

# Evaluating political decision makers: With the benefit of hindsight bias?\*

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-PRELIMINARY DRAFT-

## Abstract

In this paper we investigate the effects of biased decision evaluation in a simple two-period political agency model. We assume that voters are subjected to hindsight bias in their judgment about a politician's ability to take appropriate decisions. High ability is defined as an informational advantage over voters as to the welfare maximizing policy, creating incentives for low-ability politicians to deviate from the optimal policy choice in an attempt to be perceived as possessing superior private information. We model hindsight bias as a cognitive deficiency (imperfect memory) which makes voters, to a certain extent, immune to surprises. We show that hindsight biased policy evaluation acts as a discipline device for low-ability politicians and, under certain conditions, increases political turnover compared to fully rational evaluation. These insights may be relevant to other principal-agent relationships in which hindsight bias cannot be eliminated through explicit ex ante contracts, e.g. promotion decisions in organizations.

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# 1 Introduction

It is well documented in the psychology literature that many human beings exhibit hindsight bias, a phenomenon also known as the “I-knew-it-all-along” effect. It expresses the inability to correctly remember one’s prior expectation about an event after new information has been received. In particular, this cognitive bias may be expected to play a role in democratic elections if voters care about the competence of the politician holding office. When choosing between an incumbent and a challenger, voters use the incumbent’s history of past actions to evaluate his ability to make the right decisions. As long as this evaluation occurs after the outcome of a policy is realized, it may suffer from hindsight bias: Faced with the failure of a given policy, for instance, voters may distort their recollection of the ex ante situation and conclude that the policy was doomed to fail from the start.

Hindsight bias is one of several deviations from fully rational behavior of subjects that has been discussed in the political economy literature (cf. Frey and Eichenberger (1991) or Gowda (1999)), and conventional wisdom suggests that hindsight bias is detrimental to the evaluation of political decision makers. Unlike in other principal-agent relationships, where hindsight bias can be held in check by ex ante contracts, in a political economy context players have to rely on implicit contracts, as argued by Camerer *et al.* (1989, p. 1246):

Although the principal and agent can contract today to avoid tomorrow’s hindsight bias, when contracts are implicit, hindsight bias will cause a principal to recall the terms of yesterday’s contract incorrectly (...). This problem is especially acute in public decision making, in which principals are a diffuse group of voters and contracts are rarely explicit.

We incorporate hindsight biased voters into a simple model of political agency to investigate the consequences of behavioral decision evaluation. In the model, a politician of unknown ability has to choose between a status quo policy whose payoff is certain and a reform policy whose payoff is uncertain. While voters and low-ability politicians obtain only an imperfect signal of which policy is preferable ex ante, high-ability politicians know the state of the world with certainty. In quest of identifying a politician’s quality, voters face two sources of uncertainty, namely, uncertainty over the politician’s type and uncertainty over the right policy choice which depends on the state of the world. Our basic setup is similar to Majumdar and Mukand (2004), yet contrary to them, we assume that voters receive the same signal as low-ability politicians. This reflects the idea that voters are

exposed to a certain amount of policy relevant public information (e.g. from the media). In addition, we suppose that politicians are aware of voters' behavioral decision making. This is in line with statements from political scientists who acknowledge that "politicians typically have a strong intuitive understanding of voters' heuristics and biases" (Gowda (1999, p. 71)).

Perhaps surprisingly, we show that in this setting, hindsight biased voters may be a blessing rather than a curse. To see why, consider first what happens in the case of fully rational voters. A high-ability politician always chooses the right policy and thus disregards the publicly observed signal. In terms of welfare, a low-ability politician should always choose the policy suggested by the public signal. However, if he follows this strategy, rational voters would infer that any politician who chooses a policy that is contrary to the signal must be of high ability, so choosing an unpopular policy acts as a signal of competence. Therefore, if the signal is not too precise and the politician cares about reelection, an equilibrium where the low type always follows the signal cannot exist. The equilibrium with fully rational voters has the low-ability politician randomizing between choosing the policy suggested by the signal and doing exactly the opposite. Of course, this randomizing behavior is detrimental to welfare because policy choices are not optimal given the available information.

In the case of hindsight biased voters, policy evaluation is biased since the voters' recollection of the signal shifts in direction of the (publicly) observed outcome. We look at an extreme form of bias: with a binary signal indicating the state of the world, we assume that hindsight biased voters distort their recollection of the signal so as to make it consistent with the realized outcome. If the signal suggested that maintaining the status quo was optimal, but the politician enacts a successful reform, then voters wrongly believe that the signal had suggested all along that reform was the right choice. Therefore, with hindsight biased voters, some of the gain in reputation that follows from an unpopular policy which then turns out to be a success is destroyed, because ex post, biased voters think that it was the obvious choice anyway. As a result, the low-ability politician chooses a suboptimal policy less often when voters are hindsight biased than when they are perfectly rational. Hindsight bias on the part of voters reduces incentives for the low-ability politician to engage in costly signaling and can therefore be welfare enhancing.

The disciplining effect of hindsight biased policy evaluation is unambiguously beneficial for voters' first period welfare. However, an overall welfare assessment also has to take into

account the second (i.e., post-election) period. We analyze how hindsight bias affects the selection of the second period politician and show that, under some conditions, both the low- and the high-ability politician are less likely to be reelected. Hence hindsight biased evaluation increases political turnover. This suggests that it may very well be the case that hindsight bias decreases voters' second-period welfare. These qualifications notwithstanding, hindsight bias can be welfare-enhancing no matter what if voters discount future payoffs at a sufficiently high rate.

### **Related literature**

Early contributions to the political agency literature include Barro (1973) and Ferejohn (1986), their work restricts attention to moral hazard in the politician-voter relationship. See Persson and Tabellini (2000) or Chapter 3 in Besley (2006) for a recent overview of political agency models. Directly related to our agency problem are models of retrospective voting where past policy choices or performance measures are used by imperfectly informed voters to assess the incumbent's competence, for example see Rogoff (1990). Our model framework is partly based on Majumdar and Mukand (2004), whose main focus is on learning and policy experimentation, and on Harrington (1993), who analyzes the effects of economic performance and policy manipulation on reelection decisions. In these types of models not only is the politician's type private information but also the introduction of uncertainty over the mapping from policy choices into policy outcomes generates additional complexity for voters' evaluation of incumbent's type and policy quality. In order to judge a politician in the election, voters form beliefs regarding the appropriate policy choice and the incumbent's ability, and hence signaling issues arise quite naturally. In political economy models, for example, decision makers may engage in signaling congruence which may lead to pandering, as shown by Maskin and Tirole (2004), or wasteful spending as a signal of diligence, as in Dewatripont and Seabright (2006). This theoretical framework is also closely related to career concern models in corporate finance, where a manager's decisions under reputational pressure may lead to herding as demonstrated by Scharfstein and Stein (1990) and Zwiebel (1995). In Allen and Gorton's (1993) model of portfolio management, bad brokers cannot be distinguished from good brokers. Both types invest but good brokers buy undervalued stocks while bad brokers just speculate. An important difference to the career concern models is that in our framework decision makers know their own type perfectly.

The notion of equilibrium crucially depends on the beliefs voters hold regarding a politician's type, and to our best knowledge the impact of hindsight bias in a retrospective voting model has not been studied. The psychological bias we introduce can be understood as an internal (intrapersonal) institution that manipulates a voter's belief about the state of the world, this interpretatively resembles political agency models in which external institutions (such as media or experts) are used (as an indoctrination device) to influence the perceived reputation of the government, see for example Besley and Prat (2006). The intrapersonal view we follow in our model is also connected to the theory of collective beliefs and motivated cognition Bénabou and Tirole (2006) develop. Their theory describes the role and consequences of belief manipulation. Also following the motivated cognition approach, Levy (2007) investigates policy issues in a reputational model on political accountability in which beliefs are endogenously manipulated by voters in an intrapersonal memory game.

There also exists a small literature on hindsight bias, or related biases, in the field of political economy and political science, see for example Johns (2006) on retrospective voting and crisis outcomes. Some recent experimental studies related to our question include Viscusi and Zeckhauser (2005) on recollection bias, Camerer *et al.* (1989) on curse of knowledge, and Biais and Weber (2006) investigate the connection between hindsight biased executives (stock brokers) and investment performance.

The remainder of this section discusses the psychological underpinning of hindsight bias and its literature.

### **Psychological foundations of the bias**

Hindsight bias characterizes a systematic way in which the evaluator's judgment about the likelihood of an event departs from perfect rationality, or as Rabin (1998, p. 30) puts it, "people exaggerate the degree to which their beliefs before an informative event would be similar to their current beliefs." In our model this means that after the evaluator observes the outcome, he overestimates the extent to which the realized event was foreseeable.<sup>1</sup> A hindsight biased voter who judges the quality of the politician's decision, violates basic decision making principles because in constructing a (biased) prior he mistakenly incorporates information - which was only known after the date of decision - into his evaluation process. In judgments of decisions under uncertainty, the bias blurs the two major elements

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<sup>1</sup> For an extensive review of the literature on hindsight bias consult Hawkins and Hastie (1990) or the meta-analysis by Guilbault *et al.* (2004). An early account on hindsight bias in the psychology literature is due to Fischhoff (1975).

he initially set out to distinguish in his evaluation, namely skill and luck of the decision maker.

In general, judgment biases are explained in psychology by motivational or cognitive theories. Motivational theories rationalize the existence of a judgment bias by a deliberate (subconscious) choice of the decision maker because he may derive a (psychological) benefit from it, e.g. in the sense of a self-serving bias. In cognitive theories, meanwhile, the bias is attributed to information processing effects. Since memory and reasoning is often affected by motivation, the judgment bias generally referred to as hindsight bias may be caused by a combination of motivational and cognitive effects.

From a theoretical modeling perspective, the bias design could therefore be based on the assumption of a boundedly rational memory model, in line with a pure cognitive approach as in Mullainathan (2002), or on the assumption of motivated cognition as in Bénabou and Tirole (2006) or Levy (2007), which would give the subject room for memory manipulation, or (motivated) manipulation of own beliefs about the world.

In our model an evaluator does not judge the quality of own past decisions but past decisions of others. Hence an important dimension of self-signaling (for example to justify own past decisions) does not apply in our framework. The only motivated self-signaling possible is, for the evaluator, to think he would have been a better decision maker than the decision maker he is supposed to judge. We assume that motivational self-serving effects (self-deception) play no role for voters in our problem of political agency and rather model the hindsight bias as a by-product of knowledge updating after outcome information was received, as advocated by Hoffrage *et al.* (2000).<sup>2</sup>

This section concludes by providing some details on the idea of bounded rationality we follow with respect to the formulation of hindsight bias. The psychological literature identifies two important sources for hindsight biased judgments (cf. Hawkins and Hastie (1990) and Hoffrage *et al.* (2000) ): 1) Motivated self-presentation influences an individual's estimation of the original ex ante prior because the individual derives a benefit from appearing smart in front of others or herself.<sup>3</sup> 2) Individuals have imperfect recall and

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<sup>2</sup> Note that we do not claim that motivational effects play no role for hindsight bias per se (they may very well increase the bias), but for simplicity we exclude this possible source of distortion in the current version of the paper. Furthermore, recent experimental studies indicate that strictly motivational motives do not appear to be the main cause of the bias we study in this paper.

