A choice-based investigation of beliefs under ambiguity

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This paper extends earlier work on the effect of uncertainty on probability weighting and reports the result of an experiment designed to study how different kinds of ambiguity, i.e. uncertain situations with no given probability distribution over outcomes, affect attitudes and beliefs.

Key words: decision under uncertainty, decision weights, revealed beliefs, ambiguity, cumulative prospect theory

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1. Introduction

Imagine you are a purchasing manager who has to choose between three suppliers, whose products have the same quality and price. Because you know that delays generate high losses for your firm, you decide to seek advice from two consultants about the risk of delivery delay for each supplier. Based on available past observations, the two consultants estimate that supplier A's failure rate (the proportion of delayed delivery) is 15%. Concerning supplier B, the two consultants disagree on the failure rate: one estimates it is 5% but the other estimates it is 25%. Supplier C is new on the market and the two consultants estimate that the failure rate is between 5 and 25%. You really believe in the two consultants, who have a very and equally good reputation. If you consider the average estimations, you should be indifferent, but will you? Faced with a choice like this, which supplier would you choose?

Arguably, in most real-world situations, decision-makers only have a vague knowledge of the probabilities of potential outcomes and have to take decisions in the face of uncertainty or "ambiguity" (Ellsberg 1961). Since Ellberg (1961) the impact of ambiguity (or vaguely known probabilities) on choices has been well-documented (cf. Camerer and Weber 1992 for a review of the literature). Contrary to what the Subjective Expected Utility framework predicts (Savage 1954), there is much evidence that ambiguity affects decision-making in some systematic ways: decision makers usually exhibit ambiguity aversion for low probabilities of loss and large probabilities of gain but become ambiguity seeking for large probabilities of loss and small probabilities of gain (e.g., Cohen, Jaffray and Said 1985, 1987; Hogarth and Einhorn 1990; Lauriola and Levin 2001; Viscusi and Chesson 1999). In addition, recent experimental research on ambiguity has also shown that decision-makers are sensitive to the sources of ambiguity (Cabantous *in press*; Smithson 1999). In the literature on ambiguity, ambiguity is commonly implemented by either providing the participants with ranges of probabilities (cf. Budescu et al. 2002; Cohen, Jaffray and Said 1985; Ho, Keller and Keltika 2002) or by providing them with conflicting probabilistic estimates (cf. Einhorn and Hogarth 1985; Kunreuther, Meszaros and Spranca 1995; Viscusi and Chesson 1999 for examples of expert disagreement as a source of ambiguity). Those two implementations of ambiguity are usually assumed to be exchangeable. Smithson (1999) however has recently shown that decision-makers disentangle these two sorts of ambiguity and are most of the time averse to conflict: they tend to exhibit a preference for imprecise ambiguity (i.e., ranges of probability) over conflicting ambiguity (i.e. disagreement over the probability value of an uncertain target event).

In a model with nonlinear probability weighting, such as Cumulative Prospect Theory (Tversky and Kahneman 1992), the finding that attitude towards ambiguity depends on the location of the probability implies that the weighting function is more "inverse-S shape" for events with vaguely known probability (i.e. ambiguous events) than for their counterparts with precisely known probability (i.e. risky events). Though Kahneman and Tversky suggested, as earlier as 1979, that the psychological weights attached to ambiguous and risky outcomes do not coincide, with few exceptions only (Budescu et al., 2002; Hogarth and Einhorn, 1990) experimental research on ambiguity has not provided explanations for the behaviors under ambiguity based on such a rationale. The lack of a coherent prospect theory framework for accommodating experimental results on attitude to ambiguity is even more surprising that, since the early 90's, Tversky has conducted several joint works on the effects of uncertainty on weighting (Tversky and Fox 1995; Tversky and Wakker 1995). Since then, there is a general framework, with behavioral conditions, formalizing the "less sensitive to uncertainty than to risk" effect (cf. Tversky and Wakker 1995). In addition, assuming source dependence and greater subadditivity of the weighting functions for uncertainty than for risk have established as the way to study the effects of various sources of uncertainty on decision weights (e.g., Abdellaoui 2000; Abdellaoui, Vossman and Weber 2005; Kilka and Weber 2001; Tversky and Fox 1995; Tversky and Wakker 1995; Wakker 2004).

However, and despite their common interest in decision-making under uncertainty and ambiguity, research on weighting functions and research on attitude to ambiguity have not cross-fertilized each other. For instance no study yet has used behavioral tests (such as the one developed in the literature on weighting functions to study the "less sensitive to uncertainty than to risk" effect) for studying the effects, on decision weights and beliefs, of ambiguity as implemented in the experimental literature. This is a missed opportunity. By abridging these two perspectives together, this paper attempts to fill this gap and, more importantly, to contribute to the literature on ambiguity. A main novelty of the research is that it extends previous literature on decomposition of decision weights (Wakker 2004) to study beliefs under two sorts of ambiguous contexts commonly used in the literature on ambiguity, namely imprecise ambiguity and conflicting ambiguity. In so doing, it provides a framework for studying the effects of various kinds of ambiguity on beliefs and decision weights that is able to accommodate the pattern of behaviors to ambiguity observed in most empirical research. More specifically, it contributes to answer the following research questions: i) what are the effects of ambiguity on decision weights? Similarly, ii) what effects does ambiguity have on beliefs? In particular, are beliefs less sensitive to ambiguity than to risk? When facing ambiguous events, do decision-makers form their belief by simply averaging the end points of the range (or set) of probabilities; or, do they use a weighted linear combination of the end points of the interval of probabilities? And last, iii) does the kind of ambiguity (i.e., imprecision or conflict) have an impact on decision weights and beliefs?

The structure of the article is as follows. Section 2 sets up the theoretical framework. Section 3 describes the experimental design. The key results are presented in section 4 and section 5 discusses the major findings and concludes.

2. Theoretical framework

2.1. Behavioral definitions

For simplicity we restrict the present treatment to a single domain of outcomes and, we consider that the objects of choice are binary prospects on the outcome set $\mathbb{R}^-$ (non-mixed negative binary prospects). This article focuses on losses because vagueness of probabilistic information is quite common in the loss domain (e.g. insurance decision) and because few studies have studied probability weighting and beliefs in this domain (e.g., Etchart-Vincent 2004). We assume that the decision-maker's preferences on prospects are represented by a binary preference relation. As usual, $\succsim$ denotes weak preference, $\sim$ and $\succ$ respectively denote indifference and strict preference among binary prospects.

