Schooling regimes, and the inequality-mobility mismatch

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Abstract

Contrary to a standard theoretical approach, evidence suggests that low income inequality, and high public school funding are not systematically associated with high level of social mobility. The seminal paper of Checchi and al. (1999) offers an explanation to the puzzling situation some countries experiment in this regard. They emphasize the role of incentives in the human capital accumulation process in private versus public school funding. This paper proposes an extension of this model to account for mixed school regime. Two main findings result from numerical computations of the model. (1) More decentralized financing of school spending results in higher incentives for households to invest privately in their children's education. Consequently, the more decentralized schooling system, the higher social mobility and income inequality. (2) For a given school regime, higher level of intergenerational mobility translates into a lower level of income inequality because it strengthens households' demand for public spending in education. These two forces may explain why empirical evidence fails to detect the existence of a simple relation between public education, social mobility, and income inequality.

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1. Introduction

The foreinost, or indeed the sole condition, which is required in order to succeed in centralizing the supreme power in a democratic community, is to love equality, or to make men to believe you love it. (...) They had sought to be free in order to make themselves equal; but in proportion as equality was more established by the aid of freedom, itself was thereby rendered of more difficult attainment. Alexis de Tocqueville, *Democracy in America*, translated by H. Reeves (1851)[1]

During the last twenty years, the topic of social mobility has received a growing attention in economic literature. The high concern of economists for social mobility comes from two main reasons. On the one hand, the level of intergenerational mobility is widely used as a measure of the extend of equality of opportunity, which implies that individual social attainments should not depend on his family economic background. On the other hand, comparing intergenerational mobility between countries with different institutions and policies provides interesting insights about the mechanisms relating the social situation of one generation to the next.

About such mechanisms, Becker and Tomes (1986) seminal paper emphasizes the role of capital constraints that prevent poor people to invest in their offspring's education. Borrowing to invest in human capital is indeed more costly for poor because of credit market imperfections. Consequently, Intergenerational mobility in earnings may depend on the inheritability of endowments rather than the transmission of abilities. Such a reasoning produces two important and well-known propositions. The first one states that income equality may be positively associated with intergenerational mobility[2]. When income inequality rises, the proportion of credit constrained people in the population increases, limiting investment in their children's education, what's, finally, diminishes their prospect for upward social mobility. For this reason, the public funding of schooling is often considered as a way to reach higher level of social mobility, since it allows to lessen the credit constraint that prevent the poorest part of the population to invest in education : this is the second proposition. Hence, public education may result in

[1]La condition nécessaire pour arriver à centraliser la puissance publique dans une société démocratique est d'aimer l'égalité, ou de le faire croire. (...) Ils avaient voulu être libres pour pouvoir se faire égaux, et, à mesure que l'égalité s'établissait d'avantage à l'aide de la liberté, elle leur rendait la liberté difficile.Alexis de Tocqueville, De la démocratie en Amérique, 1848.

[2]See also Loury (1981) and Owen and Weil (1998) for dynamic extentions.

a redistribution of educational assets from richer to poorer, that enhance overall efficiency, promoting equality of opportunities, and social justice[3]. Note that this last proposition provides us with one of the main theoretical arguments in favor of public intervention on the market of education. For this reasons, it is crucial to wonder if these propositions are consistent with the empirical observation.

Although the quality and scarcity of data render important cross-country comparisons difficult and meaningless, available studies on small samples lead to question the robustness of a systematic relationship between inequality and social mobility on the one hand, and between public education and social mobility on the other hand. Thus, it seems reasonable to conclude that intergenerational mobility in earnings is not ordered by income inequality, or by the share of public spending in education. This lack of evidence stimulates the production of more complex theories along with two alternative directions. They attribute the observed mismatch between social mobility, and inequality to either differences in the educational institutions [Checchi, Ichino, and Rustichini (1999)] or to differences in the level of ability heterogeneity [see Cough and Morand (2005)], or a combination of both.

This paper follows the path introduced by Checchi and al. (1999), but an important feature of the model is to consider mixed school regimes rather than exclusive public or private school systems. This extension of Checchi and al. (1999) offers a dynamic political economic model that emphasizes the decentralization of school system as the key variable to understand the inequality-mobility mismatch. Note that, hereafter, the concept of decentralization I use refers to the way the organization of educational systems weakens or strengthens the link between parental social status, and the quality of education children receive. Thus, the concept of decentralization used in this paper does not only relate to financial decentralization, but also to the organization of school system itself. Although Checchi and al. (1999) compare the features of two alternative school systems (private versus public financing), I use a sequential choice procedure that makes me able to consider a continuum of school systems, ranging between pure public school to pure private school organizations. In other words, public and private education are not exclusive as it is usually the case in political economic models of education. Rather, children receive a bundle of public and private spending. The numerical computations of the model prompt us to consider a simple positive

[3] Becker et Tomes (1986), p.116: *The usual conflict between "equity", as measured by inequality, and efficiency is absent because a redistribution of investment toward less advantaged children is equivalent to an improvement in the efficiency of capital market.*

relationship between mobility and inequality. More I show that income inequality produces strong incentives that make people more confident to invest in their children education. If this incentive effect overcome the consequences of credit constraints, higher inequality may results in higher social mobility in the steady state.

The paper is organized as follows. Section 2 presents the empirical evidence that prevent us to consider a stable relationship between public education, inequality, and social mobility. Section 3 proposes a general theoretical background. The timing of decisions is given in section 4. Then political equilibrium is derived in section 5. Section 6 offers some numerical simulations, from which the main findings of the paper derive. Finally section 7 concludes in giving the implications of this paper, and indicates promising challenges for future research.

2. Puzzling evidences

Table 1 gathers some indicators about the income inequality, and the level of school system centralization for a sample of ten countries for which comparative figures on relative mobility are also available. Column (1) provides 90-10 income ratios, which is a very standard measure of income inequality. Although the sample is composed of developed countries, this measure suggests that income inequality varies widely. For instance, the income share of the richest 10% of the income distribution in the US represents about 16 times the income share of the poorest 10%. This is about 3 times higher than in Finland.

