Best-reply matching in an experience good model

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Abstract

The paper studies a class of experience good models in a new way. We focus on signaling games close to Akerlof’s market for lemons, in which a seller sells a good to a buyer, who ignores the quality of the good during the transaction. In this context, we first establish some properties of the mixed Perfect Bayesian Equilibria. Then we turn to the concept of best-reply matching (BRM) developed by Droste, Kosfeld & Voorneveld (2002, 2003) for games in normal form. BRM equilibria respect a consistency which is different from the Nash equilibrium one: in a BRM equilibrium, the probability assigned by a player to a pure strategy is linked to the number of times the opponents play the strategies to which this pure strategy is a best reply. We extend this logic to signaling games in extensive form and apply the new obtained concept to our experience good models. This new concept leads to a very simple rule of behavior, which is consistent, different from the Perfect Bayesian Equilibrium behavior, different from Akerlof’s result, and can be socially efficient.

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1. Introduction

The paper studies experience good models in a new way, by applying the concept of best reply matching developed by Droste, Kosfeld & Voorneveld (2002,2003). The signaling games under study are close to Akerlof's market for lemons, in which a seller sells a good to a buyer, who ignores the quality during the transaction. In section 2 we look for the characteristics of the mixed Perfect Bayesian Equilibria (PBE). We namely focus on the fact that no type of seller can play more than three prices in a PBE in which the seller earns a positive payoff regardless of the sold quality. This characteristic eliminates simple rules of behavior which respect a limited rationality. That is why we turn, in section 3, to the Best-Reply Matching (BRM) concept developed by Droste, Kosfeld and Voorneveld (2002, 2003). A BRM equilibrium respects a consistency which is different from the Nash equilibrium one. In a few words, Droste & al.(2003) and Kosfeld & al. (2002) pursue the notion of rationalizability earlier developed by Bernheim (1984) and Pearce (1984): in a BRM equilibrium, the probability that a player assigns to a pure strategy is linked to the number of times the opponents play the strategies to which this pure strategy is a best reply. Droste & al.'s concept is developed for normal form games. Therefore, in section 4, given that we are studying signaling games, we modify the definition of BRM in order to take into account the decentralized decision process allowed by the extensive form game approach. We explain why the two versions of the concept differ. In section 5 we apply Droste & al.'s normal form concept to some games close to Akerlof's market for lemons. In section 6 we apply our extensive form concept (called local BRM equilibrium) to the same games and compare the obtained results. In section 7 we generalize the results obtained in section 6. We study a model with n prices $p_1 \dots p_n$, such that the seller whose good is of quality t_i can only earn a positive payoff by selling the good at prices p_j, with j higher or equal than i. In this model the local BRM equilibrium is a very easy profile of strategies: each quality t_i is sold at each price p_j, with j higher or equal than i, with a same probability, and the consumer accepts each price with the probability 1 divided by the number of qualities possibly sold at this price. This behavior, which is far from a PBE behavior, is not only consistent with the BRM logic, but it respects common sense. Therefore it is easy to learn and to adopt. Moreover we show in section 8 that it can lead to a social surplus that is higher than the one obtained with Perfect Bayesian Equilibria. Section 9 extends the BRM concepts by allowing a more diversified behavior in case of indifference; we establish the link between the new concepts and the Nash equilibria. Section 10 concludes on further developments. It namely comes back to Akerlof's result and establishes the existence of BRM equilibria.

2. Akerlof's market for lemons, Perfect Bayesian Equilibria and limited rationality

The studied context is an experience good model close to Akerlof's context. So the studied game is a signaling game with a seller and a buyer. The seller wants to sell a car to the buyer. The car can be of different qualities. We introduce a finite number of qualities t_i, with i from 1 to n, and $t_i<t_{i+1}$ for i from 1 to n-1. The seller's reservation price for a good of quality t_i is h_i, i from 1 to n, with $h_i<h_{i+1}$ for i from 1 to n-1. The seller sets a price for her good. The buyer observes the price and accepts or refuses the transaction. The buyer's reservation price for a good of quality t_i is H_i, i from 1 to n, with $H_i<H_{i+1}$ for i from 1 to n-1. The buyer ignores the quality during the transaction, but has a prior probability distribution over the qualities, that is common knowledge of both players; the probability distribution assigns probability ρ_i to the quality t_i, with $0< \rho_i <1$ for i from 1 to n and $\sum_{i=1}^{n}\rho_i =1$. It is assumed that $H_i > h_i$ for any i from 1 to n, in order to make profitable trade for both players possible. We also introduce the assumption:

$$\frac{\sum_{i=1}^{j}\rho_i H_i}{\sum_{i=1}^{j}\rho_i} < h_j \qquad \text{for j from 2 to n} \qquad \text{(a)}$$

and even the more restrictive assumption:

for any j from 2 to n,

$$\frac{\sum_{i=1}^{j}\rho_i \alpha_i H_i}{\sum_{i=1}^{j}\rho_i \alpha_i} < h_j \qquad \text{where } \alpha_i\text{, for i from 1 to n, is equal to 1 or 0,}$$

except for the case $\alpha_j=1$and $\alpha_i=0$ for i from 1 to j-1. (b)

Assumption (a) is the heart assumption of Akerlof's comment (see below). Assumption (b) namely ensures that $H_i<h_{i+1}<H_{i+1}$ for any i from 1 to n-1. It also ensures that, if each type of seller plays a unique price, this price being higher or equal to her reservation price, then the consumer is better off accepting a price p with $h_j<p<H_j$ if and only if only t_j plays p.

The symbolic representation of the studied experience good model (with two qualities) is given in figure 1.

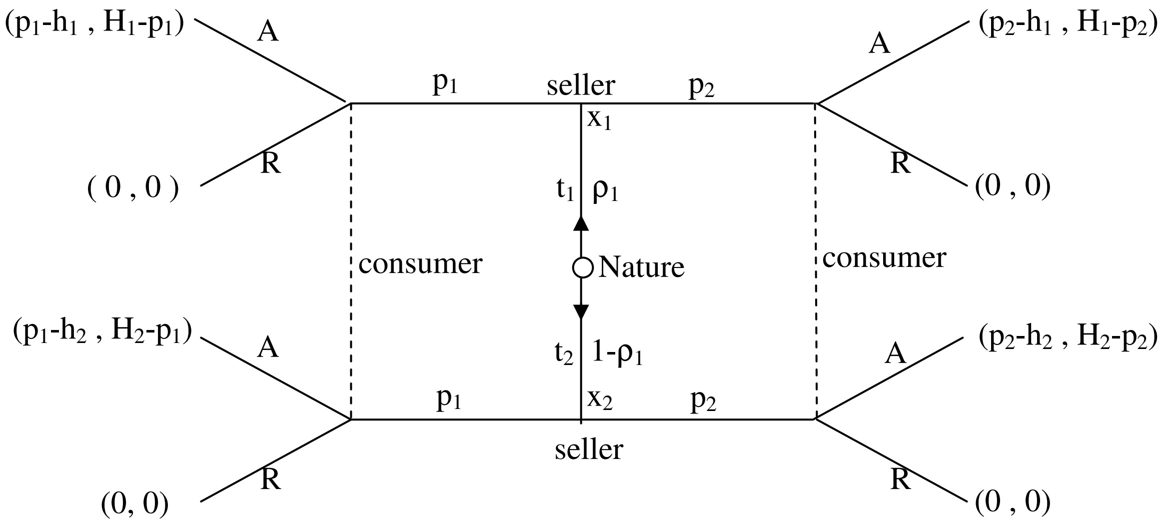

Figure 1

Legend of figure 1: A and R mean that the consumer accepts (A) or refuses (R) the trade. The first, respectively the second coordinate of each vector of payoffs, is the seller's, respectively the consumer's payoff.

Let us recall that in this game Akerlof’s comment goes as follows:

If trade occurs, the car is sold at a unique price, regardless of its quality, because any type of seller wants to sell her car at the highest price. So imagine that the observed price is p, with $h_j \leq p<h_{j+1}$, j higher or equal to 2. Only qualities lower or equal to t_j can be sold at price p. It follows that the expected quality of the sold car is $\frac{\sum_{i=1}^{j}\rho_i t_i}{\sum_{i=1}^{j}\rho_i}$ and that the highest price the

consumer accepts to pay is $\frac{\sum_{i=1}^{j} \rho_i H_i}{\sum_{i=1}^{j} \rho_i}$. Yet this price, by assumption (a), is lower than h_j and therefore lower than p. So trade will not occur at price p. As a consequence, trade can only occur at a price p lower than h_2. This price is necessarily assigned to the quality t_1 and will be accepted, provided it is lower or equal to H_1. Therefore the worst quality throws all the other qualities out of the market.

Yet Akerlof's reasoning is a pure strategy reasoning. As soon as one switches to mixed strategies, ***trade does not necessarily occur at a unique price***. Many prices can coexist on the market and this coexistence allows all qualities to be sold on the market, even in a context that satisfies assumption (b).

Let us give more insights into the Perfect Bayesian Equilibria of the studied signaling game.

Throughout the paper we use the following notations: $\pi_i(p_j)$ is the probability that the seller of type t_i (i.e. whose quality is t_i) plays p_j ; $q(p_j)$ is the probability that the consumer accepts the price p_j.

Proposition 1: existence of PBE

The studied experience good model has a huge number of PBE.

For example, there exists an infinite number of mixed strategies PBE, in which the seller of type t_i plays the prices $p_i{*}$ and $p_{i+1}{*}$, respectively with probabilities $1-\pi_i(p_{i+1}{*})$ and $\pi_i(p_{i+1}{*})$, with i from 1 to n-1; t_n plays the price $p_n{*}$ with probability 1.