We note p:x;y the usual "risky" binary prospect yielding the outcome x with probability p and the outcome y (with y>x) with probability (1-p). We then consider two special cases of ambiguity: imprecise ambiguity (A^i) and conflicting ambiguity (A^c). Imprecise ambiguity, where the uncertain target event is characterized by an imprecise probability (i.e. a probability interval) is probably the most common operationalization of ambiguity in the literature (e.g., Budescu et al. 2002). In this article, we denote [p-r;p+r]:x;y an A^i prospect which gives x with an imprecise probability that belongs to the range [p-r;p+r] and y (with y>x) otherwise. The other typical way to implement ambiguity is to provide the participants with conflicting probability estimates (e.g. Viscusi and Chesson 1999). We denote {p-r;p+r}:x;y the A^c prospect which gives x with a probability which can be either (p-r) or (p+r) and y (with y>x) otherwise. Throughout, r will be assumed as fixed and strictly positive. The sets $\Delta^i = \{[\text{p-r;p+r}] : r \le p \le 1-r\}$ and $\Delta^c = \{\{\text{p-r;p+r}\} : r \le p \le 1-r\}$ represent the two different ambiguous contexts.

> DEFINITION OF A REVEALED BELIEF: A revealed belief q is a probability such that the certainty equivalent for a risky prospect q:x;y is equal to the certainty equivalent for the A^i (A^c) prospect [p-r;p+r]:x;y, ({p-r;p+r}:x;y). Formally, we write [p-r;p+r]$\approx^R$q whenever

there exist x<y and z from $\mathbb{R}^-$ such that [p-r;p+r]:x;y~z and q:x;y~z. Similarly {p-r;p+r}$\approx^R$q whenever there exist x<y and z from $\mathbb{R}^-$ such that {p-r;p+r}:x;y~z and q:x;y~z.

The binary relation $\approx^R$ constitutes a useful tool to study attitudes towards ambiguity since it allows defining several testable preference conditions, analogous to the ones Wakker (2004) introduces (see also Tversky and Fox 1995; Tversky and Wakker 1995). By analogy with researches on weighting functions (e.g., Wu and Gonzalez 1996 and 1999), the paper focuses on two noticeable physical features of revealed beliefs: their degree of curvature and their degree of elevation. In addition, it considers that each characteristic reflects a specific psychological process at play when decision makers evaluate uncertain gambles: the degree of curvature measures the decision maker's degree of sensitivity whereas the degree of elevation reflects the decision maker's perception of attractiveness of the lottery (Gonzalez and Wu 1999).

The paper first focuses on the degree of curvature of the revealed beliefs. Equation 1 (resp. 2) defines the testable preference conditions for *less sensitivity to A^i (resp. A^c) than to risk.*

If [p-r;p+r]$\approx^R$q and [p'-r,p'+r]$\approx^R$q', then |q-q'|≤|p-p'|. (1)

If {p-r;p+r}$\approx^R$q and {p'-r,p'+r}$\approx^R$q', then |q-q'|≤|p-p'|. (2)

These two equations mean that A^i and A^c revealed beliefs vary less than the attached intervals. Typically, this indicates that a decision maker reacts less to a change in the probability level when the probabilities are ambiguous than when they are precise. This is the reason why these equations define less sensitivity to ambiguity than to risk. On the contrary, when the revealed beliefs vary more than the attached intervals, opposite inequalities hold, and decision makers exhibit *more sensitivity to ambiguity (A^i, A^c) than to risk.* Furthermore, as decision makers might disentangle the two sources of ambiguity and set up different certainty equivalents for A^i and A^c gambles, A^i and A^c revealed beliefs can also differ in terms of sensitivity. Equation 3 defines the testable preference condition for *less sensitivity to A^i than to A^c*. Note that the inverse inequality defines *more sensitivity to A^i than to A^c*.

If [p-r;p+r]$\approx^R$q, [p'-r,p'+r]$\approx^R$q', {p-r;p+r}$\approx^R$h and {p'-r,p'+r}$\approx^R$h',

then |q-q'|≤|h-h'| (3)

The second noticeable physical feature of revealed beliefs is their degree of elevation usually referred to as the degree of optimism/pessimism. The next three equations are concerned with this effect and define respectively "*more pessimism under A^i than under risk*", "*more pessimism under A^c than under risk*" and "*more pessimism under A^i than under A^c*" for negative prospects.

$$\text{If } [p\text{-}r;p\text{+}r] \approx^R q, \text{ then } q \geq p \quad (4)$$

$$\text{If } \{p\text{-}r;p\text{+}r\} \approx^R q, \text{ then } q \geq p \quad (5)$$

$$\text{If } [p\text{-}r;p\text{+}r] \approx^R q \text{ and } \{p'\text{-}r,p'\text{+}r\} \approx^R q', \text{ then } q \geq q' \quad (6)$$

In the loss domain indeed, when a revealed belief q of an A^i prospect, giving x with {p-r:p+r}, is greater (smaller) than midpoint probability p, this means that the decision-maker finds the A^i prospect less attractive (more attractive) than the risky one. This should lead him/her to exhibit ambiguity aversion (ambiguity seeking). To simplify we say the decision-makers is pessimistic (optimistic). Note that we use the midpoint p, that is the simple arithmetic mean of [p-r, p+r] and {p-r;p+r}, to define the degree of optimism/pessimism of revealed beliefs. Since no information about experts' competence is available, the midpoint p, which is the solution of lottery reduction when uniform partition holds, is indeed a useful benchmark to which we can compare revealed beliefs.

2.2. Representation

We assume Cumulative Prospect Theory (Tversky and Kahneman 1992) for risky and ambiguous contexts, with a single utility function. According to CPT, the value of a prospect p:x;y, with $x \leqslant y \leqslant 0$ is:

$$p : x; y \mapsto w(p)u(x) + (1\text{-}w(p))u(y)$$

where, u(.) is the value function satisfying u(0)=0, and w(.), called the probability weighting function, is a continuous and strictly increasing function from [0,1] to [0,1] satisfying w(0)=0 and w(1)=1. Similarly, we define the values of A^i and A^c prospects as follows:

$$[p\text{-}r;p\text{+}r] : x; y \mapsto W^i([p\text{-}r;p\text{+}r])u(x) + (1\text{-}W^i([p\text{-}r;p\text{+}r]))u(y) \text{ and}$$

$$\{p\text{-}r;p\text{+}r\} : x; y \mapsto W^c(\{p\text{-}r;p\text{+}r\})u(x) + (1\text{-}W^c(\{p\text{-}r;p\text{+}r\}))u(y)$$

where W^i and W^c are the weighting functions for A^i and A^c prospects.