<sup>3</sup> Motivational theories rationalize self-serving biases, but those biases are not restricted to self-image concerns. Hindsight bias can also be caused by affective or motivational suppression of changes in probability assessments over time because it may decrease individual perception of uncertainty in the world.

use a cognitive reconstruction process to infer the original prior. However, Hawkins and Hastie (1990) argue that the influence of motivated self-presentation may be small when compared to the impact imperfect memory may have on hindsight judgment.

Under the bounded rationality assumption on prior probabilities (imperfect recall), a subject has to follow some strategy to *reconstruct* it from the default (ex post) information which he now holds, this is what Hawkins and Hastie (1990) call “reconstruction of the prior judgment by ‘rejudging’ the outcome”. For the memory model we have in mind, we assume that an individual’s default memory consists of current, up to date probability estimates but does not stock prior probabilities formed in the past. Hoffrage *et al.* (2000) favor memory models because from today’s perspective, holding current information in memory is, for general tasks, more important and accurate than remembering past prior probabilities which are based on outdated information.<sup>4</sup>

The remainder of the paper is organized as follows. Section 2 introduces the main model while section 3 then establishes the benchmark rational policy evaluation equilibrium. In section 4 we define a hindsight bias information structure, we determine equilibrium under biased policy evaluation and compare it to the rational equilibrium. Selection and welfare implications of hindsight biased policy evaluation are studied in section 5. Finally, section 6 concludes. All proofs are delegated to the Appendix.

## 2 The basic model

In this section, we present a simple two-period political agency model. To model the effects of the strategic interaction between a decision maker (politician, agent) and a possibly biased evaluator (voter, principal) a binary state space, action space, signal space and outcome space is sufficient for our analysis.

The politician knows his type  $\theta \in \{\theta_H, \theta_L\}$ , and the prior probability  $\lambda_I$  (index  $I$  for the “incumbent”) of being of high ability ( $\theta = \theta_H$ ) is common knowledge. The state of the world is  $\omega \in \{0, 1\}$  with  $\Pr(\omega = 0) = \pi$  and  $\Pr(\omega = 1) = 1 - \pi$ , where  $\pi \in (0, 1)$ . Random variables  $\omega$  and  $\theta$  are independently distributed. Type- $\theta_H$  politicians learn  $\omega$  with certainty. Meanwhile, everybody, including type- $\theta_L$  politicians and voters receive an imperfect signal  $\sigma \in \{\sigma_0, \sigma_1\}$  about the state of the world. The distribution of this public

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<sup>4</sup> For memory-based models of bounded rationality in economic theory see Mullainathan (2002) and the literature cited therein.

signal conditional on  $\omega$  is given in Table 1, where  $x_0 \equiv Pr[\sigma_0|\omega = 0]$ .

	$\sigma_0$	$\sigma_1$
$\omega = 0$	$x_0$	$1 - x_0$
$\omega = 1$	$1 - x_1$	$x_1$

Table 1: Distribution of the signal

The politician selects an action  $a \in \{a_0, a_1\}$ . Policy  $a_0$  will be interpreted as maintaining the status quo and  $a_1$  as implementing a reform. The policy outcome (consequence) is  $y \in \{0, \Delta\}$ . Action  $a_0$  always yields a payoff of 0 to society. A reform policy costs  $c$  and delivers a payoff of  $\Delta$  with probability  $p$  (and 0 with probability  $1 - p$ ) if  $\omega = 1$ , and always yields 0 if  $\omega = 0$ . We assume  $p\Delta > c > 0$  so that  $a_1$  yields a higher expected payoff than  $a_0$  if and only if  $\omega = 1$ .

While voters only care about social welfare, politicians' preferences are given by

$$u = \phi W + (1 - \phi) \Pr[\text{reelection}],$$

where  $W$  is social welfare and  $\phi \in [0, 1]$  is a weighting factor. The game is played in two periods which are interpreted as terms in office. Period 1 is divided into four stages. At date  $t = 0$ , nature draws the incumbent's type  $\theta$  and the state of the world  $\omega$ . All types of politician, as well as voters, observe the public signal  $\sigma$ . Only type- $\theta_H$  politicians learn the state of the world  $\omega$ . At  $t = 1$ , the incumbent decides which policy  $a$  to implement. At date  $t = 2$ , the outcome of the policy is realized and learnt by all players. At date  $t = 3$ , the final stage of period 1, a challenger is drawn and the election takes place. The second term is exactly as the first except that the politician cannot be reelected, i.e. stage  $t = 3$  is omitted in the last period of the game.

In the election at the end of period 1, the voters choose between the incumbent and a challenger.<sup>5</sup> The probability that the challenger is of high ability is  $\lambda_C$ , drawn from a uniform distribution on  $[0, 1]$ . Voters learns  $\lambda_C$  before the election.

We impose the following assumption on the precision of the signal about the state of the world:

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<sup>5</sup> For simplicity we assume a representative voter in the sense of a pivotal median voter. This assumption also implies that politicians act as if confronted with homogeneous voters' beliefs, that is all voters hold the same beliefs about the government.



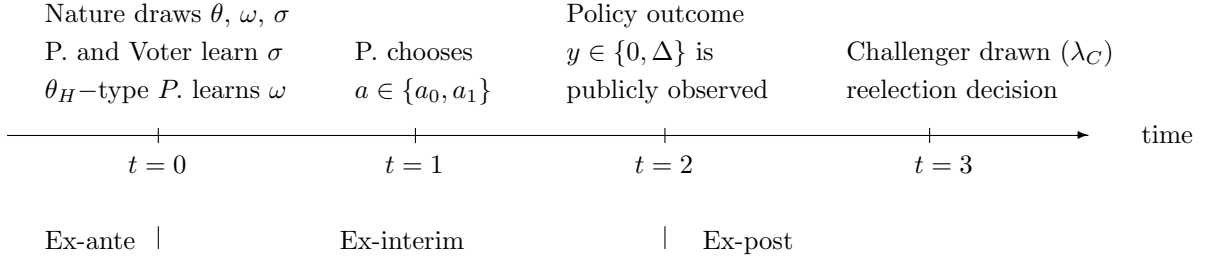


Figure 1: Timing of the game (term 1)

**Assumption 1** *The distribution of  $\sigma$  satisfies*

$$\frac{x_0}{1-x_1} > \frac{(1-\pi)(p\Delta - c)}{\pi c} > \frac{1-x_0}{x_1}.$$

This assumption ensures that, given a signal  $\sigma_\omega$ , it is welfare maximizing to implement policy  $a_\omega$ . Action  $a_0$  is optimal after observing  $\sigma_0$  if and only if

$$(1 - \nu_0)p\Delta - c < 0,$$

where  $\nu_0 \equiv \Pr[\omega = 0 | \sigma_0] = \frac{\pi x_0}{\pi x_0 + (1-\pi)(1-x_1)}$ . Action  $a_1$  is optimal after observing signal  $\sigma_1$  if and only if

$$\nu_1 p\Delta - c > 0,$$

where  $\nu_1 \equiv \Pr[\omega = 1 | \sigma_1] = \frac{(1-\pi)x_1}{(1-\pi)x_1 + \pi(1-x_0)}$ . Both conditions are implied by Assumption 1.

### 3 Equilibrium with rational voters

We now establish a perfect Bayesian equilibrium (PBE) of the above game under the assumption of complete rationality on the part of all players. We start by looking at the voters who – after observing a signal about the state of nature, the politician’s action, the outcome of the policy that is implemented, and the perceived ability of a challenger – determine whether the incumbent is reelected or replaced with the challenger. Voters’ payoff is given by their expected welfare. A strategy for the voters consists in a probability distribution over the actions “reelect the incumbent” and “elect the challenger” for each possible combination of signal, observed action, realized outcome, and the challenger’s perceived ability.

Their optimal strategy is quite simple. Since there are no reelection concerns in the second term, all politicians *try to maximize* welfare, but they are not all equally good at it. The voters' optimal strategy is therefore to elect the candidate they perceive as more competent.

Let  $\mu(\sigma, a, y)$  denote voters' posterior belief that the politician is of type  $\theta_H$  given the signal  $\sigma$ , the policy choice  $a$  and the realized outcome  $y \in \{0, \Delta\}$ . We will sometimes refer to  $\mu$  as the incumbent's reputation. Voters' optimal strategy is to reelect the incumbent if and only if  $\lambda_C \leq \mu(\sigma, a, y)$ . Since we have assumed that  $\lambda_C$  is uniformly distributed on  $[0, 1]$ , the probability that the incumbent is reelected is equal to the voters' posterior belief that he is of high ability.

We now turn to the politician. To begin, we introduce a bit of notation. The politician's payoff is given by his expected utility denoted  $U(\cdot)$ .<sup>6</sup> Let  $\alpha$  denote a mixed action such that the politician plays  $a_0$  with probability  $\alpha$  and  $a_1$  with probability  $1 - \alpha$ . Hence, expected utility given the voters' behavior and the information available to the politician is

$$U(\alpha, \mu, \Psi_\theta) = \alpha(1 - \phi)\mu(\sigma, a_0, 0) + (1 - \alpha)[\phi E(W|\Psi_\theta) + (1 - \phi)E(\mu(\sigma, a_1, y)|\Psi_\theta)],$$

where  $\Psi_\theta$  is the politician's information set which is type dependent and given by

$$\Psi_\theta = \begin{cases} (\omega, \sigma) & \text{for } \theta = \theta_H \\ \sigma & \text{for } \theta = \theta_L. \end{cases}$$

A strategy for the politician prescribes a probability  $s(\theta, \Psi_\theta)$  of playing  $a_0$  for each type  $\theta$  and for each possible realization of the information  $\Psi_\theta$ .

A PBE of this game is such that<sup>7</sup>

- strategies are optimal given beliefs, i.e.

$$\forall \theta, \forall \Psi_\theta, \quad s^*(\theta, \Psi_\theta) \in \arg \max_{\alpha} U(\alpha, \mu, \Psi_\theta)$$

and

- beliefs are derived from equilibrium strategies and observed actions using Bayes' rule, i.e.

$$\mu(\sigma, a, y) = \frac{\lambda_I \Pr[\sigma, a, y|\theta_H]}{\lambda_I \Pr[\sigma, a, y|\theta_H] + (1 - \lambda_I) \Pr[\sigma, a, y|\theta_L]}.$$

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<sup>6</sup> We assume risk neutrality.

<sup>7</sup> We omit the strategy of voters because of its simplicity.