Intergenerational income mobility is often measured as $(1-\beta)$ where β is the elasticity of child's log income on his parents' log income. Nevertheless, beyond this general principle, measures of mobility are very sensitive to the variables used, sample selection and estimation methods. As a consequence it is difficult to use such measures in cross-country comparisons because it is impossible to know whether differences are a consequence of fundamentals, or just the result of differences in measurement strategies. Corak (2004) reviewed international literature on intergenerational mobility, and proposes a database of most comparable mobility measures. Column (2) gathers the values he proposed.

Column (3) contains calculations of an index that allows us to compare the homogeneity of school quality in the different educational system. More precisely, I which measure the relative strength of the link between family background and the quality of education received by a child. Obviously, this quality should depend on the level of private investment of parent in the child's education. But, independently of the share of private spending, decentralization of public education is also an important feature of school system that may impact on the level of intergenerational mobility. Indeed, decentralization of public education may increase the role of family background in the quality of education through the consequences of parental location choices among very heterogenous communities in respect of public goods quality, tax, housing price, social capital etc.. While the countries in the sample spend about the same level of GNP on public education, the sources of public funding may is more or less centralized depending countries. For instance, in Germany, Canada or US, the share of public spending from central State is less than 8%. On contrary, State spending represents about 80% of public spending for Education in France and Italy. Moreover, beyond the way countries finance public education, fundamental differences in the schooling institution should enhance the effect of decentralization. The age of compulsory education[4], the educational curricula[5], or the recruitment and salaries of teachers are some well known illustrations of these institutional differences that matter. The values of this index equal the produce of two variables which give information about the share of public spending in education on the one hand, and the level of centralization of the school system organization on the other one. First, I consider the share of public spending in the overall spending for education in 2002[6]. Second, a centralization factor is derived from the centralization scale provided by Mons (2004)[7].

[4]For instance the age of compulsory schooling is decided at the level of states in the US (ranging from 8 to 13 years), whereas it is established by the law at the national level in France or Italy.

[5]In France, for instance, uniform curricula available in both public and private schools are established by the Parlament. This situation contrasts with Canada where each state is free to adopt his own curricula.

[6]Except for Canada, 1999

[7]Mons (2004) proposes a scale of centralization based on principal component analysis with respect to institutional qualitative data. This scale range between 8 (very centralized and homogenous educational system) and 1 (highly decentralized system of education). For instance, France is at the 8th level of the scale what implies that centralization factor is 1, i.e the maximum value. In the same way, the centralization factor for Germany is 1/8 since Germany belongs to the first of the 8 grades.

Table 1

90-10 Income ratios, intergenerational earnings correlations and educational system regimes from selected countries

Country	90/10 Earnings ratio (year) (1)[a]	Education centralization index (2)[b]	Earnings elasticity (3)[c]
Denmark	8.1 (1997)	0.64	0.15
Sweden	6.2 (2000)	0.65	0.27
Norway	6.1 (2000)	0.64	0.17
Finland	5.6 (2000)	0.49	0.18
Germany	6.9 (2000)	0.14	0.32
France	9.1 (1995)	0.92	0.41
Canada	9.4 (2000)	0.27	0.19
UK	13.8 (1999)	0.28	0.50
US	15.9 (2000)	0.12	0.47
Italy	11.6 (2000)	0.78	0.49

[a] Source : Human Develpment Report, United Nation Development Programme (2006), Table 15 (p.335).

[b] This index is based on author's calculations. Original data come from Table B3.1 of Education at a glance, OECD (2005), and Mons (2004).

[c] Comparative data on intergenerational earnings mobility from M. Corak (2004)

Table 1

A quick examination of this table suffices to identify some unexpected situations where social mobility does not match income inequality given the level of public schooling. In line with Checchi, Ichino and Rustichini (1999), table 1 suggests that Italy displays less mobility than US despite smaller income inequality and a schooling system more publicly oriented and centralized. In the same way, France exhibits a lower earnings mobility than Canada although a lower level of income inequality, and a strong and over centralized public school system. The situation of countries of Northern Europe (except Denmark) is more in line with theoretical predictions since high mobility is combined with low income inequality. Nevertheless, the school system is little centralized in those countries, what does not corroborate the argument of credit constraints as the main source of social status reproduction. The following model offers another explanation that bears on the role of incentives in human capital investment.

3. General theoretical background

3.1. Introduction

The economy is composed of a continuum of people with mass normalized to unity, and who belongs to successive generations such that population is constant over time. The timelife of each person is divided into two different periods. In the first period, people acquired a human capital denoted by a real number h. In the second period, individual supplies his acquired human capital in the labor market and receives a wage. For simplicity, I abstract from labor market, and I assume that earnings depend on the amount of human capital in a way I will describe hereafter. In each period t the distribution of human capital is denoted $\phi_t(h)$, such that for each t, the overall income of the economy is given by

$$H_t = \int_1^\infty h d\phi_t(h) \tag{1}$$

3.2. Talent, education and production

Individuals have a basic working productivity normalize to unity. Moreover, they have a natural ability with some persistence across generations, called talent, to acquire additional human capital, which enhance their productivity. Agents are thus heterogeneous with respect to their family background, and talent they receive. I assume here that school system produces information about talent such that people may use school attendance in order to discover their talent. Moreover, this information allows people to make expectations about the talent of their offsprings.

Talent is assumed to be high (H) or low (L), and is denoted a, such that $a_t \in \{L, H\}$. Moreover talent is transmitted from parents to children following a symmetric first order Markov process with $0 \leq \pi \leq 0,5$:

$$P\left(a_t = H/_{a_{t-1} = H}\right) = P\left(a_t = L/_{a_{t-1} = L}\right) = 1 - \pi$$

This process means that talent persistence is the same for both high and low talented dynasties, and the value of the parameter π ensures that parents' and children's talents are more likely to be similar than different. But individuals are not perfectly aware about their own talent before they attend to school. Ex-ante, they only have some beliefs about their abilities which depend on family history. An important by-product of school system is to provide information about talent such that individuals know perfectly their ability ex-post.