Under these assumptions we know that there exists a unique revealed belief for each element of Δ^i or Δ^c. There therefore exist a unique function q^i from Δ^i to [0;1] such that $[p\text{-}r;p\text{+}r] \approx^R q$ is equivalent to $q^i([p\text{-}r;p\text{+}r])=q$ and a similar function q^c on Δ^c such that $\{p\text{-}r;p\text{+}r\} \approx^R q$ is equivalent to $q^c(\{p\text{-}r;p\text{+}r\})=q$.

The CPT value for imprecisely ambiguous prospects can thus be rewritten:

$$[p\text{-}r;p\text{+}r]: x; y \mapsto w\left(q^{i}\left([p\text{-}r;p\text{+}r]\right)\right)u(x)+\left(1\text{-}w\left(q^{i}\left([p\text{-}r;p\text{+}r]\right)\right)\right)u(y)$$

Finally, if the prospect is a A^c prospect, its CPT value is given by:

$$\{p\text{-}r;p\text{+}r\}: x; y \mapsto w\left(q^{c}\left(\{p\text{-}r;p\text{+}r\}\right)\right)u(x)+\left(1\text{-}w\left(q^{c}\left(\{p\text{-}r;p\text{+}r\}\right)\right)\right)u(y)$$

Knowing the value function u (defined under risk) and the individual probability weighting function w of a participant (defined under risk as well), the A^i and A^c revealed beliefs can be deduced from the certainty equivalents for A^i and A^c prospects respectively. To complement the non-parametric analysis, which highlights the impact of probability levels on revealed beliefs, the study also relies on a regression line to characterize the general properties of revealed beliefs. We use a linear approximation of the functions q^i and q^c in order to define the sensitivity and pessimism indexes. These indexes are directly adapted from Kilka and Weber (2002).

First, we determine two values a and b for each context such that

$q^i([p\text{-}r;p\text{+}r])$ is approximated by a^i+b^ip

and $q^c(\{p\text{-}r;p\text{+}r\})$ is approximated by a^c+b^cp

Then, b is considered as a sensitivity index (since this slope measures the decision-maker's sensitivity to changes in probability) and the index of optimism/pessimism is defined as the average elevation (a+b/2) of the estimation. Because the linear estimation goes from 0 to 1, the value of the estimation at p=½ gives a good estimate of the elevation of the function. We can therefore determine the degree of pessimism of the revealed beliefs by assessing the departure of the pessimism index, a+b/2, from the benchmark ½. Note that those indexes will not only enable us to study attitudes under each kind of ambiguity but also to compare together the degrees of sensitivity and pessimism under A^i and A^c. (Appendix furthers the explanations of those indexes.)

3. Method

3.1. Participants

The participants in this study were 61 post-graduate students (60 men, 1 woman, median age = 22) in civil engineering at the Ecole Nationale Supérieure d'Arts et Métiers (ENSAM), Paris, France. They were invited by email to participate in a study on decision-making, and guaranteed a 10€ flat participation fee. None of them had already participated in an experiment in decision making.

3.2. Procedure

The experiment was conducted in the form of computer-based individual interview sessions, using software specifically developed for the experiment. The experimentalist and the participant were seated in front of a laptop and the experimenter entered the participant's statements into the computer after clear confirmation. After a brief explanation of the task, where the participants were asked to assume their own role and give their own preferences, and a series of three trial choices, the experiment started. On average, the participants required about 30 minutes to complete the experiment. There was absolutely no time pressure, the participants were given the time they needed and encouraged to think carefully about the questions.

3.3. Materials

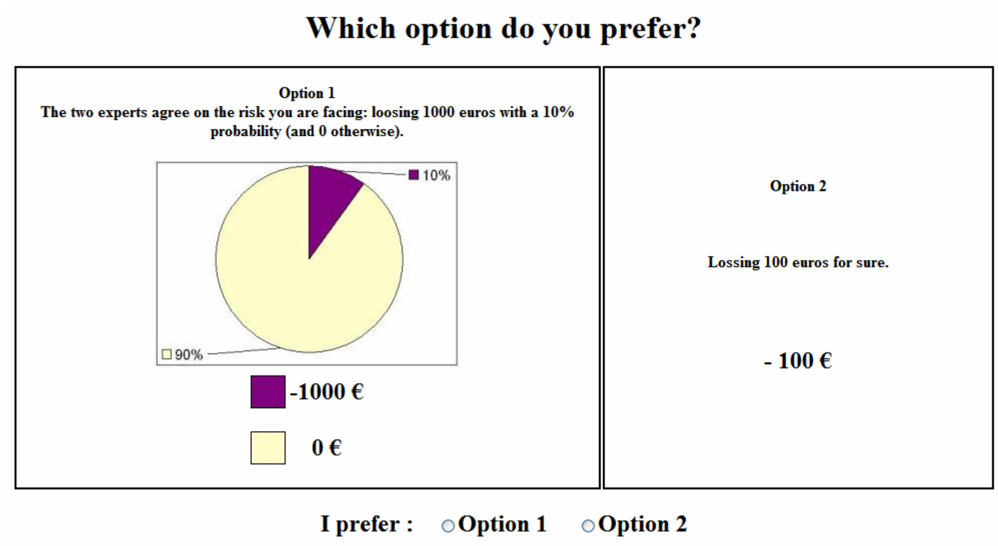
We designed the experiment to estimate participants' certainty equivalents (CEs) for three kinds of negative binary prospects: conventional risky prospects, imprecisely ambiguous (A^i) prospects and, conflictingly ambiguous (A^c) prospects (see Table 1).