In what follows, we will drop the politician's type from the specification of strategies as this cannot lead to confusion. Thus, we will write  $s(\omega, \sigma)$  instead of  $s(\theta_H, (\omega, \sigma))$  and  $s(\sigma)$  instead of  $s(\theta_L, \sigma)$ .

### Pure-strategy equilibrium

We start by looking for an equilibrium where the politician always implements the policy that, to the best of his knowledge, maximizes expected welfare. That is, a) the high type always chooses the policy that corresponds to the state of nature, and b) the low type always chooses the policy that corresponds to the signal he receives. Formally,  $s(0, \sigma) = 1$  and  $s(1, \sigma) = 0$  while  $s(\sigma_0) = 1$  and  $s(\sigma_1) = 0$ . In such an equilibrium, voters believe that a politician who chooses  $a_\omega$  in spite of a signal  $\sigma_{1-\omega}$  must be of type  $\theta_H$ .

Assume  $\sigma = \sigma_0$ , that is, the signal indicates that the state of the world is 0, and thus that policy  $a_0$  is optimal for welfare. Denote a low-ability politician's expected utility from playing  $a_0$  by  $U_0^0$ , where the superscript stands for the signal and the subscript for the policy chosen. We have

$$\begin{aligned} U_0^0 &= \phi \cdot 0 + (1 - \phi) \cdot \mu(\sigma_0, a_0, 0) \\ &= (1 - \phi) \frac{\lambda_I \pi x_0}{\lambda_I \pi x_0 + (1 - \lambda_I)[\pi x_0 + (1 - \pi)(1 - x_1)]} \\ &= (1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)/\nu_0}. \end{aligned}$$

Meanwhile, playing  $a_1$  yields  $U_1^0$  given by

$$U_1^0 = \phi[(1 - \nu_0)p\Delta - c] + (1 - \phi)[(\nu_0 + (1 - \nu_0)(1 - p))\mu(\sigma_0, a_1, 0) + (1 - \nu_0)p\mu(\sigma_0, a_1, \Delta)].$$

Since, by assumption,  $s(\sigma_0) = 1$ ,  $\mu(\sigma_0, a_1, y) = 1 \ \forall y$ . Hence, this simplifies to

$$U_1^0 = \phi[(1 - \nu_0)p\Delta - c] + (1 - \phi) \cdot 1.$$

For this to be an equilibrium, type  $\theta_L$  must indeed prefer playing  $a_0$  to playing  $a_1$ , that is, we need  $U_0^0 \geq U_1^0$ . But because  $\mu(\sigma_0, a_0, 0) < 1$ , this can be the case only if  $\nu_0$  is sufficiently large, i.e. if the signal is sufficiently precise.

Looking at the high-ability politician, we have to find conditions for him not to want to deviate, either. The interesting case is  $\omega = 0$ . For the politician to prefer  $a_0$  over  $a_1$ , we need

$$-\phi c + (1 - \phi) \cdot 1 \leq (1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)/\nu_0}.$$

This condition is automatically satisfied if the one for the low type is (if  $U_0^0 \geq U_1^0$ ).

Now assume that  $\sigma = \sigma_1$ , suggesting that a reform should be implemented. When the low-ability politician plays  $a_1$ , his expected utility is

$$U_1^1 = \phi(\nu_1 p \Delta - c) + (1 - \phi) \left[ (\nu_1(1 - p) + 1 - \nu_1) \mu(\sigma_1, a_1, 0) + \nu_1 p \mu(\sigma_1, a_1, \Delta) \right]$$

Using Bayes' rule, we have

$$\begin{aligned} \mu(\sigma_1, a_1, 0) &= \frac{\lambda_I(1 - \pi)x_1(1 - p)}{\lambda_I(1 - \pi)x_1(1 - p) + (1 - \lambda_I) \left[ (1 - \pi)x_1(1 - p) + \pi(1 - x_0) \right]} \\ &= \frac{\lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I) \left[ \frac{1}{\nu_1} - p \right]} \\ &\leq \lambda_I \end{aligned}$$

and

$$\begin{aligned} \mu(\sigma_1, a_1, \Delta) &= \frac{\lambda_I(1 - \pi)x_1 p}{\lambda_I(1 - \pi)x_1 p + (1 - \lambda_I)(1 - \pi)x_1 p} \\ &= \lambda_I. \end{aligned}$$

[Remark: Why do voters learn nothing in this case? Because  $y = \Delta$  indicates that  $\omega = 1$ , so that the high type and low type play the same strategy in this case (given  $\sigma = \sigma_1$ ).]

Hence, the politician's expected reputation when playing  $a_1$  is smaller or equal to  $\lambda_I < 1$ . Meanwhile, playing  $a_0$  would procure him utility

$$U_0^1 = (1 - \phi) \cdot 1,$$

which must be smaller or equal to  $U_1^1$  for this to be an equilibrium. Again, this is the case only if  $\nu_1$  is sufficiently large.

Moreover, type  $\theta_H$  must not have an incentive to deviate when the state of the world is  $\omega = 1$ . That is, it must be the case that

$$1 - \phi \leq \phi(p\Delta - c) + (1 - \phi) \left[ p\lambda_I + (1 - p) \frac{\lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I) \left[ \frac{1}{\nu_1} - p \right]} \right].$$

Once again, though, this is implied by the condition  $U_1^1 \geq U_0^1$  which ensures that the low type does not want to deviate.

**Lemma 1** Suppose  $\frac{1-\phi}{\phi}(1-\lambda_I) < c$ . Then there exists  $\nu_0^* < 1$  defined by

$$\phi[(1-\nu_0^*)p\Delta - c] = (1-\phi) \left[ \frac{\lambda_I}{\lambda_I + (1-\lambda_I)/\nu_0^*} - 1 \right]$$

such that, for any  $\nu_0 \geq \nu_0^*$ , the following strategy is an equilibrium when the signal is  $\sigma_0$ : the low-ability politician always chooses  $a_0$ , while the high-ability politician chooses  $a_0$  when  $\omega = 0$  and  $a_1$  when  $\omega = 1$ .

Similarly, if  $\frac{1-\phi}{\phi}(1-\lambda_I) < p\Delta - c$ , there exists  $\nu_1^* < 1$  defined by

$$\phi(\nu_1^*p\Delta - c) = (1-\phi) \left[ 1 - (1-\nu_1^*p) \frac{\lambda_I(1-p)}{\lambda_I(1-p) + (1-\lambda_I) \left[ \frac{1}{\nu_1^*} - p \right]} - \nu_1^*p\lambda_I \right]$$

such that, for any  $\nu_1 \geq \nu_1^*$ , the following strategy is an equilibrium when the signal is  $\sigma_1$ : the low-ability politician always chooses  $a_1$ , while the high-ability politician chooses  $a_0$  when  $\omega = 0$  and  $a_1$  when  $\omega = 1$ .

The proof, as well as those of all other propositions, can be found in Appendix A.

### Mixed-strategy equilibrium

We now examine what the equilibrium of the game is when the conditions for Lemma 1 to hold are not satisfied, that is, when the signal is not sufficiently precise ( $\nu_0 < \nu_0^*$  and/or  $\nu_1 < \nu_1^*$ ) or when reelection concerns are strong ( $\frac{1-\phi}{\phi}(1-\lambda_I) \geq c$  and/or  $\frac{1-\phi}{\phi}(1-\lambda_I) \geq p\Delta - c$ ). In this case, always following the signal is not an equilibrium. Consider the following alternative candidate equilibria: Type  $\theta_H$  always chooses the policy corresponding to the state of the world, but the  $\theta_L$ -type politician a) always does the opposite of what the signal suggests (so  $s(\sigma_0) = 1 - s(\sigma_1) = 0$ ), or b) randomizes between following the signal or not (in which case  $0 < s(\sigma) < 1 \ \forall \sigma$ ).<sup>8</sup>

The first of the two candidates, however, can be ruled out. To see why, note first that in this kind of equilibrium, voters believe that any politician who *does* follow the signal must be of high ability. Thus, the low type can increase his reputation by choosing the policy that the signal suggests. Moreover, in terms of expected social welfare, the low type is always better off following the signal. Therefore, choosing the “wrong” policy all the time cannot be an equilibrium.

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<sup>8</sup> There are other candidate equilibria, namely, pooling equilibria where both types of politician pool on one action (either  $a_0$  or  $a_1$ ) independent of their information, and voters attach pessimistic beliefs to deviations. In Appendix B, we demonstrate that these equilibria can be eliminated using the D1 criterion.

Conversely, we can conclude that when the signal is not too precise, an equilibrium where the high type always implements the “right” policy must have the low-ability politician randomizing between  $a_0$  and  $a_1$ . In order for him to be willing to do so, he must be indifferent between the two policies, that is, both must procure him equal utility in expectation. This requires that voters hold the appropriate beliefs.

Consider first the case where  $\sigma = \sigma_0$ . Playing  $a_0$  then yields

$$U_0^0 = (1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)s(\sigma_0)/\nu_0}, \quad (1)$$

while  $a_1$  again delivers

$$U_1^0 = \phi[(1 - \nu_0)p\Delta - c] + (1 - \phi) \left[ (\nu_0 + (1 - \nu_0)(1 - p)) \mu(\sigma_0, a_1, 0) + (1 - \nu_0)p \mu(\sigma_0, a_1, \Delta) \right], \quad (2)$$

but with

$$\begin{aligned} \mu(\sigma_0, a_1, 0) &= \frac{\lambda_I(1 - \pi)(1 - x_1)(1 - p)}{\lambda_I(1 - \pi)(1 - x_1)(1 - p) + (1 - \lambda_I)(1 - s(\sigma_0)) [\pi x_0 + (1 - \pi)(1 - x_1)(1 - p)]} \\ &= \frac{\lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s(\sigma_0)) \left[ \frac{1}{1 - \nu_0} - p \right]} \end{aligned}$$

and

$$\begin{aligned} \mu(\sigma_0, a_1, \Delta) &= \frac{\lambda_I(1 - \pi)(1 - x_1)p}{\lambda_I(1 - \pi)(1 - x_1)p + (1 - \lambda_I)(1 - s(\sigma_0))(1 - \pi)(1 - x_1)p} \\ &= \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s(\sigma_0))}. \end{aligned}$$

For the type  $\theta_L$  politician to be willing to randomize, voters’ beliefs must be such that  $U_0^0 = U_0^1$ . Moreover, these beliefs must be derived from strategies. Thus, the only  $s(\sigma_0)$  that constitutes an equilibrium is obtained by equating (1) and (2).

It follows immediately from this that the politician’s expected reputation must be greater when he chooses the opposite of what is suggested by the signal (and thus thought to be optimal by popular belief). This is because the first term in (2) is negative, implying that for  $U_1^0$  to be equal to  $U_0^0$ , the second term must compensate.