When they are highly talented (H), the earnings of individuals depend on the level of human capital they accumulate. I assume that it is possible to enhance human capital through learning effort, the quality of formal education received (school system), and the direct or indirect contributions due to parents' human capital. Formally, the production function of human capital follows a Cobb-Douglas functional form:

$$h_{t+1} = \begin{cases} 1 & \text{when} \quad a_{t+1} = L \\ A h_t^{\delta_1} E_t^{\delta_2} (1-\eta_t)^{\delta_3} & \text{when} \quad a_{t+1} = H \end{cases} \quad \text{, with} \sum_{j=1}^{3} \delta_j = 1 \tag{2}$$

where η is the share of time spent for leisure, E_t the quality of formal education received, and h_t the father's human capital.

Although the representation of human capital production technology is very standard in the literature since Glomm and Ravikumar (1992), I assume that formal education is a combination of both public (G_{t+1}) and private (S_{t+1}) spending. Moreover, public and private investment are substitutable in-puts. The quality of education received by a child is given by the following Cobb-douglas function:

$$E_{t+1} = S_{t+1}^{\alpha} G_{t+1}^{1-\alpha} \tag{3}$$

where α is the level of decentralization of the school system as defined previously. This formulation allows to consider a continuum of school system ranging between pure public and centralized school system ($\alpha = 0$), and pure private educational system ($\alpha = 1$). For instance, G could be the quality of basic compulsory education publicly provided, and S the investment in supplementary education privately founded. Since public education quality is the same for each child, private investment is necessary to get skills. In addition, higher education gives to the agent some information about his own talent. When he fails, the child knows that his talent is low, whereas if he succeeds, he knows he is talented. Then, his level of human capital will depend on both the quality of public school, and how much his parents invest privately. Finally, public education is financed with a tax rate on earnings denoted $\tau_{t+1} \in [0, 1]$ which is chosen according to a simple majority rule in each period of time such that the state budget constraints is satisfied:

$$G_{t+1} = \tau_{t+1} H_{t+1}$$

3.3. Individual preferences

Utility of an individual born in the period t depends positively on three components : the leisure time when youth, η_t, the consumption when old, C_{t+1}, and the education left to his child E_{t+1}. Education is therefore the only way to bequest. Moreover, I assume that parents' altruism depends on their expectations about the talent of the children, denoted θ_{t+1}.

$$U(\eta_t, C_{t+1}, \theta_{t+1}, S_{t+1}) = \log \eta_t + \log C_{t+1} + \theta_{t+1} \log E_{t+1} \tag{4}$$

Individual faces two constraints : the first is about the distribution of time when young (effort versus leisure), the second is the budget constraint when old:

$$\begin{cases} 0 \leq \eta_t \leq 1 \\ C_{t+1} + S_{t+1} \leq (1-\tau) h_{t+1} \end{cases} \tag{5}$$

4. The timing of the model

4.1. The history of a generation

Life of the inhabitants of the economy is composed of two periods. A person born in period t is young during a first times. Then she is old in a second time, i.e. in period $t+1$. At the beginning of his life, the individual knows the history of his family. More precisely, he knows the failures and the success of the former family's members. Moreover, he knows the quality of education he will receive since it depends on the quality of public schools determined by the oldest share of the population through the political decision making process, and the amount left by parents to supplement basic education public schools provide.

At the beginning of his life, the individual goes to public school and updates his beliefs on his own talent, but only on the basis of his family history[8]. His expectation of talent is denoted θ_t. He then decides wether he wants to supplement basic education. Put differently, he faces a point of bifurcation *à la* Boudon, where

[8] I assume that basic school experience does not change the level of self-confidence of the individual.

he has to decide whether or not to supplement basis education. If he chooses not to do so, he spends all his time in leisure activities. In contrast, if he chooses to continues to learn, he has also to determine the level of effort he devotes to accumulate additional skills. He then goes to school until the first period ends. Note that the decision of the individual to invest in additional education is taken on the basis of his talent expectation. At this stage, he has indeed no more information. This information is provided through private schooling.

At the beginning of the second period, denoted $t+1$, individuals that invest in more education know their true type (H or L). The others has no additional information about their talents. Individuals who did not go to school when young, or those who failed are low talented, i.e. L. They offer a basic level of human capital in labor market, and they receive the standard wage normalized to unity. In contrast, people with high talent receive a wage which depends on human capital accumulated in t. Then, old generation votes to choose the quality of public education through the tax rate, τ_{t+1}. Then, they choose how much to consume, and the amount of funds they bequeath to their child for his additional education. Finally, the new generation appears in the same time the parents die, such that generations do not overlap.

Finally, the dynamics of families history implies that the population of the economy is composed of four types of dynasties at any point of time :

- when both the father and his son are talented (H), then the family is said to be steady high;
- when the father is talented (H), but the son fails in additional schooling (L), the family is said to be downward mobile;
- when the son of a low talented (L) person succeed in acquiring additional schooling, then he becomes talented (H), and the families is said to be upward mobile;
- when both the father and his son are not talented (L) wether the son intents additional schooling or not, the family is said to be steady low.

4.2. The dynamic of expectations

To understand how people form their beliefs concerning their child's talent, consider the history of a family whose first member inherits an undefined level of self-confidence denoted θ_t. In beginning of the first period, this agent has two options: to continue or to stop to accumulate human capital through private schooling. In the first case, he will succeed if he is talented, otherwise he fails. In case of success, he will change his belief to $\theta_t = 1$, since he knows with certainty that he is talented. Consequently, he is able to make some expectations about his child abilities since he is aware that the probability for his child to be talented too is $(1-\pi) = \theta_{t+1}$. In case of failure, instead, $\theta_t = 0$, and $\theta_{t+1} = \pi$.