$p_1{*}=H_1$; $h_i < p_i{*} < H_i$ for i from 2 to n (and therefore $p_i{*} < p_{i+1}{*}$ for i from 1 to n-1).

The buyer accepts $p_1{*}$ with probability 1 and accepts each price $p_i{*}$, i from 2 to n, with probability $q(p_i{*})$.

$\pi_i(p_{i+1}{*})$, i from 1 to n-1, and $q(p_i{*})$, i from 1 to n, are defined by:

$$\pi_i(p_{i+1}{*}) = \rho_{i+1}\, \pi_{i+1}(p_{i+1}{*})\, (H_{i+1}-p_{i+1}{*}) / \, [\rho_i\, (p_{i+1}{*} - H_i)] \quad (1)$$

$$q(p_1{*})=1$$

$$q(p_i{*}) = (p_{i-1}{*}-h_{i-1})q(p_{i-1}{*})/(p_i{*}-h_{i-1}). \quad (2)$$

The buyer assigns each price p different from the equilibrium prices, with $H_{i-1} \leq p < H_i$, to t_{i-2}, for i from 3 to n, and each price p, with $p<H_2$, to t_1. Hence he refuses the trade at each non equilibrium price higher than H_1. He accepts all the out of equilibrium prices lower than H_1.

Proof: see appendix 1

Given that both p_i* and p_{i+1}* are higher than h_i for i from 1 to n-1 and that p_n is higher than h_n, proposition 1 ensures that, as soon as the players are allowed to play mixed strategies, trade can occur with positive probabilities at different prices and the seller's payoff can be positive at a PBE regardless of the quality of her good (for more precisions on these equilibria see Umbhauer 2007).

The PBE of the studied experience good model share some properties which are given in proposition 2.

Proposition 2

In every PBE in which each type of seller gets a positive payoff:

- each type of seller plays at most 3 prices with a positive probability;
- if t_i plays 3 prices p, p', and p'', with p<p'<p'', then p'=H_i;
- if t_i plays a price p different from H_i, then p is also played with positive probability by the adjacent type t_{i-1} or t_{i+1};
- at most 2n-1 different prices are played in a PBE path;
- the buyer's payoff is null.

In every PBE the buyer's payoff is null.

Proof: see appendix 2

The property we focus on is the fact that each type t_i can at most play two prices different from H_i. Let us illustrate the consequence of this fact on the following simplified experience good model:

There are 3 types of quality, t_1, t_2 and t_3 , with $t_1<t_2<t_3$ and only 3 possible prices, p_1, p_2 and p_3, with $h_1< p_1 < H_1 < h_2 < p_2 <H_2 < h_3 < p_3 < H_3$. We suppose that the game satisfies assumption (b), that no player plays a weakly dominated strategy, hence that no type of seller plays a price lower than her reservation price and that the consumer always accepts p_1. It follows that the studied game is given in figure 2.

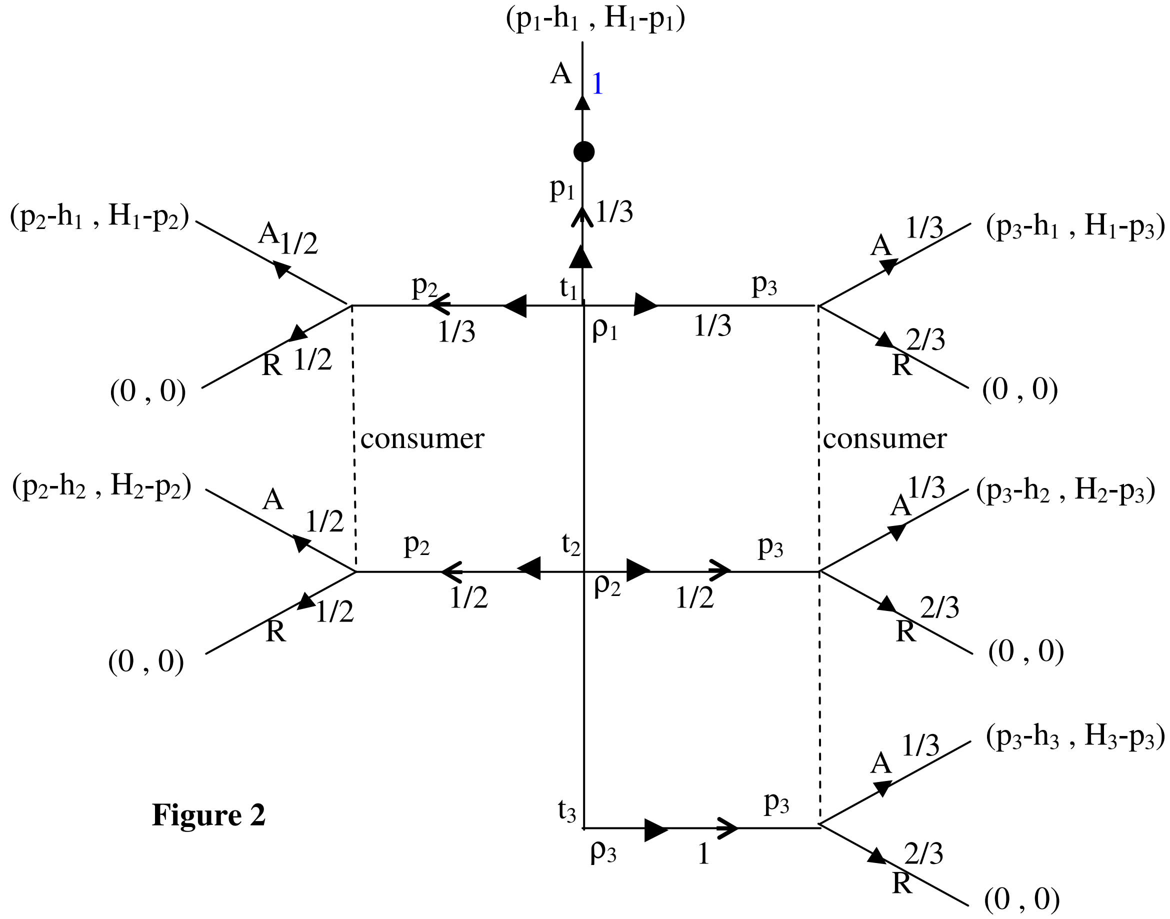

Figure 2

A PBE of this game can, for example, lead t_1 (i.e. the seller of type t_1) to play p_1 and p_2 (unfilled arrows), t_2 to play p_2 and p_3 (unfilled arrows), t_3 to play p_3 (unfilled arrow), and lead the consumer to accept p_1 (full arrow), and to accept and refuse p_2 and p_3 with positive probability (full arrows). It is impossible to find a PBE in which each type of seller earns a positive payoff, ***t_1 plays the three prices p_1, p_2, p_3,*** t_2 plays the two prices p_2 and p_3 and t_3 plays p_3 (full arrows), because t_1 can at most play 2 prices different from H_1. A fortiori it is impossible to construct a PBE (with positive payoff for each type of seller) in which t_1 plays each of the three prices p_1, p_2 and p_3 with the same probability 1/3, t_2 plays each of the two prices p_2, p_3 with probability ½, t_3 plays p_3 with probability 1, the consumer accepts p_1 with probability 1, p_2 with probability ½ and p_3 with probability 1/3 (cf. figure 2).

Yet this easy profile of strategies is not silly; it satisfies a limited rationality. As a matter of fact it is not silly for t_1 to play all the three prices with the same probability. Indeed, by comparing p_1 and p_3, she observes that it is more interesting to play p_3 when p_3 is accepted, which encourages her to put a higher probability on p_3 than on p_1; but she also knows that the consumer is more incited to refuse p_3 than p_1, which discourages her to put a higher probability on p_3 than on p_1. So, all in all, it is not silly to play both prices with the same probability. Given that she can make a similar reasoning by comparing p_1 and p_2 and p_2 and p_3, it not silly to finally assign the same probability 1/3 to each of the three prices. And of course a similar reasoning can lead her to assign the same probability ½ to p_2 and p_3 when she is of type t_2. The consumer's behavior also finds an easy justification. He prefers accepting p_2 when it is played by t_2, but he prefers refusing it when it is played by t_1; hence, given that only one of two configurations encourages him to buy the good, he buys it with probability ½. Similarly, the consumer prefers accepting p_3 if it is played by t_3, but prefers refusing it if it is played by t_1 and if it is played by t_2; hence, given that only one of three configurations encourages him to buy the good, he buys it with probability 1/3.

It derives that this easy behavior satisfies limited rationality even if, of course, it does not respect Bayesian rationality. This easy behavior has another advantage. It can easily be generalized to a higher number of types and prices. So it can become an applied rule of behavior, because it is easy to learn and therefore to adopt, regardless of the number of prices and types.

What is more, this simple rule of behavior respects a strong consistency, the best-reply matching one, to which we turn in the following sections.