We now check that, given voters’ beliefs, the high-ability politician indeed finds it optimal to choose the policy corresponding to the state of nature. As before, the interesting case arises when  $\omega = 0$ : Although playing  $a_0$  is welfare-maximizing, playing  $a_1$  *might*

improve the politician's reputation. It turns out, though, that this is not the case. For type  $\theta_H$  to prefer  $a_0$ , it must be the case that

$$(1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)s(\sigma_0)/\nu_0} \geq -\phi c + (1 - \phi) \frac{\lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s(\sigma_0)) \left[ \frac{1}{1 - \nu_0} - p \right]},$$

an inequality that is implied by the equilibrium condition  $U_0^0 = U_1^0$ . Intuitively, if the low type is indifferent, the (better informed) high type must strictly prefer the “right” action.

Turning to the case where  $\sigma = \sigma_1$ , we have the following payoffs for the low-ability politician: Playing  $a_1$  yields

$$U_1^1 = \phi(\nu_1 p \Delta - c) + (1 - \phi) \left[ (\nu_1(1 - p) + 1 - \nu_1) \mu(\sigma_1, a_1, 0) + \nu_1 p \mu(\sigma_1, a_1, \Delta) \right]$$

with

$$\mu(\sigma_1, a_1, 0) = \frac{\lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s(\sigma_1)) \left[ \frac{1}{\nu_1} - p \right]}$$

and

$$\mu(\sigma_1, a_1, \Delta) = \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s(\sigma_1))}.$$

Meanwhile, playing  $a_0$  yields

$$\begin{aligned} U_0^1 &= (1 - \phi) \mu(\sigma_1, a_0, 0) \\ &= (1 - \phi) \frac{\lambda_I \pi (1 - x_0)}{\lambda_I \pi (1 - x_0) + (1 - \lambda_I) s(\sigma_1) [\pi (1 - x_0) + (1 - \pi) x_1]} \\ &= (1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I) s(\sigma_1) / (1 - \nu_1)}. \end{aligned}$$

We can again obtain the equilibrium  $s(\sigma_1)$  by equating  $U_1^1$  and  $U_0^1$ .

For type  $\theta_H$  not to have an incentive to deviate when  $\omega = 1$ , we need

$$\phi(p\Delta - c) + (1 - \phi) [(1 - p)\mu(\sigma_1, a_1, 0) + p\mu(\sigma_1, a_1, \Delta)] \geq (1 - \phi)\mu(\sigma_1, a_0, 0)$$

which is once again implied by  $U_1^1 = U_0^1$ .

**Lemma 2** *If the conditions for Lemma 1 are not satisfied, there exists an equilibrium such that the high-ability politician always chooses the policy corresponding to the state of the world, while the low-ability politician randomizes between  $a_0$  and  $a_1$ . When the signal is*

$\sigma_0$ , the equilibrium probability of playing  $a_0$ ,  $s^*(\sigma_0)$ , is determined by

$$(1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)s^*(\sigma_0)/\nu_0} = \phi[(1 - \nu_0)p\Delta - c] + \\ + (1 - \phi) \left[ \frac{(\nu_0 + (1 - \nu_0)(1 - p)) \lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s^*(\sigma_0)) \left[ \frac{1}{1 - \nu_0} - p \right]} + \frac{(1 - \nu_0)p \lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_0))} \right].$$

When the signal is  $\sigma_1$ ,  $s^*(\sigma_1)$  is determined by

$$(1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)s^*(\sigma_1)/(1 - \nu_1)} = \phi(\nu_1 p \Delta - c) + \\ + (1 - \phi) \left[ \frac{(1 - \nu_1 p) \lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s^*(\sigma_1)) \left[ \frac{1}{\nu_1} - p \right]} + \frac{\nu_1 p \lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))} \right].$$

$s^*(\sigma_0)$  is increasing with  $\nu_0$ .  $s^*(\sigma_1)$  is decreasing with  $\nu_1$ .

**Corollary 1** *The probability that type  $\theta_L$  plays  $a_0$  ( $a_1$ ) is greater (smaller) after receiving signal  $\sigma_0$  than after receiving signal  $\sigma_1$ :  $s^*(\sigma_0) > s^*(\sigma_1)$ .*

This follows directly from Assumption 1 (which implies that  $\nu_1 > 1 - \nu_0$ ) and the monotonicity properties of the equilibrium strategies, noting that  $s^*(\sigma_0)$  would equal  $s^*(\sigma_1)$  if  $\nu_1 = 1 - \nu_0$ .

## 4 Hindsight bias as a discipline device

Voters are hindsight biased if their recollection of the prior they hold about the state of the world diverges from their actual prior once they have learned new information about which state of the world truly prevails. Hindsight biased evaluators are unable to ignore their additional information when trying to recall their original judgment. Using a fairly general notation, the bias can be formalized as follows (Camerer *et al.*, 1989; Biais and Weber, 2006):

$$E[E(\omega|\sigma)|\sigma, a, y] = bE(\omega|\sigma, a, y) + (1 - b)E(\omega|\sigma),$$

where  $b \in [0, 1]$  measures the degree of hindsight bias. Thus, the hindsight bias translates into a violation of the law of iterated expectations, and the recalled prior is assumed to be somewhere between the true prior and the posterior.



However in the binary model, there are only two possible priors, one for each realization of the signal:  $E(\omega|\sigma_0) = 1 - \nu_0$  and  $E(\omega|\sigma_1) = \nu_1$ . Thus, if we do not put any restrictions on  $b$ , the voters' recollection of their prior would not be consistent with *any* possible prior they might have had. For the purposes of our argument, we therefore adopt a formalization of hindsight bias that is extremely crude but has the advantage of being consistent with the beliefs voters may actually hold.

**Definition 1 (Hindsight bias with a binary signal)** *Suppose  $\sigma = \sigma_0$ , so that voters' prior belief that  $\omega = 1$  is  $1 - \nu_0$ . After learning that the true state of the world is 1, hindsight biased voters erroneously believe that their prior belief about the state of the world being  $\omega = 1$  was  $\nu_1 > 1 - \nu_0$ , and thus that the signal was  $\sigma_1$  rather than  $\sigma_0$ .*

The bias alters the voter's *recalled prior* in direction of the actually observed outcome  $y$ . The outcome changes – ex post (after outcome realization) – the recollection of the ex interim signal  $\sigma$  to  $\sigma^B$ . We assume, however, that this happens only in the case in which  $\sigma = \sigma_0$  and  $y = \Delta$ . In this case, the posterior belief about the state of the world is  $E(\omega|\sigma_0, a_1, \Delta) = 1$ , that is, voters are sure that the state was  $\omega = 1$ . We can then identify the parameter  $b$ , which implicitly underlies our setup, as

$$\nu_1 = b \cdot 1 + (1 - b)(1 - \nu_0) \iff b = \frac{\nu_1 + \nu_0 - 1}{\nu_0}.$$

We further assume that no distortion occurs in the other potentially bias prone case: when the signal is  $\sigma_1$ , the politician chooses  $a_1$ , and  $y = 0$ . We make this assumption for the simple reason that in this case, it is far from sure that the posterior belief,  $E(\omega|\sigma_1, a_1, 0)$ , would be lower than the prior,  $\nu_1$  – let alone lower than  $1 - \nu_0$ , which is the candidate for the hindsight biased recall.<sup>9</sup> Thus, if we assume that the voter changes the recalled signal from  $\sigma_1$  to  $\sigma_0$ , we might very well implicitly be assuming  $b > 1$ . To avoid this, we rule out any distortion of the recollection when  $\sigma = \sigma_1$ .<sup>10</sup>

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<sup>9</sup> We can compute

$$E(\omega|\sigma_1, a_1, 0) = \frac{(1 - \pi)x_1(1 - p)[\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))]}{\lambda_I(1 - \pi)x_1(1 - p) + (1 - \lambda_I)(1 - s^*(\sigma_1))[\pi(1 - x_0) + (1 - \pi)x_1(1 - p)]}.$$

This expression is increasing in  $s^*(\sigma_1)$ . Thus, the posterior about  $\omega$  is lowest when  $s^*(\sigma_1) = 0$ . Even in that case, however,  $E(\omega|\sigma_1, a_1, 0) < E(\omega|\sigma_1) = \nu_1$  only if  $\lambda_I < p$  (for which there is *a priori* no reason), and an even stronger condition is required to have  $E(\omega|\sigma_1, a_1, 0) < 1 - \nu_0$ .

<sup>10</sup> If we would introduce a similar bias in case  $\sigma = \sigma_1$  as assumed in case  $\sigma = \sigma_0$ , the constructed bias would be consistent with a pure *outcome* bias.

Table 2 summarizes a voter’s ex interim recollection of the ex ante signal in a binary world. Although the bias changes a voter’s recollection of the perceived state of the world  $\sigma$  only in a single case, it is nonetheless an extreme form of bias since the signal changes from  $\sigma_0$  to  $\sigma_1$  with probability one.<sup>11</sup>

As argued in the introduction, we assume the voter to be boundedly rational in the sense of imperfect recall. That is the evaluator cannot recall the original prior probability about the state of the world, and the bias then results from the recollection process in which a biased estimate of the prior is formed, given the evaluator’s default (ex post) information set. The bias influences the evaluator’s judgment about the quality of the politician’s decision. It is also interesting to note that the bias hinders conscious learning, conscious in the sense that the evaluator is not fully aware of his change of the probability assessment of the state of the world. This implies a reduction of surprises of any kind for the evaluator.

In solving for the equilibrium of the game with hindsight biased voters, we maintain the concept of PBE to the extent possible. We assume that politicians anticipate the voters’ hindsight bias, and that voters can compute the politician’s equilibrium strategy. The latter, of course, requires that voters be aware of their hindsight bias *ex ante*. Voters’ beliefs are based on the politician’s strategy, which they correctly anticipate. They are derived from the equilibrium strategy that the voters think the politician should have used given their recollection of the signal. Thus, voters may hold incorrect posterior beliefs, but the mistake stems solely from the erroneous recollection of the prior that is associated with hindsight bias and not from wrong expectations about strategies.

These assumptions are somewhat peculiar because in a sense we suppose that voters are *sophisticated* ex ante (they know that they are hindsight biased and use this knowledge to infer equilibrium strategies), but *naive* ex interim: When their recollected signal is  $\sigma_1$ , they do not consider the possibility that this may be due to their hindsight bias. While this may

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<sup>11</sup> According to this definition of the bias, it is independent of any action taken by a decision maker.

	$\sigma = \sigma_0$	$\sigma = \sigma_1$
$y = 0$	$\sigma_0$	$\sigma_1$
$y = \Delta$	$\sigma_1^B$	$\sigma_1$

Table 2: Recollection of priors

appear as a strong assumption, it is in line with psychological research which has shown that people are unable to avoid the hindsight bias even when they have been warned about it (for example before the experiment takes place). This being said, it would certainly be a more balanced approach to allow for sophisticated behavior at the interim stage too, i.e. that voters are aware that their recollection may be distorted due to hindsight bias.<sup>12</sup> We have chosen not to pursue such a path for reasons of tractability; nevertheless, we conjecture that the qualitative results should go through.

