \phi)f[k(w)]A = [(1 - \delta)(1 - \kappa) - \phi]f[k(w)]A\beta \]

Rearranging:

\[ w + k(w) - [(1 - \delta)(1 - \kappa)\beta_h + \kappa]f[k(w)]A = [(1 - \delta)(1 - \kappa) - \phi]f[k(w)]A(\beta - \beta_h) \]

As long as $(1 - \delta)(1 - \kappa) > \phi$, the left-hand-side is strictly negative if and only if $\beta_h > \beta$. 

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Besides, combining (24), (25) and \( \eta_t = \gamma_t \), and applying the Euler theorem, we get:

\[
\lambda = \frac{A[k + (1 - \delta)(1 - \kappa)\beta_h]f[k(w)]A - [w + k(w)]}{w + k(w) - (\kappa + \beta_h \phi_t)f[k(w)]A} \beta \gamma
\]

Therefore, the credit constraint is binding in the steady state (\( \lambda > 0 \)) whenever \( \beta_h > \beta \). This proves Proposition 2.

**D  Numerical method**

The algorithm to compute the steady-state distribution of firms is as follows:

1. We first choose a grid of wealth \( \Omega_{it} \). Our grid is a 1000-value grid over \([0, 0.9]\). We use the Chebychev nodes to make the grid more concentrated on low values of \( \Omega \).

2. We allocate an initial uniform and independent distribution to the values of \( \Omega_{i0}, \kappa_{i0} \) and \( A_{i0} \), and make an initial guess on the equilibrium wage \( w_0 \).

3. Given the initial distribution on \( \Omega_{it}, \kappa_{it} \) and \( A_{it} \) and the initial equilibrium wage \( w_0 \), we use Proposition 1 and the Markov Chain to compute the new distribution of \( \Omega_{it+1}, \kappa_{it+1} \) and \( A_{it+1} \). Using Proposition 1, we compute the corresponding distribution of labor demand \( \ell_{it+1} \). We aggregate this labor demand \( \ell_{t+1} = \sum_i \ell_{it+1} d_i \), and if \( \ell_{t+1} > \ell^*(w_t) \) (if \( \ell_{t+1} < \ell^*(w_t) \)), then we update the equilibrium wage \( w_{t+1} \) upward (downward).

4. We repeat step 3 until the equilibrium wage is reached, i.e. when aggregate labor demand is fully satisfied.

**E  Partial capital depreciation and more general production function**

The optimality condition with respect to \( \ell_{it} \) is unchanged, while the optimality condition with respect to \( K_{it} \) is now:

\[
\gamma_{it} = F_{K_{it}}[\kappa_{it} \eta_{it} + \phi_{it} \lambda_{it} + \beta (1 - \psi_{it} \kappa_{it}) E_t \gamma_{it+1}] + (1 - \delta) \beta E_t \gamma_{it+1}
\]

(36)
Replacing $\gamma_{it} = \eta_{it}$ in (25) and combining with (36), we obtain:

$$w_t = \frac{F_{it}}{F_{Kit}} \left[ 1 - (1 - \delta)^{\beta E_t \gamma_{it+1}} \right]$$

where $F_{K}(K, Al) = f'(K/Al)$ and $F_{i}(K, Al) = A[f(K/Al) - K f'(K/Al) / Al]$. Since $\gamma_{it} = u'(c_{it})$, $E_t \gamma_{it+1} = u'(c_{it+1})$, and $c_{it} = (1 - \beta) \Omega_{it}$, we obtain, using the policy function for $\Omega_{it+1}$ (17) and the definition of $Z_{it}$ (18):

$$w_t = \frac{A_{it}[f'(\frac{k_{it}}{A_{it}^2}) - k_{it} f'(\frac{k_{it}}{A_{it}}) / A_{it}]}{f'(\frac{k_{it}}{A_{it}})} \left[ 1 - (1 - \delta) \frac{r_t (k_{it+w_t}) - (\kappa_{it} + \phi) A_{it} f'(\frac{k_{it}}{A_{it}})}{r (1 - \psi \kappa_{it} - \phi) A_{it} f'(\frac{k_{it}}{A_{it}}) + (1 - \delta) k_{it}} \right]$$

Therefore, $k_{it}$, the solution to this implicit equation, now depends not only on $w_t$ but also on $r_t$, $A_{it}$, $\kappa_{it}$ and $\phi_{it}$. 