Imagine now that the individual does not supplement the basic human capital public school provides. Then, he will not have information about his own talent. In this case, his child will inherit the following Bayesian update belief $\tilde{\theta}_{t+1} = \tilde{\theta}_t(1-\pi) + \pi(1-\tilde{\theta}_t) = \tilde{\theta}_t(1-2\pi) + \pi$. If the members of this family keep on not investing in additional education say for j generations, it is straightforward to show that $\tilde{\theta}_{t+j} = \left(\tilde{\theta}_t - \frac{1}{2}\right)(1-2\pi)^j + \frac{1}{2}$. In particular, if the first decision of not supplement basic education occurred after a failure in t, the self-confidence level of the J-th member of such a dynasty is given by:

$$\tilde{\theta}_{t+j} = \frac{1}{2}\left[1 - (1-2\pi)^{j+1}\right] \tag{6}$$

Note that, whatever the initial value of π, the belief monotonically converges to $\frac{1}{2}$, which is indeed the true distribution of talent in the population . This is an important feature of the dynamics of expectations, because convergence insures that a future member of a failing dynasty will invest in education sooner or later.

Given the dynamics of beliefs, the four type of families with respect to their trajectories translate into three types of families with respect to expectation about child's talent:

- individuals that succeed in additional schooling (steady high or upward mobile families) get the highest level of expectation for his offspring, i.e. $(1-\pi)$;
- individuals that fails (downward mobile or steady low families) get the lowest level of confidence in their child's talent, i.e. π.
- individuals that keep on not investing in private schooling during j generations after a failure (steady low) get an intermediate level of confidence in their children's talent, i.e. $\tilde{\theta}_{t+j} = \frac{1}{2}\left[1-(1-2\pi)^{j+1}\right]$.

4.3. Intergenerational mobility[9]

We focus in this paper on the long run property of equilibria. This can be done by considering the invariant distribution of relevant variables such that human capital, education, and beliefs over talent.

As described in previous section, one time at least, a failure occurs in the history of each family. After this, the self-confidence level of the member who fails is 0, and his belief over his child's talent is π, i.e. the probability to be different from his father). The next generation will update this belief following the dynamics of expectation defined in (6), and then they don't go to invest in additional schooling until a critical level of self-confidence is reached. When a person reach this critical level of self-confidence, she invests in additional education, and then talent is finally revealed. If she is a low type (L), then the dynasty goes back to π, and the dynamic start again. But, if she is talented (H), the the next generation will be provided with a belief $1-\pi$. Then from this last belief two transitions exists to either π (failure) or $1-\pi$ (success).

Checci and al. (1999) shows that the dynamics described above lead to an invariant distribution which depends only on the rank of the critical agent according to the beliefs. The critical agent is a person with low type parents, who intents to invest in additional schooling. We denote the position of this agent in the distribution of expectations i^*. It is then possible to weight the different categories of families in the population in the long run as a function of i^*. I denote S_1 the share of low type agents (L) with an income of 1, S_2 the share of people with low type father, and who invests in additional schooling with success (upward mobile), and, finally, S_3 the share of type H agent for several generation. Appendix B3 of Checci and al. states that

$$S_1 = \frac{(i^*+1)\pi}{\pi(i^*+1)+\tilde{\theta}^{i^*}} \quad S_2 = \frac{\pi\tilde{\theta}^{i^*}}{\pi(i^*+1)+\tilde{\theta}^{i^*}} \quad S_3 = \frac{(1-\pi)\tilde{\theta}^{i^*}}{\pi(i^*+1)+\tilde{\theta}^{i^*}} \tag{7}$$

From the definition of the transition process, it is also possible to obtain the matrices of social mobility. In our case the simplest matrix of the mobility is sufficient to highlight the relation between the school system features, the mobility, and the inequality.

[9] The dynamics of the model I use comes from Checci et al. (1999). I only present the main findings I need for my own analysis of mixed school regimes. The formal proofs of these results can be found in the original paper.

To obtain this matrix, we divide the overall population in two classes. The first one is composed of people without additional schooling, denoted Q_1. The other part of the population forms the class denoted Q_2. With $P_{ij},\ i=1,2,\ j=1,2$ the probability that a family transits from Q_i to Q_j, the computation of the transition matrix yields to the following matrix of transition probability across classes:

$$M=\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}=\begin{pmatrix} 1-\frac{\tilde{\theta}^{i^*}}{i^*+1} & \frac{\tilde{\theta}^{i^*}}{i^*+1} \\ \pi & 1-\pi \end{pmatrix} \tag{8}$$

Note that when $i^*=0,\ \tilde{\theta}^{i^*}=\pi$. The value of P_{12} may then be interpreted as an index of upward mobility at the steady state. This index appears to be negatively associated with i^*: the higher i^*, the lesser P_{12}. This means that a society is said to be less mobile when the time to see an offspring of a low type dynasty to invest in additional schooling is long. In contrast, if people invest in additional schooling whatever the type of their parents, $i^*=0$ (children don't have to wait to accumulate sufficient confidence), the society is said mobile to the extend of π. Social mobility is then endogenously determined through the school system (upward mobility), but is also partly exogenously determined by social or genetical factors.

5. Optimal behavior and political equilibrium

5.1. Consumption and investment

In this section I determine the optimal behavior of a household denoted i using backward induction solving procedure. The ultimate decision in a generation history is about consumption and private investment in education. For parents, education and production occurred and they have a beliefs concerning the talent of their child. So, whether the situation of parents (parents' human capital and beliefs are given), the problem they must solve is the same :

$$\begin{cases} \underset{\{C_{t+1},S_{t+1}\}}{Max} \ \log C^i_{t+1}+\theta^i_{t+1}\log E^i_{t+1} \\ s.t. \quad C^i_{t+1}+S^i_{t+1}\leq(1-\tau_{t+1})h_{t+1} \end{cases}$$

Solving this program we get the optimal bundle of consumption and investment :

$$\begin{cases} C^*_{t+1}=\frac{(1-\tau_{t+1})h^i_{t+1}}{1+\alpha\theta^i_{t+1}} \\ S^*_{t+1}=\frac{\alpha\theta^i_{t+1}(1-\tau_{t+1})h^i_{t+1}}{1+\alpha\theta^i_{t+1}} \end{cases}$$