As in the rational equilibrium section, we have to look at different equilibrium candidates for the case of biased voters, and this – in principle – in each of the two subgames corresponding to the possible realizations of the signal,  $\sigma_0$  and  $\sigma_1$ . Note however that, since we assume that the hindsight bias has no bite when the signal is  $\sigma_1$ , the corresponding subgame is unchanged: When  $\sigma = \sigma_1$ , the equilibrium strategies,  $s^*(\sigma_1)$  for the low type and  $s^*(\omega, \sigma_1)$  for the high type, are the same as in the equilibrium with rational voters, specified in Lemmas 1 and 2. That is, type  $\theta_H$  always enacts the policy corresponding to the state of the world, while type  $\theta_L$  always chooses  $a_1$  if the precision of the signal,  $\nu_1$ , is sufficiently high, and randomizes between  $a_0$  and  $a_1$  if  $\nu_1$  is low. In what follows, we thus only have to analyze the more interesting case where  $\sigma = \sigma_0$ .

Another important difference to the rational case is that the subgame for  $\sigma_0$  is no longer independent of the subgame for  $\sigma_1$ . In our setting, a hindsight biased evaluator believes ex post that he would have estimated a higher prior probability for the state of the world than he actually did ex ante. This means the evaluator calculates his posterior about the politician's type with the biased prior probability. As will become clear, this means that posterior beliefs – and hence, equilibrium strategies – in the  $\sigma_0$  case depend on the equilibrium strategy in the  $\sigma_1$  case,  $s^*(\sigma_1)$ . Note that the reverse is not true given the adopted formulation of hindsight bias. Thus, when deriving the equilibrium for  $\sigma_0$ , one has to consider all possible equilibria that might arise in the  $\sigma_1$  subgame.

**Pure-strategy equilibrium.** We first look at the possible equilibrium in which type  $\theta_H$  follows the state of the world and plays  $s(0, \sigma_0) = 1$  and  $s(1, \sigma_0) = 0$ , while type  $\theta_L$  always follows his signal about the state of the world and thus chooses action  $a_0$  with probability 1, i.e.  $s(\sigma_0) = 1$ . We now check under which conditions this is an equilibrium for each of the

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<sup>12</sup> For a model where individuals forget or repress information, but are aware of the deficiencies of their memory, see Bénabou and Tirole (2002).

three possible types: Equilibrium requires that type  $\theta_L$  prefers playing  $a_0$ , type  $(\theta_H, \omega = 0)$  prefers playing  $a_0$  and type  $(\theta_H, \omega = 1)$  prefers playing  $a_1$ . The expected utility for the low type is the same as in the fully rational case if he plays  $a_0$ , that is,

$$U_0^0 = \phi \cdot 0 + (1 - \phi) \cdot \mu(\sigma_0, a_0, 0),$$

with

$$\mu(\sigma_0, a_0, 0) = \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)/\nu_0}.$$

Let the equilibrium strategy for the low type in the  $\sigma_1$  subgame be  $s^*(\sigma_1)$ . Then, playing the deviating action  $a_1$  yields  $U_1^0$ , given by

$$U_1^0 = \phi[(1 - \nu_0)p\Delta - c] + (1 - \phi)[(\nu_0 + (1 - \nu_0)(1 - p))\mu(\sigma_0, a_1, 0) + (1 - \nu_0)p\mu(\sigma_1^B, a_1, \Delta)],$$

with the posteriors

$$\begin{aligned} \mu(\sigma_0, a_1, 0) &= 1 \\ \mu(\sigma_1^B, a_1, \Delta) &= \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))}. \end{aligned}$$

This is where the hindsight bias comes in, as indicated by the superscript  $B$ . Upon observing outcome  $y = \Delta$ , voters learn that the state of the world is  $\omega = 1$ , and distort their recollection of the prior belief, which was based on  $\sigma_0$ , towards their ex post information by wrongly believing that the signal had been  $\sigma_1$ . The above posterior beliefs make clear how hindsight bias reduces the low type's incentives to deviate: while in the rational case, a politician's reputation from achieving outcome  $\Delta$  despite  $\sigma_0$  is equal to 1, it is strictly lower than 1 in the hindsight biased case. This is because hindsight biased voters consider the outcome  $\Delta$  more predictable than it actually was; in retrospect they think that  $\omega = 1$  had been more likely ex ante than was truly the case, and thus believe that playing  $a_1$  was an obvious choice that even the low type should have made with strictly positive probability (given by  $1 - s^*(\sigma_1)$ ).

For this to be an equilibrium for type  $\theta_L$ , we need  $U_0^0 > U_1^0$  to hold, which is

$$\begin{aligned} (1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)/\nu_0} &> \phi[(1 - \nu_0)p\Delta - c] + \\ &+ (1 - \phi) \left[ (\nu_0 + (1 - \nu_0)(1 - p)) \cdot 1 + (1 - \nu_0)p \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))} \right]. \end{aligned} \quad (3)$$

Looking at a decision maker of type  $\theta_H$ , we derive the conditions under which he does not want to deviate from the above stated equilibrium candidate; remember that type  $\theta_H$  has an informational advantage with respect to the low type and the evaluator since he observes both the signal and the state of the world. Here, unlike in the rational case, both states of the world need to be checked.

Based on the posterior beliefs specified above, a high-ability politician prefers playing  $a_0$  when  $\omega = 0$  if and only if

$$(1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)/\nu_0} > -\phi c + 1 - \phi. \quad (4)$$

Similarly, when  $\omega = 1$ , type  $\theta_H$  prefers playing  $a_1$  if and only if

$$(1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)/\nu_0} < \phi(p\Delta - c) + (1 - \phi) \left[ 1 - p + \frac{p \lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))} \right]. \quad (5)$$

The following lemma summarizes the conditions under which the specified set of strategies and beliefs is an equilibrium.

**Lemma 3** *Suppose voters are hindsight biased. If  $\frac{1-\phi}{\phi}(1 - \lambda_I) < c$ , there exists  $\hat{\nu}_0$  defined by*

$$\phi[(1 - \hat{\nu}_0)p\Delta - c] = (1 - \phi) \left[ \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)/\hat{\nu}_0} - 1 + (1 - \hat{\nu}_0)p \frac{(1 - \lambda_I)(1 - s^*(\sigma_1))}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))} \right]$$

*such that, for any  $\nu_0 > \hat{\nu}_0$ , the equilibrium strategy when the signal is  $\sigma_0$  is the following: the low-ability politician always chooses  $a_0$ , while the high-ability politician chooses  $a_0$  when  $\omega = 0$  and  $a_1$  when  $\omega = 1$ .*

With the results established in Lemma 3, we can now state the first proposition.

**Proposition 1** *Hindsight bias decreases the level of signal precision required for a pure-strategy equilibrium:  $\nu_0^* > \hat{\nu}_0$ .*

Proposition 1 says that the threshold value of signal precision required for a pure strategy equilibrium is lower when voters are hindsight biased than when they are fully rational. Because voters that suffer from the bias are less easily impressed by deviating behavior (if successful, they think they saw it coming), the low-ability politician has less incentive to play  $a_1$  when the signal suggests the opposite ( $\sigma_0$ ). Thus, the signal precision required for him to be disciplined and follow the signal is reduced compared to the rational case.

**Mixed-strategy equilibrium.** We now turn to the candidate equilibrium in which a  $\theta_L$ -type randomizes between the two actions  $a_0$  and  $a_1$  while a  $\theta_H$ -type plays the same pure strategy as before, that is given the state of the world he always chooses the optimal policy. For the  $\theta_L$ -type politician to be willing to randomize between action  $a_0$  and  $a_1$ , he must get the same expected utility from each action ( $U_0^0 = U_1^0$ ), which can be written as

$$(1 - \phi)\mu(\sigma_0, a_0, 0) = \phi[(1 - \nu_0)p\Delta - c] + \\ + (1 - \phi)[(\nu_0 + (1 - \nu_0)(1 - p))\mu(\sigma_0, a_1, 0) + (1 - \nu_0)p\mu(\sigma_1^B, a_1, \Delta)], \quad (6)$$

with posterior beliefs,

$$\mu(\sigma_0, a_0, 0) = \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)s(\sigma_0)/\nu_0} \\ \mu(\sigma_0, a_1, 0) = \frac{\lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s(\sigma_0)) \left[ \frac{1}{1 - \nu_0} - p \right]} \\ \mu(\sigma_1^B, a_1, \Delta) = \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))}.$$

As in the case of rational voters, the condition  $U_0^0 = U_1^0$ , equation (6), determines the equilibrium strategy  $s_B^*(\sigma_0)$ .

For the  $\theta_H$ -type, he must prefer playing  $a_0$  when  $\omega = 0$ . This is true as long as

$$(1 - \phi)\frac{\lambda_I}{\lambda_I + (1 - \lambda_I)s(\sigma_0)/\nu_0} > -\phi c + \\ + (1 - \phi)\frac{\lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s(\sigma_0)) \left[ \frac{1}{1 - \nu_0} - p \right]}. \quad (7)$$

In addition, he must prefer playing  $a_1$  when  $\omega = 1$ , which requires

$$(1 - \phi)\frac{\lambda_I}{\lambda_I + (1 - \lambda_I)s(\sigma_0)/\nu_0} < \phi(p\Delta - c) + \\ + (1 - \phi) \left[ \frac{\lambda_I(1 - p)^2}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s(\sigma_0)) \left[ \frac{1}{1 - \nu_0} - p \right]} + \frac{p\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))} \right]. \quad (8)$$

We now introduce an assumption on the relative importance of welfare and reputation.

**Assumption 2** *The payoff from a successful reform satisfies*

$$\Delta > \frac{1 - \phi}{\phi}(1 - \lambda_I).$$

This assumption says that the politician's payoff (in terms of welfare) from implementing a successful reform policy must exceed the maximum reputational loss due to hindsight bias. It ensures that the informational advantage of the high-ability politician works in favor of the welfare-maximizing policy even in the presence of hindsight bias.

**Lemma 4** *Suppose voters are hindsight biased. If the conditions for Lemma 3 are not satisfied, and under assumption 2, there exists an equilibrium such that the high-ability politician always chooses the policy corresponding to the state of the world, while the low-ability politician randomizes between  $a_0$  and  $a_1$ . The equilibrium probability of playing  $a_0$  is  $s_B^*(\sigma_0)$  determined by*

$$(1 - \phi) \frac{\lambda_I}{\lambda_I + (1 - \lambda_I) s_B^*(\sigma_0) / \nu_0} = \phi [(1 - \nu_0) p \Delta - c] + \\ + (1 - \phi) \left[ \frac{(\nu_0 + (1 - \nu_0)(1 - p)) \lambda_I (1 - p)}{\lambda_I (1 - p) + (1 - \lambda_I) (1 - s_B^*(\sigma_0)) \left[ \frac{1}{1 - \nu_0} - p \right]} + \frac{(1 - \nu_0) p \lambda_I}{\lambda_I + (1 - \lambda_I) (1 - s^*(\sigma_1))} \right].$$

We are now able to assess whether hindsight bias improves the low-ability politician's decision making. The next proposition states our main result regarding discipline.