3. Best-reply matching in a signaling game: the normal form approach

Droste, Kosfeld and Voorneveld introduced the concept of BRM equilibria in normal form games. Their definition is recalled hereby:

Definition 1 (Kosfeld & al. 2002): Normal form BRM matching equilibrium

Let $G=(N, (S_i)_{i\in N}, (\succ_i)_{i\in N})$ be a game. A mixed strategy p is a (normal form) BRM equilibrium if for every player $i \in N$ and for every pure strategy $s_i\in S_i$, :

$$p_i(s_i)= \sum_{s_{-i}\in B_i^{-1}(s_i)} \frac{1}{\text{Card } B_i(s_{-i})} p_{-i}(s_{-i})$$

In a BRM equilibrium, the probability assigned to a pure strategy is linked to the number of times the opponents play the strategies to which this pure strategy is a best reply. So, if player i' s opponents play s_{-i} with probability $p_{-i}(s_{-i})$, and if the set of player i's best responses to s_{-i} is the subset of pure strategies $B_i(s_{-i})$, then each strategy of this subset is played with the probability $p_{-i}(s_{-i})$ divided by the cardinal of $B_i(s_{-i})$.

This concept carries on the concept of rationalizability developed by Bernheim (1984) and Pearce (1984), according to which a strategy s_i is rationalizable if there exists a pure strategy profile s_{-i} played by the opponents to which s_i is a best response. Droste, Kosfeld and Voorneveld go further: they observe that, if the opponents often play s_{-i}, then s_i often becomes the best response, and therefore they argue that it is rational (rationalizable) for player i to often play s_i . More precisely, Droste & al require that, if s_{-i} is played with probability p_{-i}, s_i should be played with the same probability (if s_i is the only best reply to s_{-i}). Given that the same condition is checked for each pure strategy, each player's probability distribution (on pure strategies) is justified by the opponents' probability distributions, which ensures a strong behavior consistency.

This consistency is very different from the consistency of the Nash equilibrium concept, albeit the intersection between both concepts is not empty[2]. To see why, look at the signaling game given in figure 3.

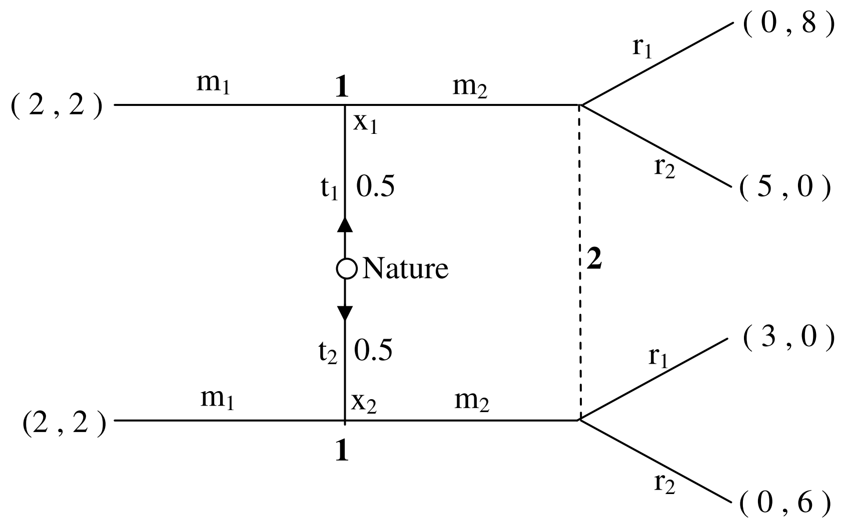

Figure 3

[2] Especially each strong Nash equilibrium is a BRM equilibrium. In fact, in a strong Nash equilibrium s*, each strategy s_i* is played with probability 1 and constitutes the unique best reply to the strategies of the other players. So according to the BRM logic, each strategy s_i* has to be played with the probability assigned to s_{-i}*, i.e. 1, which ensures that s* is also a BRM equilibrium (see also section 9 for more intersection).

The only PBE, and also the only Nash equilibria, of the game in figure 3 are such that player 1 always plays m_1 regardless of type and player 2 assigns to r_1 a probability between 0.6 and 2/3. It follows that the unique PBE outcome is the couple (2,2).

The normal form of the game is given by matrix 1. One observes in this matrix –but also directly on the extensive form of the game- that player 1 is best off playing $m_1/t_1m_2/t_2$ each time player 2 plays r_1 and that she is best off playing $m_2/t_1m_1/t_2$ each time player 2 plays r_2. Hence the BRM consistency requires that player 1 plays $m_1/t_1m_2/t_2$ as often as player 2 plays r_1, i.e. that she assigns to $m_1/t_1m_2/t_2$ a probability p_2 equal to the probability q that player 2 assigns to r_1; in the same way she has to assign to $m_2/t_1m_1/t_2$ a probability p_3 equal to the probability 1-q that player 2 assigns to r_2.

		2	
		r_1	r_2
	$m_1/t_1\ m_1/t_2$	(2 , 2)	(2 , 2)
1	$m_1/t_1\ m_2/t_2$	(2.5,1)	(1,4)
	$m_2/t_1\ m_1/t_2$	(1 , 5)	(3.5,1)
	$m_2/t_1\ m_2/t_2$	(1.5,4)	(2.5,3)

Matrix 1

	q	1-q
p_1	B_2	B_2
p_2	b_1	B_2
p_3	B_2	b_1
p_4	B_2	

Table 1

$m_1/t_1m_1/t_2$ and $m_2/t_1m_2/t_2$ are never best replies to any pure strategy of player 2. It follows that p_1 and p_4, the probabilities assigned to these strategies, are equal to 0.

Player 2's best response is r_1 each time player 1 plays $m_2/t_1m_1/t_2$ or $m_2/t_1m_2/t_2$. It is one of the two best responses when player 1 plays $m_1/t_1m_1/t_2$ (because in this case r_1 and r_2 are best responses). It follows that q has to be equal to $p_3+p_4+p_1/2$. Finally r_2 is player 2's best response each time player 1 plays $m_1/t_1m_2/t_2$; it is one of the two best responses when player 1 plays $m_1/t_1m_1/t_2$. It follows that 1-q has to be equal to $p_2+p_1/2$.

Table 1 summarizes this information. It namely tells when a strategy is a best reply: b_1 means that player 1's strategy is a best reply to player 2's strategy, B_2 means that player 2's strategy is a best reply to player 1's strategy.

We get the system of equations:

p_2 =q

p_3 =1-q

$p_1 = p_4 = 0$

q= $p_3 + p_4 + p_1/2$

The unique solution of this system is $p_2 = p_3 =$ q= 1-q= 0.5, $p_1 = p_4 = 0$.

Hence, in the BRM equilibrium, player 1 plays $m_1/t_1m_2/t_2$ half of time and $m_2/t_1m_1/t_2$ half of time and player 2 plays r_1 half of time and r_2 half of time.

This solution is very far from the Nash equilibrium solution. Let us explain the reason for this difference. The above BRM equilibrium is not a Nash equilibrium because in a Nash equilibrium, a player reacts to the ***mean behavior*** of the opponents. So, according to Nash's logic, if player 2 plays r_1 half of time and r_2 half of time, player 1 plays $m_2/t_1m_1/t_2$ with probability 1. By contrast, according to the BRM logic, player 1 takes into account that half of time, player 2 plays ***r_1 with probability 1***, in which case the best response is $m_1/t_1m_2/t_2$, and the other half of time, player 2 plays ***r_2 with probability 1***, in which case the best response is $m_2/t_1m_1/t_2$; it follows that half of time her optimal behavior is $m_1/t_1m_2/t_2$ and half of time it is $m_2/t_1m_1/t_2$.

Let us also insist on the fact that the probabilities in the BRM equilibrium have nothing to do with the probabilities of a mixed Nash equilibrium. In a BRM equilibrium a player i assigns a high probability to a pure strategy s_i if it is often a best reply (i.e. if the opponents often play the strategy profile to which s_i is a best reply). By contrast, in a mixed Nash equilibrium, the probability a player i assigns to s_i has nothing to do with the frequency with which s_i is a best reply: indeed, when she plays two strategies s_i and s_i' with positive probability, she is indifferent between both strategies and could assign any probability (summing to 1) to s_i and $s_{i'}$: in fact, the only role of the probability assigned to s_i is to justify the strategies of the opponents of player i.

Let us finally observe that, in this game, the BRM equilibrium ensures a mean payoff 2/2+5/4 >2 to t_1, a mean payoff of 2/2 +3/4 <2 to t_2 (hence a mean payoff 2 to player 1) and a payoff 11/4 >2 to player 2. It follows that, in this game, player 1 gets the same expected payoff in both the BRM equilibrium and the (PBE) Nash equilibria and player 2 gets a higher payoff in the BRM equilibrium than in the (PBE) Nash equilibria. This fact does not prove that in general BRM equilibria lead to higher payoffs, it just illustrates that both concepts are highly different and can therefore lead to completely different issues and payoffs.

4. Best-reply matching in a signaling game: the local approach

We propose in this section to apply the BRM logic in a more extensive form –decentralized- way. We know justify the play of each action at each information set. To this aim we study the game of figure 3 with local strategies. So we again call q the probability assigned by player 2 to r_1 but we call π_1, respectively π_2, the probability assigned by t_1 to m_2 and the probability assigned by t_2 to m_2.

One observes that t_1 is best off playing m_2 each time player 2 plays r_2, which leads her to play m_2 as often as player 2 plays r_2, hence π_1=1-q. t_2 is best off playing m_2 each time player 2 plays r_1, which leads her to play m_2 as often as player 2 plays r_1, hence π_2=q . Reciprocally, player 2 is best off playing r_1 if t_1 plays m_2 and t_2 plays m_1 or if t_1 plays m_2 and t_2 plays m_2. r_1 is one of the two best responses if t_1 plays m_1 and t_2 plays m_1. It follows that $q= \pi_1(1-\pi_2)+ \pi_1\pi_2+ (1-\pi_1)(1-\pi_2)/2$. Finally player 2 is best off playing r_2 each time t_1 plays m_1 and t_2 plays m_2. r_2 is one of the two best responses if t_1 plays m_1 and t_2 plays m_1. It follows that $1-q= (1-\pi_1)\pi_2+ (1-\pi_1)(1-\pi_2)/2$.