Proposition 1. *Private investment in additional education is positively related to the parental earnings, i.e. human capital, the level of self-confidence, the extend of decentralization of school system, and negatively related to the quality of public school.*

The utility function with optimal levels of investment and consumption becomes

$$W = \left(1 + \alpha\theta^i_{t+1}\right) \log\left[(1 - \tau_{t+1})h^i_{t+1}\right] + (1 - \alpha)\theta^i_{t+1} \log\left(\tau_{t+1}H_{t+1}\right) + L\left(\theta^i_{t+1}\right) \quad (9)$$

$$\text{with } L(\theta^i_{t+1}) = \theta^i_{t+1}\alpha \log(\alpha\theta^i_{t+1}) - (1 + \alpha\theta^i_{t+1}) \log(1 + \alpha\theta^i_{t+1}).$$

5.2. Public education

As stated with proposition 1, the financing of public education lowers the demand for additional education privately funded. The level of public supply results in a collective choice which is assumed to follow a simple democratic majority voting rule. I show hereafter that the median voter theorem applies such that the political process lead to a political equilibrium.

5.3. Political preferences

Given the optimal choice of the end of the second period, household i votes for his preferred tax rate. This tax rate represents the quality of public education available to the next generation, and is denoted τ^i_{t+1}. To determine his preferred tax rate, household faces the following optimization program:

$$\begin{cases} \underset{\{C^i_{t+1}, S^i_{t+1}\}}{Max} \quad \log C^{i*}_{t+1} + \theta_{t+1} \log\left(S^{i*}_{t+1}\right)^{\alpha} \left(G_{t+1}\right)^{1-\alpha} \\ s.t. \quad G_{t+1} \leq \tau^i_{t+1}H_{t+1} \end{cases}$$

We obtain the household i preferred tax rate:

$$\tau^i_{t+1} = \frac{(1 - \alpha)\theta^i_{t+1}}{1 + \theta^i_{t+1}}$$

Note that the tax rate household i prefers does not depend on his income, but, instead, is determined by the level of parents' confidence in the talent of their child. Hence, as in Piketty (1995) political preferences are related to social intergenerational trajectories. Moreover, the relationship between the tax rate and the belief depends negatively on the level of school system decentralization.

As previously stated, there is three types of dynasties in respect to beliefs. Since belief is the basis of political preferences, three different kinds of voter will arise. If the father of household i has invested in additional education and succeeded, is self-beliefs is $\theta_t^i = 1$, and the beliefs for his child is $\theta_{t+1}^i = 1-\pi$. Then, his preferred tax rate is $\tau_{t+1}^i = \frac{(1-\pi)(1-\alpha)}{(1-\pi)+1} = \tau^1$. If, instead, the father experience a failure, both beliefs become $\theta_t^i = 0$ and $\theta_{t+1}^i = \pi$, and $\tau_{t+1}^i = \frac{\pi(1-\alpha)}{\pi+1} = \tau^2$. Note that in this case only, the dynasty is downward mobile since father is highly talented (H) whereas the son is low talented (L)[10]. At last, when the son does not invest in additional schooling, he has a level of self-confidence denoted $\theta_t^i = \tilde{\theta}_t$, and his expectation for his son is $\theta_{t+1}^i = \tilde{\theta}_t(1-2\pi)+\pi$, what leads to his preferred tax rate: $\tau_{t+1}^i = \frac{\tilde{\theta}_{t+1}^i(1-\pi)}{\tilde{\theta}_{t+1}^i+1} = \tau^3$. Since we have $\pi \leq 1/2$, the following proposition derives straightforwardly from the comparison of preferred tax rates.

Proposition 2. *The ideal tax rate of a person is independent from her income. It is positively related to her level of confidence within her own child's talent. Hence, tax rates are ordered in confidence such that $\tau_1 \geq \tau_3 \geq \tau_2$.*

Proposition 2 means that the agents' support for taxation increases when their expectations grow: the higher the confidence in child's talent, the higher the preferred tax rate, i.e. the quality of public schooling.

5.3.1. Collective choice

Applying the implicit function theorem to indirect utility function (7), we obtain the expression of indifference curves of the three types of individuals populating the economy:

$$\frac{\partial \tau_{t+1}}{\partial G_{t+1}} = \frac{\theta_{t+1}^i(1-\alpha)(1-\tau_{t+1})}{G_{t+1}\left(1+\alpha\theta_{t+1}^i\right)}, \textit{ with} \left\{ \begin{array}{ll} \theta_{t+1}^i = 1-\pi & \textit{pour } i=1 \\ \theta_{t+1}^i = \pi & \textit{pour } i=2 \\ \theta_{t+1}^i = \tilde{\theta}_{t+1} & \textit{pour } i=3 \end{array} \right\} \tag{10}$$

[10] Obviously, it is possible for the son of a low talented father to fail. But, in this case, family remains with a low type, so it is a steady low family.

Expression 8 means that the slope of indifference curves differs with the type of individuals in the $[\tau, G]$ space, and is increasing with the level of confidence, θ^i_{t+1}. In such situation, a political equilibrium exists only if indifference curves single cross in $[\tau, G]$ space. To check this point it is sufficient to take the derivative of expression (8) with respect to the level of expectation, θ:[11]

$$\frac{\partial^2 \tau}{\partial G \partial \theta} = \frac{G(1-\alpha)(1-\tau)}{\left[G\left(1+\alpha\theta\right)\right]^2} \geq 0$$