In Table 1 below, the first ten prospects are risky prospects of the form p:x;y. For instance, prospect 1 is a risky prospect yielding the outcome -1000€ with probability 10% and the outcome 0€ with probability 90%. The five next prospects are A^i prospects with probability intervals. Prospect 11 for instance, is an A^i prospect, of the form [p-r;p+r]:x;y, that gives the outcome -1000€ with (a) probability belonging to the range 0% and 20%. Last, prospects 16 to 20 are A^c prospects are of the form {p-r;p+r}:x;y. They give x with probability which can be either (p-r) or (p+r) and y (with y>x) otherwise. Prospect 20 for instance gives the outcome -1000€ with probability that is either 80% or 100% and 0 otherwise. It is noteworthy that the 20 prospects are such that the probabilities varied all over the probability interval [0;1]. In addition, and so as to simplify matters, in all A^i and A^c prospects we fixed the width of the probability interval 2r to 20.

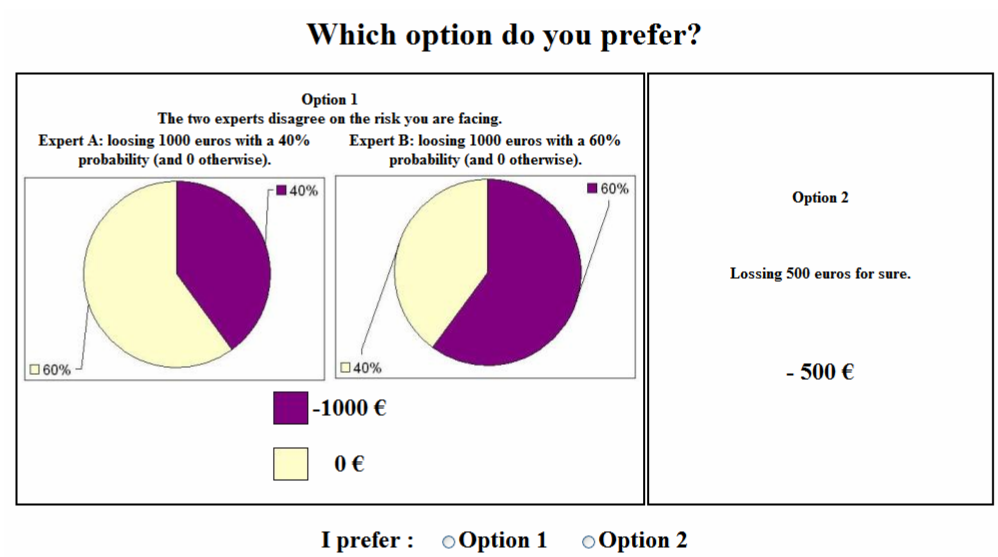
Prospect number	Context	p	x	y	Prospect number	Context	p-r	p+r	x	y
1	Risk	10	-1000	0	11	A^i	0	20	-1000	0
2	Risk	30	-1000	0	12	A^i	20	40	-1000	0
3	Risk	50	-1000	0	13	A^i	40	60	-1000	0
4	Risk	70	-1000	0	14	A^i	60	80	-1000	0
5	Risk	90	-1000	0	15	A^i	80	100	-1000	0
6	Risk	50	-500	0	16	A^c	0	20	-1000	0
7	Risk	50	-500	-250	17	A^c	20	40	-1000	0
8	Risk	50	-750	-500	18	A^c	40	60	-1000	0
9	Risk	50	-1000	-500	19	A^c	60	80	-1000	0
10	Risk	50	-1000	-750	20	A^c	80	100	-1000	0

Table 1: The twenty prospects

To estimate subjects' CEs for the twenty prospects, we constructed a bisection-like process. Such a method does not require the participants to state a precise value such that they would be indifferent between losing that amount for sure and playing a two-outcome negative lottery. It involves choices only, and is therefore easier for the participants to answer than the direct matching method. Moreover, choice method has been found to generate more reliable data (Bostic et al., 1990). With a bisection-like process, from 3 to 7 choices between a given prospect and a sure loss are required to estimate the CE of a prospect. The CE of a prospect is then determined by computing the average of the highest sure loss accepted and the lowest sure loss rejected. In this experiment, each trial started with a choice between a prospect and its expected value. Figure 2 illustrates the task the participants were presented to. Note that to simplify the participants' task, the risky, A^i and A^c screenshots had exactly the same structure: option 1 (the prospect) was systematically displayed at the left-hand side, option 2 (the sure loss) was displayed at the right-hand side of the computer screen and, whatever the informational context, x was in purple and y in yellow. In the risky context (screenshot A), we used a typical pie with a fixed line to provide the participants with a visual representation of the task. For these risky prospects, the participants – who were told they had seek advices from two independent experts - could read: "The two experts agree on the risk you are facing: loosing X euros with a p% probability (and 0 otherwise)." In the A^i context, the participants could read the following "The two experts agree on the risk you are facing: loosing X euros with probability belonging to the range (p-r)% and (p+r)% (and 0 otherwise)." In addition, to help the participants understand A^i prospects, we introduced a dynamic pie. Concretely this means that the program made the size of the pie varies slowly between (p-r) and (p+r). Last, screenshot B displays the typical choice-task in the A^c context. In that context, we introduced two different fixed pies to make clear to the participants that the two sources of information did not have the same estimate of the probability of the loss and, we told them that “The two experts disagree on the risk you are facing. Expert A: loosing X euros with (p-r)% probability (and 0 otherwise). Expert B: loosing X euros with (p+r)% probability (and 0 otherwise).”



Screenshot A: Risky context



Screenshot B: A^c context

Figure 2: Screenshots of typical choice tasks

In addition to this series of about 100 choices (i.e., 20 prospects times a number of choices between 3 and 7), we introduced 6 choice questions, at the end of the questionnaire, to check the reliability of the data. The participants were asked to give their preference for the following six choice questions: prospects 1-3-16-18-11-13 vs. their certainty equivalent. We then can check for the consistency of the answers the respondents gave to the six questions for which we have two statements per subjects.

The sequence of presentation of the twenty prospects (prospects 1-18-10-12-4-16-7-15-11-3-20-9-14) was chosen to have questions with different contexts alternating, and with different magnitudes of losses and different probability levels. It was the same for all the subjects who thus completed exactly the same questionnaire. The program did not enforce dominance and allowed the participants to modify their answer after confirmation if they wish.