Hence we get the system of equations

$\pi_1=1-q$

$\pi_2=q$

$q= \pi_1(1-\pi_2)+ \pi_1\pi_2+ (1-\pi_1)(1-\pi_2)/2$

The unique solution of this system is: $\pi_1=0.44$, $\pi_2=0.56$, $q=0.56$.

Before commenting this result, let us give the definition of the local BRM equilibrium in signaling games, we applied in the above example.

Definition 2: Local BRM equilibrium in signaling games

Let G be a finite signaling game in extensive form. Player 1 can be of n types t_i, i from 1 to n, and chooses a message in a finite set $M(t_i)$. $M=\bigcup_{i=1}^{n} M(t_i)$. Player 2 observes each message m and responds with an action r out of R(m), the finite set of actions available at message m. $\pi_{t_i}(m)$ is the probability assigned by t_i to message m and $\pi_{2m_k}(r)$ is the probability assigned by player 2 to the response r after having observed m_k. A behavioral strategy profile is a local BRM equilibrium if:

-for every type t_i of player 1, and every message m available to type t_i,

$$\pi_{t_i}(m) = \sum_{r \in B_{t_i}^{-1}(m)} \left(\frac{1}{\text{Card } B_{t_i}(r)} \prod_{j=1}^{\text{Card}M} \pi_{2m_j}(r_{m_j}) \right)$$

where $r = (r_{m1}, r_{m_2}, ... r_{m_{\text{CardM}}})$ is a profile of actions played by player 2 (one response for each possible message), and $B_{t_i}(r)$ is the set of best responses of type t_i to the profile r.

- after each message m_k, for every action r available after m_k:

$$\pi_{2m_k}(r) = \sum_{m \in B_{2m_k}^{-1}(r)} \left(\frac{1}{\text{Card } B_{2m_k}(m)} \prod_{i=1}^{n} \pi_{t_i}(m_{t_i}) \right)$$

where $m = (m_{t_1}, m_{t_2}, ..., m_{t_n})$ is the profile of messages sent by the n types of player 1 and $B_{2m_k}(m)$ is the subset of player 2's best responses to the profile m after observing m_k.

Let us now comment the difference between the normal form and the local approach of BRM. In the game of figure 3, the normal form approach led to q=0.5 and $p_1=p_4=0$, $p_2=p_3=0.5$.The Kuhn equivalent behavioral strategies are given by $\pi_1=0.5$, $\pi_2=0.5$ and q=0.5. It follows that the local BRM equilibrium, $\pi_1=0.44$, $\pi_2=0.56$ and q=0.56, albeit nor far from the normal form BRM equilibrium, is different. It follows:

Proposition 2

The local BRM equilibria and the normal form BRM equilibria do not necessarily lead to the same issues.

The reason for this difference can be understood by looking at the four configurations given in figures 3a, 3b, 3c and 3d.

The configurations given in figures 3a and 3d, respectively in figures 3b and 3c, occur with probability 0 ($p_1=p_4=0$), respectively 0.5 ($p_2=p_3=0.5$) in the normal form approach, whereas they all occur with a probability close to 0.25 in the local approach. To understand this difference, look at the configuration given in figure 3d, in which both t_1 and t_2 play m_2. This configuration is impossible in the normal approach ($p_4=0$) because there exists no pure strategy of player 2 such that both t_1 and t_2 are best off playing m_2. By contrast, this configuration makes sense in the local decentralized approach. As a matter of fact t_1 rigthly plays m_2 with probability 0.44 because player 2 plays r_2 with the same probability. And t_2 rightly plays m_2 with probability 0.56 because player 2 plays r_1 with the same probability. So it automatically follows that the event "both t_1 and t_2 play m_2" is observed with probability 0.44x0.56, which is far from 0.

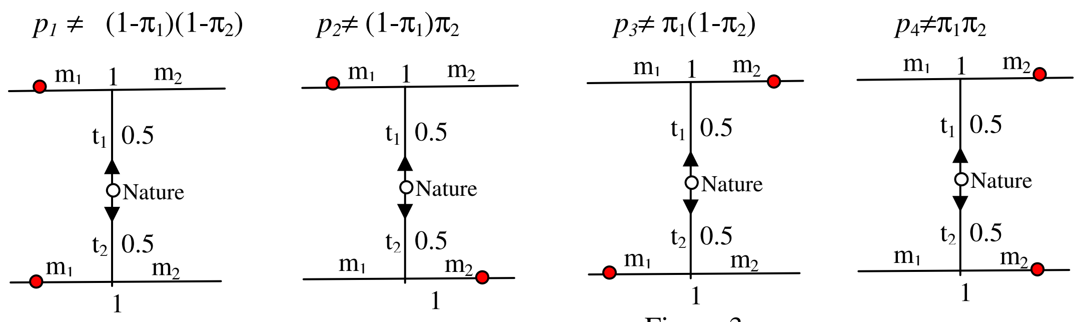

Figure 3a Figure 3b Figure 3c Figure 3d

In fact, the normal form links the actions taken at each decision node of player 1 and therefore looks for actions by player 2 that justify a profile of decisions of player 1. Hence, in the normal form -centralized- approach, the actions played at x_1 and x_2 have to be justified by the ***same*** player 2's action. By contrast, in the local, decentralized approach, the action played by t_1 is justified by a player 2's action r and the action played by t_2 is justified by a player 2's action r', ***and r' can be different from r.*** To our mind this latter fact is not problematic in a BRM context. The BRM logic nowhere requires that an action and the actions that justify it have to be played at the same moment. Hence t_1 and t_2 can both play m_2 (with probability 0.44 and 0.56) despite player 2 will not play r_2 and r_1 at the same moment, because player 2 will actually select both actions with probability 0.44 and 0.56. To our point of view, his fact advocates for the local BRM approach.

In the following sections we will apply both approaches of BRM to the experience good model.

5. Normal form best-reply matching in experience good models

Let us first consider the simplified model that satisfies assumption (b), with 2 types, t_1, t_2, and only two possible prices, p_1 and p_2, with $h_1 < p_1 < H_1 < h_2 < p_2 < H_2$. Let us also suppose that no player plays a weakly dominated strategy, so that t_2 only plays p_2 and the consumer always accepts p_1. The studied game is given in figure 4.[3]

The normal form approach leads to the best reply table 2, where p_i, i from 1 to 2, are the probabilities that the seller assigns to her pure strategies and q_i, i from 1 to 2, are the probabilities that the consumer assigns to his pure strategies.

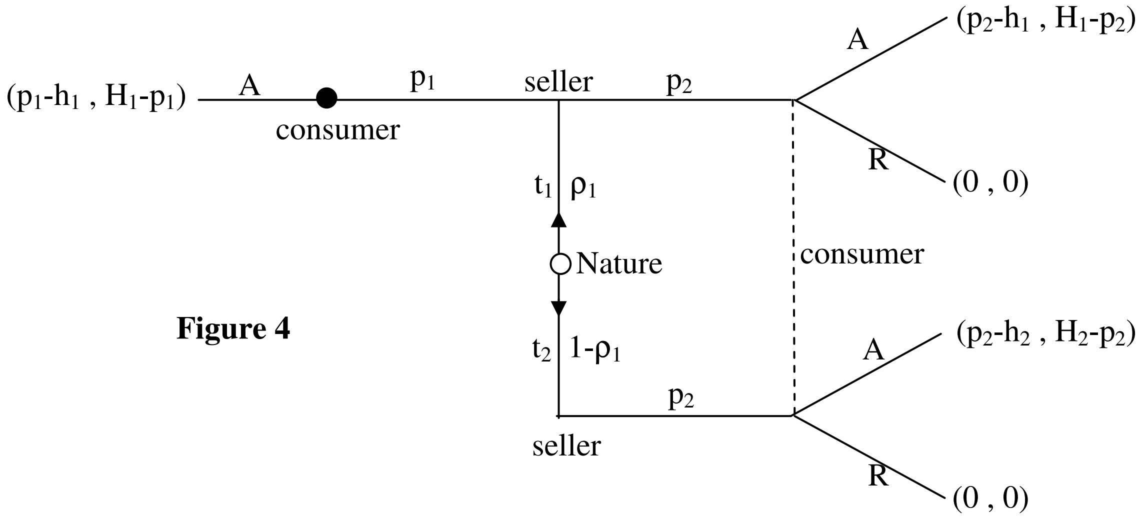

Figure 4

		q_1	q_2
		$A/p_1A/p_2$	$A/p_1R/p_2$
p_1	$p_1/t_1p_2/t_2$	B_2	$\mathbf{b_1}$
p_2	$p_2/t_1p_2/t_2$	$\mathbf{b_1}$	B_2

Table 2

The system of equations becomes:

$p_1 = q_2 \quad p_2 = q_1 \qquad q_1 = p_1 \quad q_2 = p_2$.

Therefore $p_1 = p_2 = q_1 = q_2 = ½$.

We recall that $\pi_i(p_j)$ is the probability that a seller of type t_i plays p_j and $q(p_j)$ is the probability that the consumer accepts p_j. The Kuhn behavioral equivalent strategies of the above strategy profile become:

$\pi_1(p_1) = p_1 = 0.5, \quad \pi_1(p_2) = p_2 = 0.5, \qquad \pi_2(p_2) = 1$

$q(p_1) = 1, \ \ q(p_2) = q_1 = 0.5.$

Hence t_1 plays both prices with probability ½ and the consumer accepts the high price with probability ½.