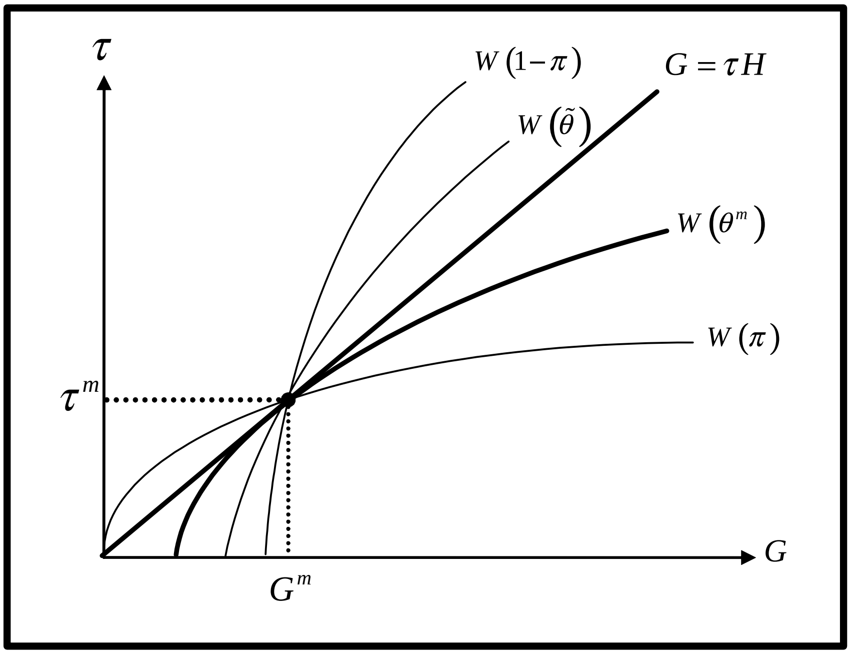
As mentioned in the proposition 2, the preferred tax rate is given by $\tau^i_{t+1} = \tau^i_{t+1}(\theta^i_{t+1})$. Thus, whoever are two agents i, and i', if their expectations are such that $\theta^{i'}_{t+1} \geq \theta^i_{t+1}$, the agent i' exhibits a higher preferred tax rate than the agent i, i.e. $\tau^{i'}_{t+1} \geq \tau^i_{t+1}$, $\forall\ (i, i')$. Besides, the dynamics of expectations implies that the agents with type H have the same ideal tax rate, which is the highest in the population. Moreover, indifference curves with respect to θ only cross once in $[\tau, G]$, then simple majority rule produces a political equilibrium [Gans and Smart (1996)]. Finally, the uniqeness of the equilibrium reacquired the concavity of indifference curves, what the following expression states:

$$\frac{\partial^2 \tau}{\partial^2 G} = \frac{-\theta(1-\alpha)(1-\tau)\left(1+\alpha\theta\right)}{\left[G\left(1+\alpha\theta\right)\right]^2} \leq 0$$

Proposition 3. *A political equilibrium exists. It is unique and depends on the agent getting the median level of expectation denoted* θ^m.

The following figure depicts such equilibrium. It is located to the point of tangency between the median voter's indifference curve, and the state budget constraints.

[11] The sequence of decision being now well understood, we omit indices to simplify notations starting from here.



5.3.2. Who is the decisive voter ?

A this stage of the model, we know that a political equilibrium exists, and that it matches the preferred tax rate of the agent having the median level of expectation. But to which category of agent belongs this decisive voter? From the features of the invariant distribution, it comes the following proposition.

Proposition 4. *The pivotal voter of the economy is a low type agent with non additional schooling.*

Proof. From expression (7), the decisive voter is low talented (H) if $\frac{(i^*+1)\pi}{(i^*+1)\pi+\tilde{\theta}^{i^*}} \geq 0,5$ where i^* is the rank of the critical agent, i.e. an individual with low type parents who decides to invest in additional schooling. Simplifying and rearranging, expression (8) is verified if $(1-2\pi)^{i^*+1} > 1-2\pi(i^*+1)$. With $x = 2\pi$, and $n = i^*+1$, we have $f(x) = (1-x)^n$, where $0 \leq x \leq 1$. This function is convex and its derivative for 0 is negative, such that $f(x) > f(0) + f'(0)x \Rightarrow (1-2\pi)^{i^*+1} > 1-2\pi(i^*+1)$ ■

5.4. Individual optimal strategy

At the beginning of the first period, individuals have to choose whether they want to invest in additional schooling. For convenience, to invest in additional education represents option 1 whereas not to represents option 2. So individuals must choose one of these two options by comparing the welfare expectations both options provide. Moreover, if they choose to invest privately in human capital, they have to determine the level of effort $(1-\eta_t)$ they will do according to their level of self-confidence, θ_t^i. Supposing that they subsequently choose the optimal levels of consumption and investment, the program of the agent may be written:

$$\begin{cases} \underset{\{\eta_t\}}{Max} \ \log \eta_t + \theta_t \left(1+(1-\pi)\alpha\right) \log(1-\tau)h_t + (1-\alpha)\log\left(\tau H_t\right) \\ \qquad +L(1-\pi) + (1-\theta_t)\left[(1-\alpha)\log(\tau H_t) + L(\pi) + \log(1-\tau)\right] \\ s.t. \quad 0 \leq \eta_t \leq 1 \end{cases}$$

Simplifying the expression, we seek to define the optimal level of leisure time, denoted η_t^*, such that:

$$\eta_t^* = Arg\max\left\{\log \eta_t + \theta_t\left(1+(1-\pi)\alpha\right)\delta_3 \log(1-\eta_t)\right\}$$

Computation gives that:

$$\eta_t^* = \frac{1}{1+\theta_t\left(1+(1-\pi)\alpha\right)\delta_3}$$

This expression shows that the level of agent's effort is increasing with the self-confidence level which is given by parents to the child. Moreover, the strength of the relationship between talent expectation and the effort level is related to the marginal productivity of the effort in the production function of the human capital on the one hand, and the degree of decentralization of the school system in the other hand. This last property is in line with a standard result of the literature of the endogenous growth [Lucas (1988)].