3.4. Elicitation technique

In this experiment, five risky prospects of the form .50:x;y and five risky prospects of the form p:-1000;0, where the probability p of losing -1000€ varied from 10 to 90 were used to simultaneously elicit parametric estimations of the value function u(.) and of the probability weighting function w(.). We used the five A^i prospects and the five A^c prospects, with the normalization conditions u(-1000)=-1 and u(0)=0, to estimate the decision weights under imprecise ambiguity (W^i) and under conflicting ambiguity (W^c). Note that under the representation previously assumed (see 2.3), [p-r;p+r]:x,y~z is equivalent to $W^i([p-r;p+r])=-u(z)$ and {p-r;p+r}:x,y~z is also equivalent to $W^c(\{p-r;p+r\})=-u(z)$. This means that decision weights are equal to the utility of the certainty equivalents. Obtaining decision weights is necessary to answer the first research question, i.e. the impact of ambiguity on decision weights. Then, to proceed with the analysis, revealed beliefs can be computed using the following equivalence:

$$W^i([p-r;p+r])=-u(z) \Leftrightarrow w(q^i([p-r;p+r]))=-u(z) \Leftrightarrow q^i([p-r;p+r])=w^{-1}(-u(z))$$

$$W^c(\{p-r;p+r\})=-u(z)v \Leftrightarrow w(q^c(\{p-r;p+r\}))=-u(z) \Leftrightarrow q^c(\{p-r;p+r\})=w^{-1}(-u(z))$$

Consequently, knowing w, we can deduce revealed beliefs from decision weights. We will thus be able to study revealed beliefs for several levels of probability. Last we will use these values to obtain a linear approximation of q^i and q^c so as to compute the sensitivity indexes and the pessimism indexes.

4. Results

4.1. Data reliability

In this article reliability refers to participants' stability (or consistency) for the six questions that were presented twice (prospects 1-3-16-18-11-13 in Table 1). Across questions the mean reliability rate is 77.32%. This means that on average about 3/4 of the participants gave the same answer when the identical choice task was presented twice. Table 2 gives the consistency rate for each question. A Friedman test reveals that the consistency rate does not significantly depend on the informational context (χ^2_2=2.15; p=0.341). Similarly, a Cochran test for dichotomous data shows that reliability does not significantly depend on the question (χ^2_5=9.98; p=0.076). The overall picture thus suggests that participants were consistent in their responses and that the elicited preferences are stable.

Context	**Risk**		**A^c**		**A^i**	
Prospect number	1	3	16	18	11	13
Number of consistent subjects	42	54	45	44	51	47
Consistency rate	69%	89%	74%	72%	84%	77%

Table 2. Consistency check

4.2. Utility function and probability weighting function

For each participant, the utility function and the probability transformation function were simultaneously obtained from the ten certainty equivalents under risk using standard nonlinear least square regression (Levendberg Marquadt algorithm). Parametric estimation of the utility function in the loss domain was conducted using the power functional form $u(x)=-(-x)^{\beta}$, $x\leq 0$. Table 3 reports the estimates for mean and median utility function. A two-tailed t-test on the mean estimate β reveals that it is significantly greater than 1 (t_{60}=3.99; p=0.000) indicating concavity of the utility function. Though one might expect to obtain a convex utility function, it is noteworthy that in the loss domain, results on utility functions tend to be rather mixed. Recent experimental studies for instance have reported convex utility function but have also show that, at the individual level, there are always some subjects exhibiting concave utility functions (e.g., Abdellaoui, Bleichrodt and Paraschiv 2007; Abdellaoui 2000; Tversky and Kahneman 1992; Fennema and Van Assen 1999; Etchart-Vincent 2004). Abdellaoui, Bleichrodt and L'Haridon (2007) for instance have reported linear utility functions for losses between 0 and -10,000€; and in Abdellaoui, Bleichrodt and Paraschiv (2007), the utility function is convex between 0 and -100,000FF (0 and -15,000€). As pointed out by Köbberling, Schwieren and Wakker (2007), diminishing sensitivity is strongly related to the numerosity effect (that is why they use the introduction of Euro to isolate this phenomenon). More generally, in the loss domain, two phenomena generate different effects: one effect, called diminishing sensitivity (Tversky and Kahneman 1992) implies convexity of the utility function but the neoclassical decreasing marginal utility generates concavity. Our results therefore suggest that for small amounts (between 0 and -1000€), the impact of diminishing marginal utility can exceed the impact of diminishing sensitivity.

Function	**Parameter**	**Median**	**Mean**	**SD**
$u(x) = -(-x)^{\beta}$	β	1.13	1.26	0.52
$w(p) = \delta p^{\gamma}/(\delta p^{\gamma})+(1-p)^{\gamma}$	δ	0.72	0.75	0.33
	γ	0.73	0.86	0.49

Table 3. Summary statistics for parameters of the utility and the probability weighting functions

Parametric estimations of individual probability weighting functions were conducted using Goldstein and Einhorn (1987) two-parameter specification, $w(p)=\delta p^{\gamma}/(\delta p^{\gamma})+(1-p)^{\gamma}$. This specification has been frequently employed in recent experimental studies (e.g., Latimore et al. 1992; Tversky and Fox 1995; Abdellaoui 2000; Etchart-Vincent 2004) because it provides a clear separation between two physical properties of the function, elevation and curvature, each of which is captured independently by a parameter (Gonzalez and Wu 1999). The δ parameter mainly controls the elevation of the function and thus the attractiveness of the gamble, whereas the γ parameter essentially governs the curvature of the function and captures the decision-makers' ability to discriminate between probabilities. Table 3 gives the median and mean estimates of the parameters. A two-tailed *t*-test shows that δ is significantly smaller than 1 (t_{60}=-6.00; *p*=0.000). This indicates that the probability weighting function exhibits a small degree of elevation and reflects the fact that on average the participants perceived the negative risky gambles as attractive ones. Although such a small degree of elevation may be surprising in the loss domain, Abdellaoui (2000) obtained a similar result with $\delta = 0.84$; and Etchart-Vincent (2004) reported δ smaller than 1 for both small and large losses ($\delta = 0.84$ and $\delta = 0.85$ respectively).

Concerning the curvature of the probability weighting function, the estimate of γ is significantly smaller than 1 (t_{60}=-2.24; p=0.029, two-tailed t-test), indicating that the probability weighting function exhibits the usual inverse S-shape. This estimate of γ is in accordance with previous empirical estimates in the loss domain: Abdellaoui (2000) for instance reported γ = 0.65 and Etchart-Vincent found γ = 0.836 and γ =0.853 for small and large losses respectively.