---

[3] Working with the weakly dominated strategies changes the numerical values of the probabilities but not the nature of the results (see Umbhauer 2007 for the approach with weakly dominated strategies).

Let us now consider the simplified model that satisfies assumption (b), with 3 types, t_1, t_2 and t_3 and only 3 possible prices, p_1, p_2 and p_3, with $h_1 < p_1 < H_1 < h_2 < p_2 < H_2 < h_3 < p_3 < H_3$ Let us again suppose that no player plays a weakly dominated strategy, so that no type of seller plays a price lower than her reservation price and so that the consumer always accepts p_1. It follows that the studied game is given in figure 3 (without the arrows and the probabilities on the arrows).The best-reply table becomes table 3.

		q_1	q_2	q_3	q_4
		$A/p_1A/p_2A/p_3$	$A/p_1A/p_2R/p_3$	$A/p_1R/p_2A/p_3$	$A/p_1R/p_2R/p_3$
p_1	$p_1/t_1p_2/t_2\ p_3/t_3$	B_2			$\mathbf{b_1}$
p_2	$p_1/t_1p_3/t_2\ p_3/t_3$		B_2		$\mathbf{b_1}$ B_2
p_3	$p_2/t_1p_2/t_2\ p_3/t_3$		$\mathbf{b_1}$	B_2	
p_4	$p_2/t_1p_3/t_2\ p_3/t_3$				B_2
p_5	$p_3/t_1p_2/t_2\ p_3/t_3$		B_2		
p_6	$p_3/t_1p_3/t_2\ p_3/t_3$	$\mathbf{b_1}$	B_2	$\mathbf{b_1}$	B_2

Table 3

The system of equations is given by:

$p_1 = p_2 = q_4/2$, $\quad p_3 = q_2$, $\ p_4 = p_5 = 0$, $\ p_6 = q_1 + q_3$

$q_1 = p_1$, $\ q_2 = p_2/2 + p_5 + p_6/2$, $\ q_3 = p_3$, $\ q_4 = p_2/2 + p_4 + p_6/2$

It follows that $p_1 = p_2 = 1/7$, $p_3 = 2/7$, $p_4 = p_5 = 0$, $p_6 = 3/7$, $q_1 = 1/7$, $q_2 = q_3 = q_4 = 2/7$.

The Kuhn equivalent behavioral strategies are:

$\pi_1(p_1) = 2/7$, $\pi_1(p_2) = 2/7$, $\pi_1(p_3) = 3/7$

$\pi_2(p_2) = 3/7$, $\ \pi_2(p_3) = 4/7$, $\ \pi_3(p_3) = 1$

$q(p_1) = 1$, $\ q(p_2) = 3/7$, $\ q(p_3) = 3/7$.

It immediately follows:

Proposition 3

The BRM equilibrium and the PBE (see proposition 2) are different. ***The main difference is that t_1 does not only play p_1 and p_2, but she also plays p_3 with a significant probability***. This difference is linked to another one. In a PBE, the probability of accepting a price strictly decreases in the price. This fact is no longer true with the BRM concept ($q(p_2)=q(p_3)$). More generally, in a model with n types of seller, each type t_i, i from 1 to n, in a normal form BRM equilibrium, plays each price p_j, j from i to n, with a positive probability (a fact which is impossible in any PBE with a positive payoff for the seller cf. proposition 2).

Proof: see Appendix 3

6. Local best-reply matching in experience good models

We now study the same models than in section 4, but with the local BRM concept. In the first model (given in figure 4), one immediately obtains:

$\pi_1(p_1)= 1-q(p_2)$ and $\pi_1(p_2)= q(p_2)$

given that t_1's best response is p_2 each time the consumer accepts p_2, and p_1 in the remaining case.

$\pi_2(p_2)= 1$

$q(p_1)= 1$

$q(p_2) = 1-\pi_1(p_2)$ given that player 2's best response when he observes p_2 is to accept p_2 if and only if t_1 plays p_1.

It immediately follows that:

$\pi_1(p_1)= \pi_1(p_2)= ½$, $\pi_2(p_2)= 1$, $q(p_1)=1$ and $q(p_2)=1/2$.

Hence, in the two type game, we get exactly the same result regardless of the employed BRM concept.

Unfortunately, this equality of results does not generalize.

Indeed, in the second model (given in figure 3), one obtains:

$\pi_1(p_3)=q(p_3)$

$\pi_1(p_2)= (1-q(p_3))q(p_2)$

given that t_1's best response is p_3 each time the consumer accepts p_3 and it is p_2 each time the consumer refuses p_3 but accepts p_2. With the remaining probability (not written here) t_1 plays p_1.

$\pi_2(p_3)= q(p_3) + (1-q(p_3))(1-q(p_2))/2$

given that t_2's best reply is to play p_3 each time p_3 is accepted and also each time both p_3 and p_2 are refused. In the latter case, both p_2 and p_3 are best replies, which explains the division by 2. t_2 plays p_2 with the remaining probability (not written here).

$\pi_3(p_3)= 1$

$q(p_1)= 1$

$q(p_2)= (1- \pi_1(p_2)) \pi_2(p_2)+(1-\pi_1(p_2))(1-\pi_2(p_2))/2$

because accepting p_2 is optimal if only t_2 plays p_2 or if neither t_1 nor t_2 play p_2. In the latter case, player 2 can also refuses p_2, which explains the division by 2. The consumer refuses p_2 with the remaining probability.

$q(p_3)= (1-\pi_1(p_3))(1-\pi_2(p_3))$

because accepting p_3 is optimal only if t_1 and t_2 do not play p_3. The consumer rejects p_3 with the remaining probability.

Solving the system of equations leads to:

$\pi_1(p_1)= \pi_1(p_2)=\pi_1(p_3)=1/3$, $\pi_2(p_2)= \pi_2(p_3)=1/2$, $\pi_3(p_3)=1$, $q(p_1)=1$, $q(p_2)=1/2$ and $q(p_3)=1/3$.

Let us comment this result.

First, even if the seller's behavior is not far from the one in the normal form game (2/7, 2/7, 3/7 become 1/3,1/3,1/3 and 3/7 becomes ½), the results obtained in the extensive form are different from the ones obtained in the normal form**.** This difference clearly derives from the decentralization which is possible in the extensive form and impossible in the normal form.

Second, the obtained result is worth of interest in that the obtained behaviors are quite simple: t_1 can play 3 prices and plays each of them with probability 1/3, t_2 can play 2 prices and plays each of them with probability ½, t_3 can only play one price and of course plays it with probability 1; the buyer accepts p_1 –which can only be played by t_1- with probability 1, he accepts p_2- which can be played by 2 types- with probability ½, and he accepts p_3 –which can be played by 3 types- with probability 1/3. ***So we precisely obtain the simple behavior we talked about in section 2. It follows that this easy behavior for which we found a limited rationality explanation, respects a strong consistency, the best-reply matching one. What is more, we prove in the next section that this behavior can be generalized.***

7. Generalization: a simple behavior rule

In this section we prove that the above behavior generalizes as soon as one smoothly changes the behavior of the consumer when he is indifferent between buying and not buying. We indeed agree with Droste & al.(2003) who tell that, *if there are several best responses to* *a strategy profile, there is no real motivation to assign to each best response the same probability (by dividing by the cardinal of the subset of best responses).*

So let us turn to the general case with n types, after elimination of the weakly dominated strategies. We focus on a game with n types, n prices p_1, p_2, ..p_n, with $h_i<p_i<H_i$, i from 1 to n, which satisfies assumption (b). It follows that, for each pure strategy profile of the seller, the consumer is better off accepting p_i if only t_i plays p_i and he is indifferent between accepting and refusing p_i only if nobody (i.e. no type lower or equal to t_i) plays p_i. In this latter case, ***we now suppose that, instead of accepting and refusing p_i with the probability of the event "no type lower or equal to t_i plays p_i" divided by 2, the consumer accepts p_i only with the probability of this event divided by i.*** Given that i is the cardinal of the set of types who can play p_i, we introduce in some way a kind of risk aversion that grows with higher prices. This is not a silly assumption but we admit that we only introduce it in order to get the generalization of the result obtained in the three type case.

The system of equations in the general case becomes:

$\pi_1(p_n)=q(p_n)$

$\pi_1(p_i)= q(p_i)\prod_{j=i+1}^{n}(1-q(p_j))$ for i from 2 to n-1

$\pi_1(p_1)= 1- \sum_{i=2}^{n}\pi_1(p_i)$

$\pi_i(p_n)= q(p_n) +[\prod_{j=i}^{n}(1-q(p_j))]/ (n-i+1)$ for i from 2 to n-1

$\pi_i(p_k)= q(p_k) \prod_{j=k+1}^{n}(1-q(p_j)) +[\prod_{j=i}^{n}(1-q(p_j))]/ (n-i+1)$ for i from 2 to n-1 and k from i+1 to n-1

$\pi_i(p_i)= 1- \sum_{j=i+1}^{n}\pi_i(p_j)$

$\pi_n(p_n)= 1$

$q(p_1)= 1$

$q(p_i)= \pi_i(p_i) \prod_{j=1}^{i-1}(1-\pi_j(p_i))+ [\prod_{j=1}^{i}(1-\pi_j(p_i))] / i$ for i from 2 to n

It is easy to check that the solution for this system of equations is given by:

Proposition 4

In the n type case, the BRM behavior is given by:

$\pi_i(p_j)= 1/(n-i+1)$ for i from 1 to n and j from i to n.