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References


Table 1. Calibration Strategy

<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th>Value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.97</td>
</tr>
<tr>
<td>( r )</td>
<td>Gross interest rate on bonds</td>
<td>( \frac{0.99}{\beta} )</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Liquidity cost</td>
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</tr>
<tr>
<td>( \eta )</td>
<td>Frisch parameter</td>
<td>1</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Elasticity of output wrt capital</td>
<td>0.36</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Output collateral share for debt</td>
<td>0.3266</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>s.s output collateral share for liquidity</td>
<td>0.5866</td>
</tr>
<tr>
<td>( \kappa_i )</td>
<td>Firm-specific collateral share for liquidity</td>
<td>[0.55; 0.635]</td>
</tr>
<tr>
<td>( A )</td>
<td>Steady-state productivity shock</td>
<td>1.00</td>
</tr>
<tr>
<td>( A_i )</td>
<td>Firm-specific productivity shock</td>
<td>[0.988; 1]</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Targeted Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\text{Cash}}{\text{Total Asset}} ) \text{ average}</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>( \frac{\text{Cash}}{\text{Total Asset}} ) \text{ 25%}</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>( \frac{\text{Cash}}{\text{Total Asset}} ) \text{ 75%}</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>( \frac{\text{Sales}<em>{75%}}{\text{Sales}</em>{25%}} )</td>
<td>17</td>
<td>17</td>
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</table>
Table 2. Simulated Moments

<table>
<thead>
<tr>
<th>Benchmark Calibration</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (m)_{\text{average}} ) \hspace{5mm} \text{Cash ratio average}</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>( (m)_{\text{std}} ) \hspace{5mm} \text{Cash ratio standard-deviation}</td>
<td>0.13</td>
<td>0.23</td>
</tr>
<tr>
<td>( \ell_{25%} \hspace{5mm} \ell_{25%} ) \hspace{5mm} \text{Interquartile ratio of labor}</td>
<td>15.75</td>
<td>17.36</td>
</tr>
<tr>
<td>( corr(m, \ell) ) \hspace{5mm} \text{Correlation(cash ratio; labor)}</td>
<td>-0.29</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Credit-Constrained Firms</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( corr(m, \ell) ) \hspace{5mm} \text{bottom 25%} \hspace{5mm} \text{Corr(cash ratio; labor)_{low}}</td>
<td>-0.24</td>
<td>-0.08</td>
</tr>
<tr>
<td>( corr(m, \ell) ) \hspace{5mm} \text{top 25%} \hspace{5mm} \text{Corr(cash ratio; labor)_{high}}</td>
<td>-0.35</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

**Notes:** In both panels, the empirical correlation between \( m \) and \( \ell \) is computed after removing the firm-specific linear trend. In the lower panel, this empirical correlation is computed for two groups of firms. In the first (second, resp.) line, we select the 25 percent of firms with the lowest (highest, resp.) average debt-to-sales ratio. From the model, we set the share of output collateral, \( \phi \), to \( 0.06 \times r \) (first group) and \( 0.31 \times r \) (second group).
Table 3. Average value of labor and cash ratio by class of firms.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$\ell$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_i$</td>
<td>bottom 50%</td>
<td>0.72</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>top 50%</td>
<td>11.83</td>
<td>0.10</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>bottom 50%</td>
<td>0.64</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>top 50%</td>
<td>1.33</td>
<td>0.04</td>
</tr>
<tr>
<td>$A_i$</td>
<td>bottom 50%</td>
<td>0.79</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>top 50%</td>
<td>1.18</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Notes:** All the values of labor and the cash ratio are weighted by the distribution probability.
Figure 1: Corporate Liquidity and Employment.

Note: The upper panel displays the liquidity ratio and employment. The lower panel displays the inflation-adjusted liquidity (in level) and employment. Employment and inflation-adjusted liquidity are expressed in logarithm. All variables are HP-filtered.
Figure 2: Cross-section correlation between employment and the cash ratio by year.

Note: For both variables, we remove the firm-specific linear trend. Markers with circle corresponds to correlation coefficients significant at 1%.

Figure 3: Cash ratio by class of firms

Note: The solid (dashed, resp.) line corresponds to the cash ratio for firms with labor share below (above, resp.) the median. The labor share is industry-specific and it is computed as the ratio between payroll and production.
Figure 4: Impulse response functions to a negative aggregate liquidity shock ($\kappa_1$).

Figure 5: Impulse response functions to a negative aggregate technology shock ($A_t$).
Figure 6: Impulse response functions to a negative aggregate credit shock ($\phi_t$).

- **Wealth**
- **Labor**
- **Capital**
- **Debt**
- **Cash**
- **Cash Ratio**
- **Output**
- **Wage**
Figure 7: Value of the labor to wealth ratio \((l_i/\omega_i)\) and the cash ratio \((m_i)\).

Note: All values of \(l_i/\Omega_i\) and \(m_i\) are weighted by the distribution probability.
Figure 8: Model with partial depreciation of capital.

Note: The solid lines correspond to the benchmark model. The dashed lines correspond to the model with partial depreciation of capital. All IRFs are expressed in percentage deviation from the steady-state.

Figure 9: Model with credit-unconstrained firms.

Note: The solid lines correspond to the benchmark model. The dashed lines correspond to the model with unconstrained firms. All IRFs are expressed in percentage deviation from the steady-state.

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