4.3. Decision weights for risky and ambiguous prospects

Although the main objective of this article is to study the properties of revealed beliefs, it is useful to estimate decision weights under risk (called w) and under both kinds of ambiguity (called W^i and W^c). Previous empirical work on decision weights has indeed focused on decision weights for uncertain events (i.e., the description of the event does not comprise any probabilistic information), and no work has yet computed decision weights for ambiguous lotteries of the kind operationalized in this experiment. In this subsection, to have a meaningful comparison of the impact of the informational context on decision weights, we computed the three decision weights, w and W^i and W^c, with a unique non-parametric method. This means that rather than comparing the parametric estimation of w reported in Table 3 with non parametric estimations of W^i and W^c, we converted the risky, A^i and A^c CEs into decision weights using the elicited utility and by considering minus the utility of the certainty equivalent (see paragraph 3.4 for a description of the non-parametric method). Table 4 reports the mean and median (standard deviations) values of these estimations and the results of a series of two-tailed t-tests designed to test for differences with the midpoint probabilities (i.e. the simple average of the two end points of the range of probabilities). These tests confirm that participants transform probabilities under risk and weigh their beliefs under ambiguity.

Midpoint		Decision weights		
Probability		**w**	**W^c**	**W^i**
0.1	**Mean**	0.11	0.08^{*}	0.18^{***}
0.3		0.29	0.29	0.29
0.5		0.43^{***}	0.41^{***}	0.43^{***}
0.7		0.59^{***}	0.60^{***}	0.61^{***}
0.9		0.74^{***}	0.80^{***}	0.76^{***}
0.1	**Median**	0.07 (0.11)	0.06 (0.07)	0.14 (0.15)
0.3		0.27 (0.15)	0.26 (0.16)	0.29 (0.13)
0.5		0.42 (0.15)	0.41 (0.17)	0.43 (0.15)
0.7		0.62 (0.15)	0.62 (0.14)	0.63 (0.13)
0.9		0.77 (0.14)	0.82 (0.11)	0.77 (0.14)

* : $p<0.05$; ** : $p<0.01$; *** : $p<0.001$

Table 4. Mean, Median (SD) values for decision weights

Table 5 furthers the analysis by reporting a series of two-tailed paired *t*-tests testing for the effects of the informational context (risk, A^i, and A^c) on decision weights.

Midpoint	Decision weights		
Probability	**w - W^c**	**w - W^i**	**W^c - W^i**
0.1	$t_{60}=2.15^{*}$(AS)	$t_{60}=-3.25^{**}$(AA)	$t_{60}=-6.52^{***}$ (CS)
0.3	$t_{60}=-0.01$	$t_{60}=-0.48$	$t_{60}=-0.56$
0.5	$t_{60}=0.51$	$t_{60}=-0.38$	$t_{60}=-0.84$
0.7	$t_{60}=-0.87$	$t_{60}=-1.15$	$t_{60}=-0.29$
0.9	$t_{60}=-4.98^{***}$(AA)	$t_{60}=-0.83$	$t_{60}=2.54^{*}$ (CA)

* : $p<0.05$; ** : $p<0.01$; *** : $p<0.001$.

AA/AC: Ambiguity Aversion/Seeking; CA/CS: Conflict Aversion/ Seeking

Table 5. Decision weights: results of two-tailed paired *t*-tests

A comparison between w and W^c and W^i first shows that ambiguity has no impact on decision weights associated with medium probability of loss but tend to impact decision weights associated with extreme probabilities of loss (p=0.1 and p=0.9). This trend is very clear in the A^c context where $W^c(\{0;0.2\})$ is significantly smaller (p=0.03) than w(0.1) and $W^c(\{0.8;1\})$ is significantly larger (p=0.000) than w(0.9). This suggests that participants are ambiguity seeking for low probability of loss – more weight is given to low probability risky losses than to low probability A^c losses – but become ambiguity averse for high probability of loss – less weight is given to low probability risky losses than to low probability A^c losses. Such results are quite surprising as most experimental studies have shown that the opposite pattern of behaviour is prevalent in the loss domain. They have usually reported that participants are ambiguity seeking for low probabilities of loss but tend to become ambiguity neutral (or even ambiguity seeking) when the probability of loss increases (Camerer and Weber 1992; Viscusi and Chesson 1999). The effects of A^i on decision weights are more in accordance with previous experimental studies as in this experiment participants exhibit significant ambiguity seeking behaviour for very unlikely losses (i.e., they give on average more weight to A^i losses close to impossibility than to risky losses close to impossibility, W^i ([0;0.2])>w(0.1)). Then, when the probability of the negative outcome increases, ambiguity seeking disappears: participants are neutral to imprecise ambiguity for medium and high probability of loss.

Second, (and to complement the analysis), it is worth comparing the A^c and A^i decision weights with each other. The series of two-tailed *t*-test for paired samples reported in Table 5 reveals that the way ambiguity is implemented has an impact on decision weights. In particular, such tests clearly indicate that W^c and W^i differ for very unlikely as well as very likely losses: they show that participants prefer conflicting ambiguity over imprecise ambiguity (i.e. conflict seeking) for low probability losses but are conflict averse for high probability of losses – $W^c(\{0;0.2\}$ is significantly smaller than $W^i([0;0.2])$ and $W^c(\{0.8;1\})$ is significantly larger than $W^i([0.8;1])$.

4.4. Revealed beliefs

One main novelty of this study is that estimated degrees of beliefs are not “judged probabilities” (given through a direct judgment) but revealed beliefs (derived from choices). In this article, participants’ beliefs are indeed determined through choices and directly inferred from certainty equivalents using Wakker’s (2004) theorem. Table 6 reports the revealed beliefs’ mean and median values (as well as the standard deviations) of the revealed beliefs in the two ambiguous contexts (called q^i and q^c). It also gives the results of two-tailed *t*-tests with midpoint probabilities.