$q(p_j)= 1/j$ for j from 1 to n.

In other words, each type plays each available price with the same probability and the consumer accepts each price with the probability 1 divided by the number of types who can play this price.

Given that this behavior can also be explained with limited rationality (cf. section 2), we claim that it is difficult to find a more easy behavior that satisfies the same amount of consistency.

It follows that we conclude that it would be worth testing this behavior experimentally, in order to see if it is sometimes adopted.

8. Best-reply matching and social surplus

The preceding behavior rule is not only simple and consistent but it can lead to positive payoffs for both the consumer and the seller, at least if the number of types is low.

Proposition 5

In the simplified experience good model with two types and two prices p_1 and p_2 examined in sections 5 and 6, ***best reply matching can lead to positive payoffs for both the consumer and the seller. Moreover the social surplus can be higher than the highest PBE social surplus in the experience good model with two types examined in section 2.***[4]

To prove this proposition, we first observe that in the experience good model with two types studied in section 2, the highest social surplus limits to the highest seller's payoff (given that the consumer's surplus is null cf. proposition 2). By usual maximization, one establishes that the highest social (seller) surplus is equal to $\rho_1(H_1-h_1)+\rho_2(H_2-h_2)(H_1-h_1)/(H_2-h_1)$.

[4] See Umbhauer (2007) for stronger results on social surplus.

We now turn to the BRM equilibrium in the simplified experience good model with 2 prices p_1 and p_2 examined in sections 5 and 6. We know that in this case the normal form BRM concept and the local BRM concept lead to the same result, i.e. t_1 plays each price with probability ½ and the buyer accepts p_2 with probability ½. It follows that the surplus of the seller is equal to $\rho_1[(p_1-h_1)1/2 + (p_2-h_1)1/2.1/2]+ \rho_2 (p_2-h_2)1/2$. The consumer's surplus is equal to $\rho_1[(H_1-p_1)1/2 + (H_1-p_2)1/2.1/2]+ \rho_2 (H_2-p_2)1/2$. It follows that the total surplus is equal to $\rho_1(H_1-h_1)3/4 + \rho_2 (H_2-h_2) 1/2$.

Let us set: $H_1=50$, $h_1=49$, $H_2=70$, $h_2=61$, $\rho_1=\rho_2=0.5$. The values of the parameters check the assumptions given in section 2; it follows that the highest PBE social surplus is 5/7. By contrast, the BRM social surplus, for example for p_1 very close to 50[5] and $p_2=62$, is equal to 10.5/4, which is much higher than 5/7. The maximal consumer surplus for a p_1 close to 50 is obtained for p_2 very close to 61 and is equal to 3.5/4. The highest BRM seller payoff is obtained for p_1 very close to H_1 and p_2 very close to H_2 and is equal to 20.5/4 (the surplus of the consumer being negative in this case).

Moreover it is easy to find values for p_1 and p_2 that lead to positive payoffs for both players, both payoffs being higher that the highest PBE payoffs. For example, for p_1 very close to 50 and $p_2=62$, the consumer surplus is equal to 2/4 (>0) and the seller surplus is equal to 8.5/4 (>5/7).

It follows that the BRM approach can be socially efficient .This fact is not astonishing given that Nash equilibria (and PBE) are not necessary Pareto efficient and given that the Nash equilibrium consistency and the BRM equilibrium consistency are different.

9. Best-Reply Matching and behavior in case of indifference

An interesting development of BRM concerns the treatment of indifference. In both definitions of BRM (normal form and local), a player is supposed to share equally a probability between all the strategies that are best responses. For example, suppose that A_1 and B_1 are best replies for player 1 only if player 2 plays C_2. Suppose also that player 2 plays

[5] We choose p_1 very close (approximately equal) to H_1 in order to show that the result is not linked to the fact that p_1 can be chosen lower than H_1 in the BRM approach whereas it has to be higher or equal to H_1 in any PBE.

C_2 with probability q. In that case, the BRM concept assigns probability q/2 to A_1 and to B_1. Yet there is no reason to divide equally q between A_1 and B_1 (this fact led us to choose another division to get the result in proposition 4).

More precisely, one should examine the whole set of equilibrium possibilities, in which the probability assigned to A_1 is p, with $0 \le p \le q$, the probability assigned to B_1 being 1-p. So we get the following new versions of BRM equilibria.

Definition 3: New normal form BRM equilibrium

Let G=(N, $(S_i)_{i\in N}$, $(\succ_i)_{i\in N}$) be a game. A mixed strategy p is a new normal form BRM equilibrium if for every player i $\in$ N and for every pure strategy $s_i \in S_i$, :

$$p_i(s_i) = \sum_{s_{-i} \in B_i^{-1}(s_i)} \delta_{s_i}\, p_{-i}(s_{-i})$$

with $\delta_{s_i} \in [0, 1]$ for any s_i belonging to $B_i(s_{-i})$ and $\sum_{s_i \in B_i(s_{-i})} \delta_{s_i} = 1$

Definition 4: New local BRM equilibrium in signaling games

Let G be a finite signaling game in extensive form. Player 1 can be of n types t_i, i from 1 to n, and chooses a message in a finite set $M(t_i)$. $M(t_i)$. $M = \bigcup_{i=1}^{n} M(t_i)$. Player 2 observes each message m and responds with an action r out of R(m), the finite set of actions available at message m. $\pi_{t_i}(m)$ is the probability assigned by t_i to message m and $\pi_{2m}(r))$ is the probability assigned by player 2 to the response r after having observed m. A behavioral strategy profile is a new local BRM equilibrium if:

-for every type t_i of player 1, and every message m available to type t_i,

$$\pi_{t_i}(m) = \sum_{r \in B_{t_i}^{-1}(m)} (\delta_m \prod_{j=1}^{\mathrm{Card}M} \pi_{2m_j}(r_{m_j}))$$

where $r = (r_{m1}, r_{m_2}, ... r_{m_{\mathrm{Card}M}})$ is a profile of actions played by player 2 (one response for each possible message), $B_{t_i}(r)$ is the set of best responses of type t_i to the profile r, $\delta_m \in$ [0, 1] for any m belonging to $B_{t_i}(r)$ and $\sum_{m \in B_{t_i}(r)} \delta_m = 1$.

- after each message m_k, for every action r available after m_k:

$$\pi_{2m_k}(r) = \sum_{m \in B_{2m_k}^{-1}(r)} (\delta_r \prod_{i=1}^{n} \pi_{t_i}(m_{t_i}))$$

where $m=(m_{t_1}, m_{t_2}, ..., m_{t_n})$ is the profile of messages sent by the n types of player 1, $B_{2m_k}(m)$ is the subset of player 2's best responses to the profile m after observing m_k, $\delta_r \in [0, 1]$ for any r belonging to $B_{2m_k}(m)$, and $\sum_{r \in B_{2m_k}(m)} \delta_r = 1$.

Proposition 6

Each pure strategy Nash equilibrium is a new normal form BRM equilibrium. Each pure strategy Nash equilibrium in a signaling game is a new local BRM equilibrium.

Proof:

Consider a pure strategy Nash equilibrium s*. For each player i, s_i* is a (possibly among others) best reply to s_{-i}*. Given that s_{-i}* is played with probability 1, it is now possible, in a new normal form BRM equilibrium, to put probability 1 on s_i* (even if s_i* is not the unique best reply). It automatically follows that s* is a new BRM equilibrium, hence that the set of pure strategy Nash equilibria is included in the set of new normal form BRM equilibria.

Consider a pure strategy Nash equilibrium s* in a signaling game. s* bijectively corresponds to a behavioral Nash equilibrium $\pi^*= (\pi_1(.)^*,\pi_2(.)^*)$. For each type t_i, $\pi_{t_i}(.)^*$ assigns probability 1 to a message m_i (because s* is a pure strategy Nash equilibrium), m_i being a best reply (possibly among others) to $\pi_2(.)^*$. Given that player 2, after each message m, assigns probability 1 to only one response (because s* is a pure strategy Nash equilibrium), the new local BRM concept allows t_i to put probability 1 on m_i because player 2 assigns probability 1 to all the played responses. Reciprocally, for each possible message m, $\pi_{2m}(.)^*$ assigns probability 1 to one response r_m after the message m. Given that r_m is a best reply (possibly among others) after m to $\pi_1(.)^*=(\pi_{t_i}(.)^*,...,\pi_{t_n}(.)^*)$, given that $\pi_{t_i}(.)^*$ assigns probability 1 to the unique message played by t_i, it follows that the new local BRM concept allows to put probability 1 on r_m. It derives that π^*, and therefore s*, is a new local BRM equibrium. So the set of pure Nash equilibria is included in the set of new local BRM equilibria. ☺

But of course, this extension does in no way bring nearer together mixed Nash equilibria and BRM equilibria given that the consistency of both criteria differs.

10. Conclusion: existence and further developments, a comeback to Akerlof's result

Let us first check the consequences of the above extension on the experience good model. Unfortunately this extension is not sufficient for the set of normal form BRM equilibria to become Kuhn equivalent to the set of local BRM equilibria.

Let us come back to the simplified experience good model given in figure 3.

According to table 3, the new normal form BRM concept leads to the set of equations I:

<u>Set of equations I</u>

$p_1 = \boldsymbol{\alpha q_4}$, $p_2 = \mathbf{(1-\alpha)q_4}$, $p_3 = q_2$, $p_4 = p_5 = 0$, $p_6 = q_1 + q_3$

$q_1 = p_1$, $q_2 = \boldsymbol{p_2\beta} + p_5 + \boldsymbol{p_6\gamma}$, $q_3 = p_3$, $q_4 = \boldsymbol{p_2}\mathbf{(1-\beta)} + p_4 + \boldsymbol{p_6}\mathbf{(1-\gamma)}$

where $\alpha, \beta, \gamma \in [0, 1]$.