**Proposition 2** *Suppose Assumption 2 holds. For any  $\nu_0 < \nu_0^*$ , hindsight bias improves the low-ability politician's discipline:  $s_B^*(\sigma_0) > s^*(\sigma_0)$ .*

Proposition 2 means that, in terms of first-period welfare, voters may benefit from being hindsight biased. Anticipating the voters' bias, the low-ability politician knows that he has relatively little to gain from deviating to reform, and accordingly, will do so less often. The intuition for this result is the following. We know that the low type is always more likely to choose reform after observing  $\sigma_1$  than after observing  $\sigma_0$ . When in spite of a signal  $\sigma_0$ , the politician chooses reform and succeeds, hindsight biased voters change their recollection of the signal to  $\sigma_1$ , and thus believe that the low type should have played reform with a higher probability than was actually the case (given  $\sigma_0$ ). As a result, voters exaggerate the likelihood that the observed event came from a low type, which reduces their esteem for the incumbent. For the low-ability politician, deviation is therefore associated with diminished reelection prospects compared to the rational voter case.

## 5 Selection and welfare

Propositions 1 and 2 show that hindsight bias improves the discipline of the (low-ability) incumbent. Therefore, the effect of hindsight bias on voters' first-period welfare is unambiguously positive. To make a general statement about the welfare consequences of hindsight bias, however, we must also take into account second-period welfare. This means we have to investigate the effect on selection: Because in the second period, the politician always implements the policy that, according to the information he possesses, is best, voters are always weakly better off in the second period if a high-ability politician is in office.

The effect of hindsight bias on selection works through two channels. The first is that voters sometimes have erroneous posteriors, so that they don't always elect the politician that is truly more able (in expected terms). The second is more indirect: since the anticipation of voters' hindsight bias changes the low-ability politician's behavior, the inferences that can be drawn from a given event are modified too.

While the first effect clearly is bad for welfare, the second is more complex. Hindsight bias increases the low type's equilibrium probability of playing  $a_0$  after observing  $\sigma_0$ . This means that the politician's reputation when playing  $a_0$  decreases, while his reputation when playing  $a_1$  increases, and this regardless of his type. Thus, for instance, the  $(\theta_H, \omega = 0)$  type is less likely and the  $(\theta_H, \omega = 1)$  type more likely to be reelected, which has conflicting welfare implications.

Because of the ambiguous effects of improved discipline on selection, performing a general welfare analysis is a daunting task. In this section, we instead opt for a less ambitious approach that consists in separately considering the reelection chances of low-ability and high-ability politicians. Obviously, second-period welfare is higher if, *ceteris paribus*, high types are reelected more often and low types less often. Thus, if both go in the right direction – that is, if we are able to show that, for example, hindsight bias improves the reelection chances of the high-ability politician while hurting those of the low-ability one – then we can make a clear welfare statement. However, this is only a sufficient and not a necessary condition, and it turns out that both effects do not typically point in the same direction so that we are unable to make such a statement for the general case.

We consider first the  $\theta_L$ -type politician. His *ex ante* probability of reelection when the



signal is  $\sigma_0$ , which we denote  $\mathcal{R}_L$ , is given by

$$\mathcal{R}_L = s(\sigma_0) \mu(\sigma_0, a_0, 0) + (1 - s(\sigma_0)) [(1 - p(1 - \nu_0)) \mu(\sigma_0, a_1, 0) + p(1 - \nu_0) \mu(\sigma_0, a_1, \Delta)]$$

To simplify notation, let us denote  $\mu_0 \equiv \mu(\sigma_0, a_0, 0)$ ,  $\mu_{10} \equiv \mu(\sigma_0, a_1, 0)$  and  $\mu_{1\Delta} \equiv \mu(\sigma_0, a_1, \Delta)$ . In addition, we use the superscript  $R$  (respectively,  $B$ ) to indicate that we are evaluating posterior beliefs at the equilibrium strategy ( $s^*$ ) in the presence of rational (hindsight biased) voters.

**Lemma 5** *If  $\nu_0 < \nu_0^*$ , the following inequalities hold for the politician's reputation:*

$$\mu_0^R > \mu_0^B \tag{9}$$

$$\mu_{10}^R < \mu_{10}^B \tag{10}$$

$$\mu_{1\Delta}^R > \mu_{1\Delta}^B. \tag{11}$$

We are now ready to compare the low type's reelection chances in the presence of rational and hindsight biased voters.

**Proposition 3** *Suppose  $\nu_0 < \nu_0^*$ . The low-ability politician is less likely to be reelected when facing hindsight biased voters than when facing rational voters:  $\mathcal{R}_L^B < \mathcal{R}_L^R$ .*

Thus, the result for the low-ability politician is unambiguous: his reelection chances are hurt by the voters' bias. To see why this is the case, note first that the low type's expected utility must be equal to  $U_0^0 = (1 - \phi)\mu_0$  (if he is to play a mixed strategy, both  $a_0$  and  $a_1$  must provide him with the same utility). His expected utility is a weighted sum of two parts: expected welfare when he reforms (negative because the signal is  $\sigma_0$ ) times the probability of reforming, and his reelection prospects,  $\mathcal{R}_L$ . Expected utility is greater with rational voters because  $\mu_0$  is larger. At the same time, the welfare part is more negative because  $s(\sigma_0)$  is smaller. To compensate for this, the probability of reelection must necessarily be higher with rational voters.

We now turn to the  $\theta_H$  type whose probability of reelection is

$$\mathcal{R}_H = \nu_0 \mu_0 + (1 - \nu_0) [(1 - p) \mu_{10} + p \mu_{1\Delta}]. \tag{12}$$

It is difficult to evaluate this expression in general. The next proposition looks at a special case.

**Proposition 4** *Suppose  $p = 1$  and that out-of-equilibrium beliefs are pessimistic. Then, the high-ability politician is less likely to be reelected when facing hindsight biased voters than when facing rational voters ( $\mathcal{R}_H^B < \mathcal{R}_H^R$ ); hindsight bias increases political turnover.*

In the special case where  $p = 1$  (meaning that reform always succeeds when  $\omega = 1$ ) the high type never faces the possibility of failing and thus having reputation  $\mu_{10}$ . Since in the two remaining cases, rational voters have a higher opinion of the politician than hindsight biased ones ( $\mu_0^R > \mu_0^B$  and  $\mu_{1\Delta}^R > \mu_{1\Delta}^B$ ), the high type stands to lose from hindsight bias.

For the case  $p = 1$ , our model therefore predicts that political turnover – defined as the rate of replacement of the politician holding office – is larger when voters are hindsight biased. The reason is that both the low and the high type are less likely to be reelected. This result is in line with conventional wisdom which holds that, when evaluating somebody else’s performance, a person suffering from hindsight bias gives less credit than is due in case of success, and more blame than is warranted in case of failure.

The result also means that our analysis of the overall effect of hindsight bias on second-period welfare is inconclusive. We nevertheless point out that hindsight bias can be welfare-enhancing regardless of what happens in the second period: because voters discount the future, discipline is more important than selection for a sufficiently low discount factor.

## 6 Conclusion

We have constructed a political agency model where voters exhibit a cognitive deficiency known as hindsight bias: after the uncertainty about an event is resolved, they think that the realized outcome was more foreseeable than it actually was. In our model, voters have to evaluate the incumbent politician in order to decide whether to reelect him or replace him with a challenger. Politicians are assumed to differ in their ability, where ability is taken to mean the quality of the information they have about the welfare-maximizing policy. In this setup, low-ability politicians have incentives to disregard public information on what the optimal policy is in order to appear to have superior private information, as high-ability politicians do. We have shown that, in this context, hindsight bias on the part of voters can act as a discipline device. This is because hindsight biased voters are less easily impressed by a successful reform – they think it was the obvious choice to make from the outset, even if the available information had actually suggested otherwise. Therefore,

they give an incumbent who succeeds with a reform policy in spite of public pessimism less credit than rational voters who perfectly recall their prior. Anticipating this, low-ability politicians are less likely to deviate from the welfare-maximizing policy.

The disciplining effect of hindsight bias is unambiguously beneficial for voters' first-period welfare. However, an overall welfare assessment also has to take into account the second (i.e., post-election) period. We have analyzed how hindsight bias affects the selection of the second-period official and shown that, under some conditions, both the low- and the high-ability politician are less likely to be reelected when voters are hindsight biased than when they are rational. This suggests that it may very well be the case that hindsight bias does not serve voters well in terms of second-period welfare. These qualifications notwithstanding, hindsight bias can be welfare-enhancing no matter what if voters discount future payoffs at a sufficiently high rate.

Our framework may be applicable to problems other than the political economy issues we have studied here. For example, our analysis may be relevant for promotion decisions in organizations (which, much like democratic elections, do not follow rules set forth in an explicit *ex ante* contract). Consider a human resource department that has to decide whether to promote an employee from inside the firm, whose actions and performance have been observed, or to hire an outsider for the job. In a firm, there typically will be some amount of public information concerning the right action to take, but employees may also have superior information on their specific assignment. Our model would predict that, if anticipated, hindsight bias on the part of the human resource manager may prevent low-ability employees from deviating to suboptimal actions in order to appear smart, but not necessarily help in choosing the right candidate.

We close by noting that, with the benefit of hindsight, all of our results are, of course, obvious.

# Appendix

## A Proofs of propositions

### Proof of Lemma 1:

The assumption that  $(1 - \phi)(1 - \lambda_I) < \phi c$  means that in case  $\sigma = \sigma_0$ , neither type has an incentive to deviate for  $\nu_0 = 1$ . By continuity,  $U_0^0 > U_1^0$  for  $\nu_0$  close to 1, so that for those values of  $\nu_0$ , type  $\theta_L$  doesn't want to deviate. This, in turn, implies that type  $\theta_H$  has no incentive to deviate either. By contrast, we know from the above discussion that for  $\nu_0$  close to  $1 - \frac{c}{p\Delta}$ , and given the voters' beliefs, the low type strictly prefers to deviate and play  $a_1$ . Since the expressions of  $U_0^0$  and  $U_1^0$  are monotonic in  $\nu_0$ , it follows that there must be a cutoff  $\nu_0^*$  above which the specified set of strategies and beliefs is an equilibrium. The proof in the case where  $\sigma = \sigma_1$  is analogous. ■

### Proof of Lemma 2:

It follows from Lemma 1 that if  $\frac{1-\phi}{\phi}(1-\lambda_I) \geq c$  or if  $\nu_0 < \nu_0^*$  (respectively, if  $\frac{1-\phi}{\phi}(1-\lambda_I) \geq p\Delta - c$  or if  $\nu_1 < \nu_1^*$ ), it is not an equilibrium for the low type to play a pure strategy where he follows the signal, i.e.  $s(\sigma_0) = 1$  (respectively,  $s(\sigma_1) = 0$ ). Since the low type playing  $a_1$  with probability 1 when  $\sigma = \sigma_0$  ( $a_0$  with probability 1 when  $\sigma = \sigma_1$ ) can never be an equilibrium (the politician could increase both welfare and his reputation by deviating to  $a_0$  ( $a_1$ )), the only possible equilibrium has the low type randomizing. For a mixed action to be optimal, the politician must be indifferent between playing  $a_0$  and playing  $a_1$ . Thus, voters must hold beliefs which ensure that  $U_0^0 = U_1^0$  ( $U_0^1 = U_1^1$ ); moreover, these beliefs must be derived from equilibrium strategies. Thus, the only  $s(\sigma_0)$  ( $s(\sigma_1)$ ) that constitutes an equilibrium is the claimed one.