Option 1 leads to the following expected level of welfare:

$$\begin{aligned} &\Delta_1 = \theta_t\delta_3\left[1+(1-\pi)\alpha\right] L\left(1+\theta_t\delta_3\left[1+(1-\pi)\alpha\right]\right)^{-1} \\ &+\left(1+\theta_t\alpha(1-\pi)\right)\log(1-\tau) + \theta_t\left[1+(1-\pi)\alpha\right]\log A h_t^{\delta_1} E_t^{\delta_2} \\ &+(2-\theta_t)(1-\alpha)\log G_t + L\left(1-\pi\right) + (1-\theta_t)L(\pi) \end{aligned} \tag{11}$$

In option 2, agents decide not to invest in higher education. Then, they have not to determine an effort level, and $\eta_t^* = 1$. Besides, we know that expectations take in this case a transitional value denoted $\tilde{\theta}_t$. The expected welfare is given by the following expression.

$$\Delta_2 = (1-\alpha)\log G_t + L\left(\tilde{\theta}_t\right)$$

Hence, it is straightforward to see that individuals will invest in additional human capital only if option 1 gives a higher level of expected welfare than option 2, i.e. $\bar{\Delta} = \Delta_1 - \Delta_2 > 0$. Arranging the expression, we obtain that people will invest privately in human capital only if the following condition is fulfilled.

$$\begin{aligned}
&\bar{\Delta} = \theta_t\delta_3\left[1+(1-\pi)\alpha\right]L\left(1+\theta_t\delta_3\left[1+(1-\pi)\alpha\right]\right)^{-1} \\
&+\left(1+\theta_t\alpha(1-\pi)\right)(1-\alpha)\log G_t + \theta_t(1+\alpha)(1-\pi)\log(1-\tau) \\
&+\alpha\theta_t\left[1+(1-\pi)\alpha\right]\log\left(\tfrac{\alpha\theta_t}{1+\alpha\theta_t}\right) + \theta_t\left[1+(1-\pi)\alpha\right]\log A \\
&+\theta_t\left[1+(1-\pi)\alpha\right]\delta_1\log h_t + \alpha\theta_t\left[1+(1-\pi)\alpha\right]\log h_{t+1} \\
&+\theta_t L(1-\pi) + (1-\theta_t)L(\pi) - L(\tilde{\theta}_t)
\end{aligned} \tag{12}$$

6. Numerical computations

In this section I provide the results of two sets of numerical computations that allow to understand the main implications of the model. Firstly, I quickly describe the methodology I use. Then the results of two different computations are proposed. In the first one, I how the level of school system decentralization may explain the mobility-inequality mismatch. In the second one, I observe the consequences of an exogenous change in intergenerational mobility.

6.1. Methodology

To determine the characteristics of the invariant distribution corresponding to various values of the exogenous variables, it is necessary to identify the position of two special agents: the pivotal voter, and the critical agent as defined previously. Once these two stages carried out, it is easy to deduce from it the values of other variables, and in particular the level and the structure of the educational supply according to the institutional background.

- Identifying the decisive voter

From proposition 4, it derives that the rank of the decisive voter denoted i^m may be such that $\frac{(1-2\pi)^{i^m-1}}{1-2\pi(i^m)} < 1$ and $\frac{(1-2\pi)^{i^m}}{1-2\pi(i^m+1)} > 1$. Note that i^m only depends on the level of exogenous persistence in abilities, π. The smaller the persistence in abilities across generations (π increases), the lesser the median value of beliefs, i^m. Finally, according to proposition 2, when the confidence of median voter decreases, social demand for education is lower.

- Identifying the critical agent

Now, we turn to identifying position of he critical agent, i^*. From expression 7, the critical agent is such that $\bar{\Delta}^{i^*+1} < 0$ and $\bar{\Delta}^{i^*} \geq 0$. But, in order to check this, it is necessary to evaluate the aggregate income and the level of public supply according to the tax rate of the decisive voter that we previously identified. Aggregate income is obtained from the following general expression:

$$h_n = \begin{cases} 1 \text{ if } n < 0 \\ A\left(\frac{\alpha\tilde{\theta}^{i^*}(1-\tau)}{(1+\alpha\tilde{\theta}^{i^*})}\right)^{\alpha\delta_2}(\tau H)^{(1-\alpha)\delta_2}\left(1-\frac{1}{1+\tilde{\theta}^{i^*}(1+(1-\pi)\alpha)\delta_3}\right)^{\delta_3} \text{ if } n = 0 \\ Ah_{n-1}^{(\delta_1+\alpha\delta_2)}\left(\frac{(1-\pi)\alpha(1-\tau)}{1+\alpha(1-\pi)}\right)^{\alpha\delta_2}(\tau H)^{(1-\alpha)\delta_2}\left(1-\frac{1}{1+(1-\pi)(1+(1-\pi)\alpha)\delta_3}\right)^{\delta_3} \text{ if } n > 0 \end{cases}$$

Otherwise, applying the results of section 4.3, we obtain the income share of the three different components of the population, respectively the uneducated, the upward mobile, and the steady high families:

$$Z_1 = S_1,\ Z_2 = h_0 \times S_2, \text{ and } Z_3 = h_n \times \sum_{n=1}^{\infty}\frac{\pi(1-\pi)^n\tilde{\theta}^{i^*}}{\pi(i^*+1)+\tilde{\theta}^{i^*}}$$

Since the mass of the population is normalized to unity, it is interesting to note that S_2 is also the probability of upward mobility in the economy.

Combining these results with the production function of human capital, we have to find the aggregate income denoted H, which satisfies $H = \sum_{j=1}^{3} Z_j$. Then, , it is possible to determine the value of i^* from a series of iteration such that $\bar{\Delta}^{i+1} < 0$ and $\bar{\Delta}^{i} \geq 0$.

- Other calculations

When the position of the critical agent is known, others calculations are straightforward. For instance, the overall spending for education per capita is given by:

$$\begin{aligned}\text{DIE} &= G + \left(\left(\frac{\pi\tilde{\theta}^{i^*}}{(1-i^*)\pi+\tilde{\theta}^{i^*}}\right)\left(\frac{\alpha\tilde{\theta}^{i^*}(1-\tau)}{1+\alpha\tilde{\theta}^{i^*}}\right)\right)\\ &\quad + \sum_{n=1}^{\infty}\left[\left(\frac{(1-\pi)^n\pi\tilde{\theta}^{i^*}}{(1-i^*)\pi+\tilde{\theta}^{i^*}}\right)\left(\frac{\alpha(1-\pi)(1-\tau)}{1+\alpha(1-\pi)}\right)h_n\right]\Big/\tilde{h}\end{aligned}$$

The share of public spending in education is given by:

$$TS = \frac{G}{DIE}$$

To measure income inequality, I consider the ratio of the income share of educated people (type H) and the income share of uneducated people (type L):

$$INE = \frac{Z_2 + Z_3}{Z_1}$$

Finally, the level of upward mobility is the probability for an agent with type L to become talented (H), what corresponds to the value of P_{12} in expression 8.