Midpoint probability		**Revealed belief** $\mathbf{q^c}$	$\mathbf{q^i}$
0.1	**Mean**	0.06^{***} (AS)	0.19^{***} (AA)
0.3		0.31	0.33
0.5		0.49	0.53
0.7		0.73	0.73
0.9		0.90	0.86^{**} (AS)
0.1	**Median**	0.04 (0.07)	0.13 (0.16)
0.3		0.30 (0.15)	0.31 (0.15)
0.5		0.50 (0.19)	0.54 (0.13)
0.7		0.75 (0.13)	0.73 (0.15)
0.9		0.92 (0.08)	0.88 (0.11)

* : $p<0.05$; ** : $p<0.01$; *** : $p<0.001$. AA/AC: Ambiguity Aversion/Seeking

Table 6. Mean, Median (SD) values for revealed beliefs

Patterns depicted in Table 6 show that for medium probabilities, revealed beliefs do not differ from midpoint probabilities. In such cases, revealed beliefs are almost equal to p, the probability of the risky loss, leading participants to be "neutral to ambiguity" (cf. W^c and W^i are not different from w). However, such neutrality to ambiguity is no more present when participants are exposed to ambiguous losses with extremes probabilities. This is true in particular in the A^i context, where the revealed belief associated with the lowest range of probability is significantly above the corresponding midpoint probability, indicating that participants acted "as if" the probability of the A^i loss was higher than the probability of the risky loss (inducing ambiguity aversion). On the contrary, the A^i revealed belief associated with the highest range of probability is significantly below the corresponding midpoint probability, inducing ambiguity seeking behaviour. It is noteworthy that while the finding that $q^i([0; 0.2])$ is significantly ($p<0.01$) greater than 0.1 confirms previous findings on decision weights that participants are averse to imprecise ambiguity, for high probability of loss decision weights and revealed beliefs do not point exactly in the same direction. For such high probability losses, the analysis of decision weights indeed concluded that participants are neutral to imprecise ambiguity but the fact that $q^i([0.8; 1])$ is significantly (*p*<0.01) smaller than 0.9 should lead to ambiguity seeking behaviour. The difference between W results and q results may come from the fact that we use a parametric fitting of w to obtain revealed beliefs whereas decision weights under risk were non-parametrically estimated from the certainty equivalents.

Concerning the A^c context, the series of two-tailed *t*-test reveals that revealed beliefs associated with medium probability losses are not significantly different from the midpoint probability. This indicates, once again, that ambiguity does not have any impact for medium probabilities losses (neutrality to ambiguity) but does affect extreme probability losses. More specifically, the participants are ambiguity seeking for low probability losses but are ambiguity neutral for high probability losses – $q^c(\{0;0.2\})$ is significantly below p=0.1 but $q(\{0.8;1\})$ is not significantly different from midpoint probability. These findings are therefore in line with the results reported in the decision weight subsection, though it is noteworthy that for high probability of losses, the analysis of the decision weights concluded that participants are ambiguity averse (rather than ambiguity neutral).

To complement the analysis of the impact of ambiguity on revealed beliefs, we also tested for differences between the two revealed beliefs. The series of *t*-tests for paired samples reported in Table 7 confirm previous findings on decision weights. They show again that for extreme events, where ambiguity has an impact on revealed beliefs, the kind of ambiguity matters. For instance, for very unlikely losses, q^i is significantly greater than q^c, reflecting a net preference for A^c (over A^i). For very likely losses, the kind of ambiguity also matters but the respective effects of A^i and A^c on revealed-beliefs are reversed: q^i is significantly smaller than q^c, suggesting that participants prefer A^i over A^c (conflict aversion) when facing very likely losses.

Midpoint probability	**Revealed beliefs** $q^c - q^i$
0.1	$t_{60}=-6.37^{***}$ (CS)
0.3	$t_{60}=-0.91$
0.5	$t_{60}=-1.43$
0.7	$t_{60}=0.05$
0.9	$t_{60}=3.5^{***}$ (CA)

* : p<0.05 ; ** : p<0.01 ; *** : p<0.001. CA/CS: Conflict Aversion/ Seeking

Table 7. Revealed beliefs: results of two-tailed paired *t*-tests

The following figure illustrates these results graphically. It first shows that for medium probabilities, revealed beliefs are not different from midpoint probabilities. This means that ambiguity has no impact on revealed beliefs associated with medium probabilities. Second, the figure makes clear that for extreme probability losses (i.e., very likely and very unlikely losses), where ambiguity has an impact on revealed beliefs, the source of ambiguity does matter. The figure indeed shows that whereas q^i starts above the 45° (leading to ambiguity aversion), crosses the line near 0.9 and ends below the 45° diagonal (leading to ambiguity seeking); q^c starts below the 45° line (leading to ambiguity seeking) and tends to finish above it (reflecting a tendency to ambiguity aversion). Third, the figure also clearly depicts the finding that even if both the q^i and q^c revealed beliefs belong to the range [p-r;p+r], represented by the two parallel dashed lines above and below the 45° line, they do not look like a constant linear combination of the two end points of the range or set of probabilities. This finding will be confirmed by the analysis of the sensitivity indexes in paragraph 4.5.

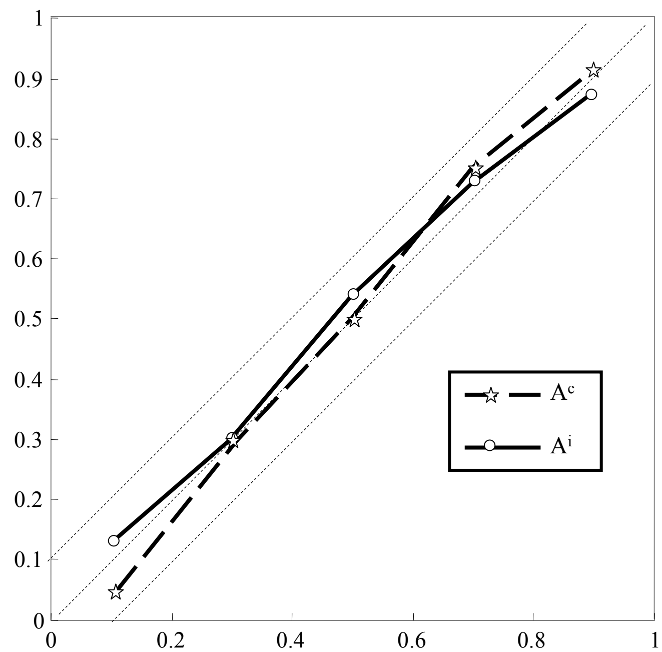


Figure 3. Revealed beliefs (q^i and q^c median values)

4.5. Indexes of sensitivity and optimism

This subsection proceeds with the analysis conducted in 4.3 and tries to understand something of the causes of participants' attitude to ambiguity by analysing the sensitivity index and the optimism index (see 2.3). Participants' non neutrality to ambiguity can indeed result from two distinct but complementary mechanisms (see Wakker 2004): they can exhibit a dispreference (or a preference) for ambiguity because they consider that ambiguous gambles are inherently less (or more) attractive than risky gambles (cf. pessmism index). But, their reaction to ambiguous gambles can also result from a more "cognitive" effect of vaguely known probabilities on their ability to discriminate between different levels of likelihood (cf. sensitivity index). Table 8 (below) reports the mean and median values of the sensitivity and optimism indexes we obtained using linear optimization: $q^i([p-r;p+r])=a^i+b^i*p$ and $q^c(\{p-r;p+r\})=a^c+b^c*p$.