As argued in the text, the high type has no incentive to deviate because his informational advantage makes sure that whenever the low type is indifferent between  $a_0$  and  $a_1$ , he strictly prefers playing the policy corresponding to the state of the world.

Finally, we prove the claimed monotonicity properties of the equilibrium strategies. These follow from applying the implicit function theorem. Let  $F_0 \equiv U_0^0 - U_1^0$  and  $F_1 \equiv$

$U_0^1 - U_1^1$ . We have

$$\begin{aligned}\frac{\partial s^*(\sigma_0)}{\partial \nu_0} &= -\frac{\partial F_0/\partial \nu_0}{\partial F_0/\partial s} \\ \frac{\partial s^*(\sigma_1)}{\partial \nu_1} &= -\frac{\partial F_1/\partial \nu_1}{\partial F_1/\partial s}.\end{aligned}$$

It is straightforward to see that  $\partial F_0/\partial s < 0$  and  $\partial F_1/\partial s < 0$ . By contrast, the sign of  $\partial F_0/\partial \nu_0$  and  $\partial F_1/\partial \nu_1$  is *a priori* ambiguous. Computations yield, respectively:

$$\begin{aligned}\frac{\partial F_0}{\partial \nu_0} &= \phi p \Delta + (1 - \phi) \lambda_I \left[ \frac{s^*(\sigma_0)(1 - \lambda_I)}{\nu_0^2 (\lambda_I + (1 - \lambda_I) s^*(\sigma_0)/\nu_0)} + \right. \\ &\quad \left. + \frac{(1 - p)(1 - s^*(\sigma_0))(1 - p(1 - \nu_0))(1 - \lambda_I)}{(1 - \nu_0)^2 [\lambda_I(1 - p) + (1 - \lambda_I)(1 - s^*(\sigma_0))(1/(1 - \nu_0) - p)]^2} - \right. \\ &\quad \left. + \frac{p}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_0))} - \frac{(1 - p)p}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s^*(\sigma_0))(1/(1 - \nu_0) - p)} \right],\end{aligned}$$

$$\begin{aligned}\frac{\partial F_1}{\partial \nu_1} &= -\phi p \Delta - (1 - \phi) \lambda_I \left[ \frac{s^*(\sigma_1)(1 - \lambda_I)}{(1 - \nu_1)^2 (\lambda_I + (1 - \lambda_I) s^*(\sigma_1)/(1 - \nu_1))} + \right. \\ &\quad \left. + \frac{(1 - p)(1 - s^*(\sigma_1))(1 - p\nu_1)(1 - \lambda_I)}{\nu_1^2 [\lambda_I(1 - p) + (1 - \lambda_I)(1 - s^*(\sigma_1))(1/\nu_1 - p)]^2} + \right. \\ &\quad \left. + \frac{p}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))} - \frac{(1 - p)p}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s^*(\sigma_1))(1/\nu_1 - p)} \right].\end{aligned}$$

The sum of the last two terms in the square bracket in each of those expressions is positive because, respectively,  $1/(1 - \nu_0)$  and  $1/\nu_1$  are greater than 1. We can conclude that  $\partial F_0/\partial \nu_0 > 0$ , and hence that  $\partial s^*(\sigma_0)/\partial \nu_0 > 0$ , while  $\partial F_1/\partial \nu_1 < 0$ , and hence  $\partial s^*(\sigma_1)/\partial \nu_1 < 0$ . ■

### Proof of Lemma 3:

We proceed in two steps. First, we show that under the assumption of the Lemma, the claimed  $\hat{\nu}_0$  exists and is smaller than 1 and that for any  $\nu_0$  that is larger, the low type prefers  $a_0$  to  $a_1$ . Second, we show that the low type preferring  $a_0$  to  $a_1$  implies that the high type prefers implementing the policy corresponding to the state of the world.

The assumption that  $\frac{1-\phi}{\phi}(1 - \lambda_I) < c$  means that neither type has an incentive to deviate for  $\nu_0 = 1$ . By continuity,  $U_0^0 > U_1^0$  for  $\nu_0$  close to 1, so that for those values of

$\nu_0$ , type  $\theta_L$  doesn't want to deviate. By contrast, for  $\nu_0 = 1 - \frac{c}{p\Delta}$ , and given the voters' beliefs, the low type strictly prefers to deviate and play  $a_1$  since  $\mu(\sigma_0, a_0, 0) < \lambda_I$  while  $\mu(\sigma_0, a_1, 0) = 1$  and  $\mu(\sigma_0, a_0, \Delta) \geq \lambda_I \forall s(\sigma_1) \in [0, 1]$ . Moreover, if  $\frac{1-\phi}{\phi}(1 - \lambda_I) < \Delta$  the expressions of  $U_0^0$  and  $U_1^0$  are monotonic in  $\nu_0$ .  $U_0^0$  is (unconditionally) increasing in  $\nu_0$ .  $U_1^0$  is decreasing in  $\nu_0$  since

$$\frac{\partial U_1^0}{\partial \nu_0} = p \left[ -\phi\Delta + (1 - \phi) \left( 1 - \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))} \right) \right] < 0$$

for any  $s^*(\sigma_1) \in [0, 1]$  if  $\frac{1-\phi}{\phi}(1 - \lambda_I) < \Delta$ , which is implied by  $\frac{1-\phi}{\phi}(1 - \lambda_I) < c$ . It follows that there must be a cutoff  $\nu_0^*$  above which type  $\theta_L$  strictly prefers playing  $a_0$ .

This, in turn, implies that type  $\theta_H$  has no incentive to deviate either. To prove this, we first show that (3) implies (4), so that the high type prefers  $a_0$  when  $\omega = 0$ . The “worst case” for this to be true is when  $s(\sigma_1) = 1$  because then the low type's expected reputation from playing  $a_1$  is lowest relative to the high type's. In that case, (3) implies (4) if and only if

$$\begin{aligned} \phi(1 - \nu_0)p\Delta + (1 - \phi)[1 - p(1 - \nu_0)(1 - \lambda_I)] &> 1 - \phi \\ \iff \frac{1 - \phi}{\phi}(1 - \lambda_I) &< \Delta, \end{aligned}$$

which, again, is true by the assumption that  $\frac{1-\phi}{\phi}(1 - \lambda_I) < c$ . Second, we show that (5) is sure to be satisfied, so that the high type prefers  $a_1$  when  $\omega = 1$ . This is true for the same reason as the one invoked for the low type preferring  $a_1$  when  $\nu_0 = 1 - \frac{c}{p\Delta}$  (see the argument above). ■

### Proof of Proposition 1:

This follows directly from comparing the equations defining  $\nu_0^*$  (see Lemma 1) and  $\hat{\nu}_0$  (see Lemma 3). They differ only in that the equation defining  $\hat{\nu}_0$  includes an additional term on the RHS, namely,  $(1 - \phi)(1 - \nu_0)p \frac{(1 - \lambda_I)(1 - s^*(\sigma_1))}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))}$ . This term is strictly positive for any  $\nu_0 < 1$  because  $s^*(\sigma_1) < 1$ . Hence,  $\nu_0^*$  must necessarily be greater than  $\hat{\nu}_0$ . ■

### Proof of Lemma 4:

It follows from Lemma 3 that if  $\frac{1-\phi}{\phi}(1 - \lambda_I) \geq c$  or if  $\nu_0 < \hat{\nu}_0$ , it is not an equilibrium for the low type to play a pure strategy ( $s(\sigma_0) = 1$ ). Since the low type playing  $a_1$  with probability 1 can never be an equilibrium (the politician could increase both welfare and reputation by deviating to  $a_0$ ), the only possible equilibrium has the low type randomizing.

For a mixed action to be optimal, the politician must be indifferent between playing  $a_0$  and playing  $a_1$ . Thus, voters must hold beliefs which ensure that  $U_0^0 = U_1^0$ ; moreover, these beliefs must be derived from equilibrium strategies. Thus, the only  $s(\sigma_0)$  that constitutes an equilibrium is the claimed one.

Next, we show that under Assumption 2, the high type prefers implementing the policy corresponding to the state of the world given that the low type is indifferent between  $a_0$  and  $a_1$ . Regarding the case where  $\omega = 0$ , equation (6) implies (7) if and only if

$$\begin{aligned} (1 - \phi)\mu(\sigma_0, a_1, 0) &< \phi(1 - \nu_0)p\Delta + (1 - \phi) [(\nu_0 + (1 - \nu_0)(1 - p))\mu(\sigma_0, a_1, 0) + \\ &\quad + (1 - \nu_0)p\mu(\sigma_0, a_1, \Delta)] \\ \iff \Delta &> \frac{1 - \phi}{\phi} [\mu(\sigma_0, a_1, 0) - \mu(\sigma_0, a_1, \Delta)]. \end{aligned}$$

The term in square brackets can be written as

$$\begin{aligned} \mu(\sigma_0, a_1, 0) - \mu(\sigma_0, a_1, \Delta) &= \frac{\lambda_I(1 - p)}{\lambda_I(1 - p) + (1 - \lambda_I)(1 - s(\sigma_0)) \left[ \frac{1}{1 - \nu_0} - p \right]} - \\ &\quad - \frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))} \\ &\leq 1 - \lambda_I \end{aligned}$$

Thus, the difference in reputation is bounded above by  $1 - \lambda_I$ . Then, by Assumption 2, type  $\theta_H$  prefers playing  $a_0$  when  $\omega = 0$ .