6.2. School system decentralization

In this section I analyze the effect of decentralization on both income inequality and intergenerational mobility. Table 2 gathers the results of numerical computations for two economies that exhibit the same level of abilities persistence across generations. The only difference between them, is the organization of the school system. In case 1, education is highly centralized and publicly funded as it is in France, whereas, in case 2, education is very decentralized as in Germany, for instance[12]

[12] Values used for computations : $A = 5$; $\delta_1 = \delta_3 = 0,25$; $\delta_2 = 0,5$.

Table 2
Results with different levels of decentralization (α)

	Case 1	Case 2
Example	France, Italy	Germany
Decentralization level (α)	0.2	0.7
Social mobility (π)	0.2	0.2
Median voter position (i^m)	3	3
Critical agent position (i^*)	6	4
Share of agents with type L (S_3)	0.74	0.68
Upward mobility probability (P_{12})	0.07	0.09
Aggregate income (H)	1.21	1.59
Tax rate (τ)	0.22	0.08
Public spending for education (G)	0.27	0.14
Overall school spending in GDP	0.31	0.27
Public share in total spending	0.86	0.52
Income inequality index	1.8	3.1

Several lesson comes out from this comparison. Firstly, upward mobility is higher in a more decentralized school system. For instance, the number of generations necessary so that a dynasty invests in additional schooling after a failure is clearly weaker in the decentralized system. This result corroborates only partly those of Checchi and al.. (1999) who show that the best kind of education financing depends on the relative values of factors in the production function of human capital. I show that when several forms of supplies of education coexist, the superiority of the decentralized system compared to public education does not depend on the marginal productivity of the components of the production function of human capital. This advantage remains, whatever the degree of decentralization, except for a particular case: the pure public system. In other words, incentives to invest in education is lesser when education is more publicly financing and centralized. This negative effect on incentives outweighs the positive effect of public education in alleviating credit constraints on the poorest people. Hence, equality lead lower the opportunity cost of education for poor people, but, in the same time, equalization also lessen the returns to education, what is detrimental for poor people to invest in additional education. Moreover, the decline of returns to education diminish not only for the poorest, but for the overall population, what results in a fall of the number of agents which invest additional schooling.

In fact, comparing the two cases, overall investment in education seems to be higher in case 2 (note that GDP is clearly higher so that, in absolute terms, overall investment in education is about 0.43 in case 2 versus 0.37 in case 1). Moreover, if the share of the GDP each economy devotes to education is overall identical, the structure of spending varies according to the type of education system. It is not only due to a simple substitution effect. This effect comes from the higher upward mobility in case 2 compared to case 1: people are more willing to invest in their own education, and then to make effort, in this case.

Finally the income inequality, and the aggregate income are higher in an educational system decentralized. This results seems to be quite standard in the literature [Glomm and Ravikumar (1992)]. However, it should be stressed that here, income inequality and social mobility are compatible. For instance, Germany is at the same time more mobile and more unequal than France, what is one of the main point we wish to clear up in this paper.

6.3. Exogenous shocks on mobility

Because education is only one of the main mechanisms linking social status across generations, it would be interesting to wonder how an exogenous difference in abilities transmission efficiency translates into income inequality, upward mobility, and aggregate out-put. For this purpose, I consider that the school system is closed (same level of decentralization), and I compute the model for different values of π. Results are reported in table 3[13].

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[13] On peut par exemple penser aux caractéristiques du marché du travail qui sont très différentes selon les pays, ce qui peut modifier les stratégies d'insertion des acteurs, et donc, *in fine*, la MI.

Table 3
Results with different values for π

	Case 3	Case 4
Example	UK, US	Canada
Decentralization level (α)	0.7	0.7
Social mobility (π)	0.1	0.3
Median voter position (i^m)	4	1
Critical agent position (i^*)	6	3
Share of agents with type L (S_3)	0.64	0.71
Upward mobility probability (P_{12})	0.06	0.12
Aggregate income (H)	2.09	1.34
Tax rate (τ)	0.07	0.09
Public spending for education (G)	0.15	0.12
Overall school spending in GDP	0.32	0.22
Public share in total spending	0.50	0.54
Income inequality index	4.02	2.2

The comparison of these two cases suggests that, for identical school systems, intergenerational mobility goes hand in hand with income equality. But, the more interesting result is that the share of public spending in overall investment in education is higher in the case that exhibits the highest level of intergenerational mobility.

7. Conclusion

This paper intents to explain why empirical literature does not succeed in detecting the positive relationship proposed by the standard economic theory between income equality and intergenerational mobility. An extension of the seminal paper of Checci and al. (1999) is developed to deal with the inequality-mobility mismatch with mixed school regimes.

Firstly, I show that more decentralized financing of school spending results in higher incentives for households to invest privately in their children's education. More precisely, if return to effort is important enough, income inequality generates incentives to invest in education that overcome credit constraints effects, such that private education results in higher social mobility. Consequently, the more decentralized the school system, the higher the social mobility and the income inequality.

Secondly, I find that, for a given school regime, exogenous higher level of intergenerational mobility translates into a lower level of income inequality, because incentives to invest in education are growing with social mobility. These two forces combined with credit constraints may explain why countries with low income inequality and high social mobility exhibit school system mildly decentralized.

My investigations suggest that the existence of an optimal level of decentralization of educational organization may be able to produce both income equality and social mobility. However formal demonstration is not provided here, what sets a clear direction for future research.

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