The new local BRM concept leads to the set of equations II:

<u>Set of equations II</u>

$\pi_1(p_3) = q(p_3)$ $\pi_1(p_2) = (1-q(p_3))q(p_2)$ $\pi_2(p_3) = q(p_3) + \boldsymbol{\delta}\mathbf{(1-q(p_3))(1-q(p_2))}$

$\pi_3(p_3) = 1$ $q(p_1) = 1$ $q(p_2) = (1-\pi_1(p_2))\, \pi_2(p_2) + \boldsymbol{\mu}\mathbf{(1-\pi_1(p_2))(1-\pi_2(p_2))}$

$q(p_3) = (1-\pi_1(p_3))(1-\pi_2(p_3))$ with $\delta, \mu \in [0,1]$

The Kuhn equivalent behavioral strategies to the mixed strategies (p, q) are given in the set of equations III:

<u>Set of equations III</u>

$\pi_1(p_1) = p_1 + p_2$, $\pi_1(p_2) = p_3 + p_4$, $\pi_1(p_3) = p_5 + p_6$

$\pi_2(p_2) = p_1 + p_3 + p_5$, $\pi_2(p_3) = p_2 + p_4 + p_6$, $\pi_3(p_3) = 1$

$q(p_1) = 1$, $q(p_2) = q_1 + q_2$ $q(p_3) = q_1 + q_3$.

The intersection between the set of new normal form BRM equilibria and the set of new local BRM equilibria, are the values p_i, i from 1 to 6, q_i, i from 1 to 4, α,β,γ, μ and δ, that satisfy simultaneously the three sets of equations I, II and III.

Yet it is easy to establish that the only solution satisfying all the equations is given by :

$\alpha=\beta=\gamma=\mu=0$, $\delta=1$

$p_2=q_4=1$, $p_1=p_3=p_4=p_5=p_6=q_1=q_2=q_3=0$

hence

$\pi_1(p_1) = 1 \quad \pi_1(p_2) = \pi_1(p_3) = 0$

$\pi_2(p_2) = 0, \quad \pi_2(p_3) = 1$

$\pi_3(p_3) = 1$

$q(p_1) = 1 \quad q(p_2) = q(p_3) = 0$

In other terms, the intersection of the sets of new normal form and new local BRM equilibria only contains one equilibrium, which, surprisingly, is compatible with Akerlof's result i.e.: only the lowest quality is sold on the market.

More generally it is easy to establish the following result:

Proposition 7

In the general n-type case (described in section 7), Akerlof's result is a new normal form and a new local BRM equilibrium. More precisely, the strategy profile such that the seller sets the price p_1 if she is of type t_1 and the price p_n if she is of type t_i, i from 2 to n, and the consumer only accepts the price p_1, is a new normal form and a new local BRM equilibrium.

Proof:

It is immediate that the strategy profile such that the seller sets the price p_1 if she is of type t_1 and the price p_n if she is of type t_i, i from 2 to n, and the consumer only accepts the price p_1, is a pure strategy Nash equilibrium. It follows from proposition 6 that it is a new normal form and a new BRM equilibrium. ☺

It is interesting to observe that the values of the parameters δ_m and δ_r, in case of indifference, that ensure that the tested strategy profile is a new local BRM equilibrium, are the ones given in bold letters below.

$\pi_1(p_n)=q(p_n)= 0$ $\qquad$ $\pi_1(p_i)= q(p_i)\prod_{j=i+1}^{n}(1-q(p_j))=0$ for i from 2 to n-1

$\pi_1(p_1)= 1- \sum_{i=2}^{n}\pi_1(p_i) =1$ $\qquad$ $\pi_i(p_n)= q(p_n) +[\prod_{j=i}^{n}(1-q(p_j))]\ \mathbf{1} = 1$ for i from 2 to n-1

$\pi_i(p_k)= q(p_k) \prod_{j=k+1}^{n}(1-q(p_j)) +[\prod_{j=i}^{n}(1-q(p_j))]\ \mathbf{0} = 0$ for i from 2 to n-1 and k from i+1 to n-1

$\pi_i(p_i)= 1- \sum_{j=i+1}^{n}\pi_i(p_j) =0$ $\qquad$ $\pi_n(p_n)= 1$ $\qquad$ $q(p_1)= 1$

$q(p_i)= \pi_i(p_i) \prod_{j=1}^{i-1}(1-\pi_j(p_i))+ [\prod_{j=1}^{i}(1-\pi_j(p_i))]\ \mathbf{0}$ $\qquad$ for i from 2 to n

Of course, this result should not make forget that in the two type case, the intersection of new normal form and local BRM equilibria does not shrink to Akerlof's result, because in this model both sets of equilibria are equal, and do only contain the profile which leads t_2 to play each of the two prices half of time and the consumer to accept the high price half of time.

Let us now turn to the question of existence of BRM equilibria. Droste & al.(2003) proved the existence of normal form BRM. It follows:

Proposition 8: existence

Each signaling game has at least one local BRM equilibrium.

Each game in normal form has at least one new normal form BRM equilibrium, and each signaling game has at least one new local BRM equilibrium

Proof:

Looking for local BRM equilibria in the extensive form of the signaling game is equivalent to looking for normal form BRM equilibria in ***the agent normal form of the game*** (because in the agent normal form each agent plays only one time).

Therefore, given that the agent normal form is a normal form game, it has a normal form BRM equilibrium; this normal form BRM equilibrium bijectively corresponds to a local BRM equilibrium (each strategy of an agent of player 1 becomes the local strategy of a type of player 1, and each strategy of an agent of player 2 becomes the local strategy of player 2 after observing a message). It derives that each signaling game has at least one local BRM equilibrium.

The existence of at least one new normal form BRM equilibrium and of at least one new local BRM equilibrium immediately derives from the fact that the sets of new normal form and new local BRM equilibria respectively include the sets of normal form and local BRM equilibria. ☺

Let us finally conclude on the fact that it would be worth developing a local approach in more general extensive form games, especially games in which a same player has to play at several information sets. These games include traditional games like the centipede game and the prisoner's dilemma. More generally it would be worth looking for the BRM equilibria in these traditional games because the new consistency behind BRM concepts allows to get new solutions that can better fit with real observed behavior.

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Appendix 1

(2) implies that t_i, with i from 1 to n-1, is indifferent between p_i^* and p_{i+1}^*, i.e. :

$(p_{i+1}^*-h_i).q(p_{i+1}^*)= (p_i^*-h_i)q(p_i^*)$

Given the definition of p_i^*, it follows that $q(p_i^*)$ decreases in i .

Let us prove that, for i from 2 to n-1, t_i prefers p_i^* and p_{i+1}^* to any p_j^*, with j higher than i+1:

$(p_{j+1}^*-h_j).q(p_{j+1}^*)= (p_j^*-h_j)q(p_j^*)$ for j from i+1 to n-1

Hence $(p_{j+1}^*-h_i).q(p_{j+1}^*) =(p_{j+1}^*-h_j+h_j-h_i).q(p_{j+1}^*)= (p_j^*-h_j)q(p_j^*) +(h_j-h_i)q(p_{j+1}^*)$

$< (p_j^*-h_j)q(p_j^*) +(h_j-h_i)q(p_j^*)$ (given that $h_j>h_i$ and that $q(p_i^*)$ decreases in i) .

Hence $(p_{j+1}^*-h_i).q(p_{j+1}^*)<(p_j^*-h_i)q(p_j^*)$ for any j from i+1 to n-1 and therefore:

$(p_j^*-h_i).q(p_j^*)<(p_{i+1}^*-h_i)q(p_{i+1}^*) = (p_i^*-h_i)q(p_i^*)$ for any j from i+2 to n.

Let us now establish that t_i, for i from 2 to n-1, prefers p_i^* and p_{i+1}^* to any p_j^*, with j lower than i.

We have, for any j, with $1<j \leq i$:

$$\begin{aligned}
(p_{j-1}^*-h_i).q(p_{j-1}^*) &= (p_{j-1}^*-h_{j-1}).q(p_{j-1}^*)+(h_{j-1}-h_i)q(p_{j-1}^*) \\
&= (p_j^*-h_{j-1}).q(p_j^*)+(h_{j-1}-h_i)q(p_{j-1}^*) \\
&=(p_j^*-h_i).q(p_j^*)+ (h_i-h_{j-1})q(p_j^*)+(h_{j-1}-h_i)q(p_{j-1}^*) \\
&=(p_j^*-h_i).q(p_j^*)+ (h_i-h_{j-1})(q(p_j^*)-q(p_{j-1}^*)) \\
&< (p_j^*-h_i).q(p_j^*) \text{ because } (h_j-h_{j-1})(q(p_j^*) -q(p_{j-1}^*))<0.
\end{aligned}$$

It follows that $(p_j^*-h_i).q(p_j^*) < (p_i^*-h_i)q(p_i^*)$ for j, with $1\leq j<i$.

It follows that t_i's behavior is optimal, for i from 1 to n.

Let us now turn to the consumer. Given his out of equilibrium path beliefs, his reaction to out of equilibrium prices is optimal. We consider now his behavior after equilibrium prices:

It is optimal to accept H_1.

Only t_{i-1} and t_i play p_i^* for any i from 2 to n.