As for the case where  $\omega = 1$ , (6) implies (8) if and only if

$$\begin{aligned} \phi\nu_0p\Delta + (1 - \phi) [(1 - p)\mu(\sigma_0, a_1, 0) + p\mu(\sigma_0, a_1, \Delta)] \\ > (1 - \phi) [(\nu_0 + (1 - \nu_0)(1 - p))\mu(\sigma_0, a_1, 0) + (1 - \nu_0)p\mu(\sigma_0, a_1, \Delta)] \end{aligned}$$

which can again be simplified to

$$\Delta > \frac{1 - \phi}{\phi} [\mu(\sigma_0, a_1, 0) - \mu(\sigma_0, a_1, \Delta)],$$

so that the same argument as in the case of  $\omega = 0$  applies. Hence type  $\theta_H$  prefers playing  $a_1$  when  $\omega = 1$ . ■

### Proof of Proposition 2:

We first prove this claim for the case where the  $\sigma_1$  subgame has a pure-strategy equilibrium and then move on to the mixed-strategy case. For a pure-strategy equilibrium,

$s^*(\sigma_1) = 0$ , we need  $p\Delta - c > \frac{1-\phi}{\phi}(1 - \lambda_I)$  and  $\nu_1 \geq \nu_1^*$  (cf. Lemma 2). By Proposition 1,  $\nu_0^* > \hat{\nu}_0$ . Hence, for  $\nu_0 \in (\hat{\nu}_0, \nu_0^*)$ , we have  $s_B^*(\sigma_0) = 1$  while  $s^*(\sigma_0) < 1$  so that the claimed result holds. What remains to be shown is that the result is also true for  $\nu_0 \leq \hat{\nu}_0$ . Comparing the equations determining  $s^*(\sigma_0)$  and  $s_B^*(\sigma_0)$  (see Lemmas 2 and 4), one notices that they differ only in the term representing the politician's reputation when he implements a successful reform:  $\frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s(\sigma_0))}$  in the case of rational voters, and  $\frac{\lambda_I}{\lambda_I + (1 - \lambda_I)(1 - s^*(\sigma_1))}$  in the case of hindsight biased voters. Since  $\nu_1 \geq \nu_1^*$  by assumption,  $s^*(\sigma_1) = 0$  by Lemma 1. Therefore, the last expression simplifies to  $\lambda_I$ .

The payoff  $U_0^0$  from playing  $a_0$  is the same in both cases and decreases with  $s(\sigma_0)$ . The payoff  $U_1^0$  from playing  $a_1$  increases with  $s(\sigma_0)$  and differs between the rational and hindsight biased cases. It coincides only at  $s(\sigma_0) = 0$ . At any other  $s(\sigma_0)$ ,  $U_1^0$  is greater with rational voters than with hindsight biased voters. Therefore, the intersection of  $U_0^0$  and  $U_1^0$  lies further to the left in the rational case than in the case of hindsight bias, which is what we needed to show.

We now turn to the case where the  $\sigma_1$  subgame has a mixed-strategy equilibrium,  $s^*(\sigma_1) > 0$ , which requires  $\nu_1 < \nu_1^*$ . The proof is similar to the pure-strategy case. Now, however, the payoff from deviating,  $U_1^0$ , coincides not at 0 but at  $s(\sigma_0) = s^*(\sigma_1)$ . For any  $s(\sigma_0)$  that is greater than  $s^*(\sigma_1)$ ,  $U_1^0$  is larger with rational voters than with hindsight biased voters. Thus, for the claimed result to hold we need the intersection of  $U_0^0$  and  $U_1^0$  in the rational case – i.e.,  $s^*(\sigma_0)$  – to be situated to the right of  $s^*(\sigma_1)$ . But this is exactly the subject of Corollary 1. ■

### Proof of Lemma 5:

Inequalities (9) and (10) follow directly from the fact that  $s^*(\sigma_0) < s_B^*(\sigma_0)$  whenever  $\nu_0 < \nu_0^*$ , a result established in Proposition 2. Meanwhile, (11) holds because  $s^*(\sigma_0) > s^*(\sigma_1)$ ; see Corollary 1. ■

### Proof of Proposition 3:

In a mixed-strategy equilibrium, the low type's expected utility for a given strategy is  $s(\sigma_0)U_0^0 + (1 - s(\sigma_0))U_1^0$  or

$$s(\sigma_0)(1 - \phi)\mu_0 + (1 - s(\sigma_0))[\phi((1 - \nu_0)p\Delta) + (1 - \phi)[(1 - p(1 - \nu_0))\mu_{10} + p(1 - \nu_0)\mu_{1\Delta}]]$$



which we can rewrite to highlight the probability of reelection:

$$(1-\phi) \left[ \underbrace{s(\sigma_0)\mu_0 + (1-s(\sigma_0))[(1-p(1-\nu_0))\mu_{10} + p(1-\nu_0)\mu_{1\Delta}]}_{=\mathcal{R}_L} \right] + (1-s(\sigma_0))\phi((1-\nu_0)p\Delta).$$

Since, for a mixed strategy equilibrium,  $U_0^0 = U_1^0$  must hold, we have

$$\mathcal{R}_L = \mu_0 + (1-s(\sigma_0))\frac{\phi}{1-\phi}(c - (1-\nu_0)p\Delta).$$

We are in a mixed-strategy equilibrium under both regimes iff  $\nu_0 < \hat{\nu}_0$ . Using Lemma 5 and  $s^*(\sigma_0) < s_B^*(\sigma_0)$  (cf. Proposition 2), it is then immediate to establish the claimed result. What remains to be shown is that it holds also for  $\hat{\nu}_0 \leq \nu_0 < \nu_0^*$ . In that case,  $s_B^*(\sigma_0) = 1$  while  $0 < s^*(\sigma) < 1$ , so

$$\mathcal{R}_L^B = \mu_0^B < \mu_0^R + (1-s^*(\sigma_0))\frac{\phi}{1-\phi}(c - (1-\nu_0)p\Delta) = \mathcal{R}_L^R. \quad \blacksquare$$

#### Proof of Proposition 4:

By replacing  $p = 1$  in (12) and applying Lemma 5, we immediately get the required result. The restriction on out-of-equilibrium beliefs is needed because, contrary to the case where  $p < 1$ , there can now be events that do not arise in equilibrium. For example, if  $s(\sigma_0) = 1$  in equilibrium, the event  $(\sigma_0, a_1, 0)$  is off the equilibrium path. The assumption of pessimistic beliefs, i.e.  $\mu(\sigma, a_1, 0) = 0 \quad \forall \sigma$ , makes sure that all of the previous analysis remains valid.  $\blacksquare$

## B Elimination of alternative equilibria with criterion D1

It is a well-known fact that, because it does not pin down out-of-equilibrium beliefs, the PBE concept is often plagued by multiple equilibria. In our case, there effectively exist alternative equilibria; namely, pooling equilibria where all types of politician choose the same policy irrespective of their information. Consider the following sets of strategies and beliefs:

- All types pool on  $a_0$ , and voters believe that any politician who plays  $a_1$  is of type  $\theta_L$  with probability one, i.e.  $\mu(\sigma, a_1, y) = 0$ ;
- all types pool on  $a_1$ , and voters believe that any politician who plays  $a_0$  is of type  $\theta_L$  with probability one, i.e.  $\mu(\sigma, a_0, 0) = 0$ .

The first of these candidates requires  $\frac{1-\phi}{\phi}\lambda_I > p\Delta - c$ , the second  $\frac{1-\phi}{\phi}\lambda_I > c$ , to be an equilibrium.

Both of these equilibria can be eliminated using a refinement known as the D1 criterion which puts restrictions on out-of-equilibrium beliefs.<sup>13</sup> We show this for the first of the two candidates (pooling on  $a_0$ ); the argument can be applied in an analogous way to the other.

Whatever his type, the politician's equilibrium payoff is  $(1-\phi)\lambda_I$ . Let  $\gamma$  denote a mixed action for the voters, i.e.  $\gamma$  is the probability of voting for the incumbent in the election at the end of period 1. Define  $D((\theta, \Psi_\theta), a_1)$  as the set of mixed best responses to action  $a_1$  that make a politician of type  $\theta$  and with information  $\Psi_\theta$  strictly better off playing  $a_1$  than with his equilibrium strategy,

$$D((\theta, \Psi_\theta), a_1) = \bigcup_{\mu} \{\gamma \in MBR(\mu, a_1) \text{ such that } (1-\phi)\lambda_I < \phi E(W|\Psi_\theta) + (1-\phi)\gamma\},$$

where  $MBR(\mu, a_1)$  is the set of mixed best responses to action  $a_1$  for posterior beliefs  $\mu$ . Similarly, let  $D^0((\theta, \Psi_\theta), a_1)$  denote the set of responses for which the politician is indifferent. According to the D1 criterion, a type  $(\theta, \Psi_\theta)$  can be deleted for action  $a_1$  if there exists another type  $(\theta, \Psi_\theta)'$  (i.e., of different ability and/or with different information) such that

$$D((\theta, \Psi_\theta), a_1) \cup D^0((\theta, \Psi_\theta), a_1) \subset D((\theta, \Psi_\theta)', a_1)$$

where  $\subset$  denotes a strict inclusion.

Let us derive the sets  $D((\theta, \Psi_\theta), a_1)$  for the different types in the most interesting case where  $\sigma = \sigma_1$ . We have

$$E(W|\Psi_\theta) = \begin{cases} \nu_1 p\Delta - c & \text{for } \theta_L \\ -c & \text{for } (\theta_H, \omega = 0) \\ p\Delta - c & \text{for } (\theta_H, \omega = 1) \end{cases}$$

The voters' best response to  $a_1$  depends on  $\mu$ . Suppose that the perceived ability of the challenger is  $\lambda_C$ . The voters' best response is "vote for incumbent" if  $\mu > \lambda_C$ , "vote for challenger" if  $\mu < \lambda_C$ , and any mixed action  $\gamma \in [0, 1]$  if  $\mu = \lambda_C$ . Thus, any  $\gamma$  is a mixed

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<sup>13</sup> D1, developed by Cho and Kreps (1987), is a slightly stronger version of Banks and Sobel's (1987) "divinity" concept.

best response for some belief  $\mu$ , and

$$\begin{aligned} D(\theta_L, a_1) &= (\lambda - \frac{\phi}{1-\phi}(\nu_1 p \Delta - c), 1] \\ D((\theta_H, \omega = 0), a_1) &= (\lambda + \frac{\phi}{1-\phi} c, 1] \\ D((\theta_H, \omega = 1), a_1) &= (\lambda - \frac{\phi}{1-\phi}(p \Delta - c), 1]. \end{aligned}$$

Clearly, if  $\nu_1 < 1$ ,  $D(\theta_L, a_1) \cup D^0(\theta_L, a_1) \subset D((\theta_H, \omega = 1), a_1)$ , so that type  $\theta_L$  (and, *a fortiori*, type  $(\theta_H, \omega = 0)$ ) can be pruned based on criterion D1. That is, voters should believe that a deviation to  $a_1$  is infinitely more likely to come from type  $(\theta_H, \omega = 1)$  than from  $\theta_L$ , in which case they should reelect the incumbent. Anticipating this, the high-ability politician will not stick to his prescribed equilibrium strategy when observing  $\omega = 1$ , and the equilibrium breaks down.

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