Accepting p_i^* leads to the expected payoff:

$\rho_{i-1}\, \pi_{i-1}(p_i^*)(H_{i-1}-p_i^*)+\rho_i\, \pi_i(p_i^*)(H_i-p_i^*)$

Given (1) this payoff is equal to 0, which justifies the buyer's mixed strategy.

Appendix 2

Let us focus on a PBE path in which each type of seller gets a positive payoff.

Let us first prove that if t_i plays 3 prices p, p' and p", then $p'=H_i$.

We necessarily have $(p-h_i)q=(p'-h_i)q'=(p"-h_i)q"$ where q, q' and q" are the probabilities of buying at prices p, p' and p". Necessarily $q >q'>q">0$ (given the positive payoff of each type of seller). It follows that, for each type t_j with $j<i$, $(p-h_j)q>(p'-h_j)q'>(p"-h_j)q"$ and that for each type t_j with $j>i$, $(p-h_j)q<(p'-h_j)q'<(p"-h_j)q"$. Therefore p' and p" can not be played by any type lower than t_i and p and p' can not be played by any type higher than t_i. It derives that p' is only played by t_i. Given that q' is different from 0 and 1, the consumer is indifferent between buying and not buying; this is only possible if $p'=H_i$.

It follows in the same way that, if t_i plays 4 prices p, p', p" and p"', with $p<p'<p"<p"'$, then $p'=p"=H_i$. Hence each type of seller sets at most 3 prices. Moreover, if she sets three prices, the middle price is H_i.

We now show that if a price p is only played by t_i, then it is necessarily equal to H_i. As a matter of fact, if $p>H_i$, p is refused and t_i's payoff is null (a contradiction to the positivity of the payoff of each type of seller). If $p<H_i$ then p is accepted with probability 1. It follows that p is necessarily the lowest price played in the game. Moreover, given that t_i (weakly) prefers p to any higher equilibrium price, any type lower than t_i also prefers p to the higher prices. Hence, either t_i is different from t_1 and p is played by several types (a contradiction to our assumption), either $t_i=t_1$; but the lowest price played by t_1, in each PBE, is at least H_1 (a contradiction to our assumption), given that any price lower or equal to H_1 is accepted by the consumer. It follows that if a price p is only played by t_i, then it is necessarily equal to H_i.

It derives from the above observation that if t_i plays a price p different from H_i, then p is necessarily played by another type. Let us be more precise by showing that an adjacent type, t_{i-1} or t_{i+1}, plays p.
If p is played by t_j with $j<i-1$, than t_{i-1} prefers p to any lower price. And, given that t_i plays p, t_{i-1} prefers p to any higher price. It follows that t_{i-1} only plays p.
Symmetrically, if p is played by a type t_j with $j>i+1$, than t_{i+1} prefers p to any higher price. And, given that t_i plays p, t_{i+1} prefers p to any lower price. It follows that t_{i+1} only plays p.

It immediately follows that at most (2n-1) different prices are played in the game. As a matter of fact, given that a type t_i can at most play 3 different prices, and given that, in this case, the middle price is necessarily H_i, t_1 can only play 2 different prices H_1 and $p_1>H_1$. Hence p_1 is necessarily played by t_2. It follows that t_2 can at most play the three prices, p_1, H_2 and $p_2>H_2$. It follows that t_3 plays p_2 and that t_3 can at most play the three prices p_2, H_3 and $p_3>H_3$. And so on, till to t_{n-1} who can at most play three prices, p_{n-2},H_{n-1} and p_{n-1}. Hence t_n plays p_{n-1} and she can at most play 2 different prices, p_{n-1} and H_n. The number (2n-1) follows.

Let us finally prove that in a PBE path in which each type of seller gets a positive payoff, the buyer's payoff can only be equal to 0.
It follows from the positivity of the payoff of each type of seller that the consumer accepts each equilibrium price with a positive probability. Let us suppose that the buyer accepts an equilibrium price p* with probability 1. In that case, p* is necessarily the lowest price played in the equilibrium. Call t_i the highest type playing p*. Necessarily, $p^*\geq h_i$ and t_i plays p* with at most probability 1. Yet assumption (b) ensures that $\frac{\sum_{j=1}^{i}\rho_j H_j}{\sum_{j=1}^{i}\rho_j} < h_i \leq p^*$ for i from 2 to n. It follows that the consumer refuses p* (a contradiction), unless i is equal to 1. Yet, in that case, p* is necessarily equal to H_1 and the buyer's payoff is null . Hence each price different from H_1 is accepted with a probability lower than 1. It follows that the buyer is indifferent between buying and not buying at every equilibrium price different from H_1. This means that his payoff is equal to 0 for any equilibrium price.

In fact the buyer's payoff is null in any PBE of the studied experience good model. Consider any price p* of the PBE path. Either p is refused with probability 1, in which case the buyer's payoff is null. Either it is accepted with a positive probability, in which case the preceding observations ensure that the buyer's payoff is also equal to 0.

Appendix 3

It is immediate that in a PBE the probability of accepting a price strictly decreases in the price, because, if else, the types who play a low price would be better off switching to a high price.

Let us now turn to the generalized simplified experience good model with n prices p_i, i from 1 to n, with $h_i<p_i<H_i$, i from 1 to n, which satisfies assumption (b), and in which the

consumer always accepts p_1 and no type of seller plays a price lower than her reservation price. We prove that in this context, each type t_i, i from 1 to n, in the normal form BRM equilibrium, plays each price p_j, j from i to n, with a positive probability.

To do so, one first proves that in a normal form BRM equilibrium, the consumer's strategy **(A/p_1,R/p_j), j from 2 to n,** is played with positive probability.
Let us suppose the contrary, i.e. that (A/p_1,R/p_j), j from 2 to n, is played with probability 0. In that case, the consumer necessarily accepts a price p_j, j>1, with positive probability. Suppose that p_k is the highest price accepted with positive probability.
If k<n-1, this means that the consumer plays the strategy (A/p_1, $./p_i$, A/p_k, R/p_j) with positive probability, with i from 1 to k-1, j from k+1 to n, and the point meaning either A or R. A seller's best reply to this strategy is (p_k/t_i,p_n/t_j), i from 1 to k and j from k+1 to n; this strategy is therefore also played with positive probability. The consumer's strategy **(A/p_1,R/p_j), j from 2 to n,** is a best reply to (p_k/t_i,p_n/t_j), i from 1 to k and j from k+1 to n. It follows that **(A/p_1,R/p_j), j from 2 to n,** is also played with positive probability, a contradiction to our assumption.
If k=n-1, the consumer plays the strategy (A/p_1, $./p_i$, A/p_{n-1}, R/p_n) with positive probability, with i from 1 to n-2, the point meaning either A or R. A seller's best reply to this strategy is (p_{n-1}/t_i,p_n/t_n), i from 1 to n-1; hence this strategy is played with positive probability. The consumer's strategy (A/p_1,R/p_j,A/p_n), j from 2 to n-1, is a best reply to **(**p_{n-1}/t_i,p_n/t_n), i from 1 to n-1; hence it is played with positive probability. The seller's strategy (p_n/t_i), i from 1 to n, is a best reply to (A/p_1,R/p_j,A/p_n), j from 2 to n-1; hence it is played with positive probability. Finally **(A/p_1,R/p_j), j from 2 to n,** is a best reply to (p_n/t_i), i from 1 to n. It follows that **(A/p_1,R/p_j), j from 2 to n,** is played with positive probability, a contradiction to our assumption.
If k=n, the consumer plays the strategy (A/p_1, $./p_i$, A/p_n) with positive probability, with i from 1 to n-1, the point meaning either A or R. A seller's best reply to this strategy is (p_n/t_i), i from 1 to n; this strategy is therefore played with positive probability. The consumer's strategy **(A/p_1,R/p_j) j from 2 to n,** is a best reply to (p_n/t_i), i from 1 to n. It follows that **(A/p_1,R/p_j), j from 2 to n,** is played with positive probability, a contradiction to our assumption.

Let us now observe that:
The seller's strategy **(p_i/t_i), i from 1 to n** is a best reply to the buyer's strategy **(A/p_1,R/p_j), j from 2 to n.**
In turn (A/p_1,R/p_j), j from 2 to n, is a best reply to the seller's strategy **(p_n/t_i), i from 1 to n**.
In turn, (p_n/t_i), i from 1 to n, is the best reply to the buyer's strategy **(A/p_i), i from 1 to n.**
Finally, in turn, (A/p_i), i from 1 to n, is the best reply to the seller's strategy **(p_i/t_i) i from 1 to n.**
It follows from this circularity in best replies, and from the fact that (A/p_1,R/p_j) j from 2 to n, is played with positive probability, that **each of the above mentioned strategy is played with positive probability**. Hence, given that (p_i/t_i), i from 1 to n and (p_n/t_i), i from 1 to n, are played with positive probability, **each type t_i, i from 1 to n, plays both p_i and p_n with positive probability.**

One also observes that:
Take any j lower than n. The seller's strategy **(p_j/t_i, p_k/t_k),** is the best reply to the consumer's strategy **(A/p_i, R/p_k), i from 1 to j and k from j+1 to n.**

In turn $(A/p_i, R/p_k)$, i from 1 to j and k from j+1 to n, is a best reply to the seller's strategy **(p_n/t_s), s from 1 to n**.
Hence, given that (p_n/t_s), s from 1 to n, is played with positive probability (cf. above), $(p_j/t_i, p_k/t_k)$, i from 1 to j and k from j+1 to n, is played with positive probability.
Given that j goes from 1 to n-1, it follows that **each type t_i plays any price p_j, with j from i to n-1, with positive probability.**