First, a series of two-tailed *t*-test on the pessimism index, which measures the global elevation of revealed beliefs, indicates that A^i generates significant pessimism ($a^i+b^i/2$ is significantly higher than ½; $t_{60}=2.94$; $p=0.005$). In the loss domain indeed, the higher the index, the more pessimistic the participants are. These *t*-tests also show that contrary to A^i, the A^c context does not induce any specific effect ($a^c+b^c/2=0.50$; $t_{60}=0.04$; $p=0.97$). An additional *t*-test for paired sample confirms that participants are significantly more pessimistic under A^i than under A^c ($a^i+b^i/2>a^c+b^c/2$; $t_{60}=2.75$; $p=0.008$). In this experiment, thus, A^i clearly engenders higher beliefs than risk and A^c do. Since the participants were presented with negative outcome, this finding indicates that participants found, on average, the A^i prospects less attractive than the A^c and risky prospects.

Index of	**Comparison to**	**A^c**		**A^i**	
		Mean	**Median (SD)**	**Mean**	**Median (SD)**
Pessimism (a+b/2)	½ (neutrality)	0.50	0.50 (0.06)	0.53**	0.53 (0.08)
Sensitivity(b)	1 (neutrality)	1.05*	1.04 (0.20)	0.87***	0.94 (0.27)

* : p<0.05 ; ** : p<0.01 ; *** : p<0.001.

Table 8. Optimism and Sensitivity indexes: mean, median (SD) values and results of two-tailed *t*-test

Second, the analysis reveals that the two sensitivity indexes are significantly different from 1. This indicates that both sources of ambiguity had an impact on participants' discriminability. There is nevertheless a key difference between the two sensitivity indexes: while the sensitivity index is significantly smaller than 1 in the A^i context ($t_{60}=-3.84$; $p=0.000$), it is significantly higher than 1 in the A^c context ($t_{60}=2.07$, $p=0.042$). This finding suggests that A^i decreases the participants' ability to distinguish among various levels of likelihoods (by comparison with their ability to discriminate between precise probabilities). The effect of A^i on revealed beliefs therefore corresponds to "less sensitivity under imprecise ambiguity than under risk". On the other hand, the finding that the sensitivity index is greater than 1 in the A^c means that the participants are more sensitive to changes in conflicting probabilities than they are to changes in precise probabilities. This "over-sensitivity" phenomenon results from a strong sensibility to extreme cases (i.e., cases where one expert says that the loss is sure and cases when one expert says it is impossible). An additional *t*-test (for paired sample) confirms that both indexes are significantly different from each other (t=6.83; *p*=0.000). We can therefore conclude that, in this experiment, the participants are less sensitive to changes of probability levels when receiving imprecise probabilities of the form "both sources consider the probability of the loss belongs to the range [p-r;p+r]" than when they face an A^c situation where one source of information considers the probability of the target event is p-r but the other source considers it is p+r.

To conclude, the analysis interestingly reveals that the results we obtained for decision weights and revealed beliefs can be explained by i) the negative impact of imprecision on the attractiveness of prospects and ii) by the opposite impacts of imprecise and conflicting ambiguities on sensitivity. In other words, under A^c, the "non-neutrality" towards ambiguity is mainly due to a stronger sensitivity; but under A^i, it results from the combined effects of imprecise probability on both the attractiveness of the gamble (i.e., pessimism) and on participants' ability to discriminate between different levels of likelihood (i.e., weaker sensitivity than under risk).

5. Discussion and Conclusion

5.1. Summary and major findings

The purpose of this paper was to investigate the potential effects on decision weights and revealed beliefs, of different kinds of ambiguity, namely Imprecise Ambiguity or A^i (where the decision maker learns that the probability of the uncertain target event belongs to a probability interval) and, Conflicting Ambiguity or A^c (where the decision-maker receives precise but different estimates of the likelihood of an uncertain target event). To achieve this objective, the paper first provided a general framework based on the Cumulative Prospect Theory for studying decision weights and revealed beliefs under different informational contexts. Second it developed an experimental design to test several research questions regarding the features of decision weights and beliefs under ambiguity. By providing a coherent framework, that is able to accommodate the pattern of behavior under ambiguity observed in most experimental studies, this paper contributes to the literature on ambiguity (Camerer and Weber 1992; Ellsberg 1961). The second contribution of the paper is to extend Wakker (2004)'s revealed-preference study of decision weights and beliefs to two specific kinds of uncertain contexts which, even though they are common operationalizations of ambiguity in the experimental literature on ambiguity, have been neglected in the literature on decision weights. The paper therefore also contributes to the literature on decision weights (e.g. Abdellaoui et al. 2005; Fox and Tversky 1995; Wakker and Tversky 1995) by extending its scope of investigation to new informational contexts.

We return to the series of research question stated in the introduction to assess the contributions of the research.

i) What are the effects of ambiguity on decision weights?

Though most experimental research on ambiguity have implicitly considered that "non neutrality" to ambiguity comes from the impacts that vaguely known probabilities have on decision weights and beliefs, few studies have actually developed an explanation of behaviors towards ambiguity based on such a rationale (for two exceptions, see Hogarth and Einhorn 1990 and Budescu et al. 2002). In this article, we use Wakker's (2004) framework to assess the impacts on decision weights of the two most common sources of ambiguity (i.e., imprecise ambiguity and conflicting ambiguity). Our experimental results clearly show that for events close to impossibility and to certainty, decision weights for ambiguous events differ from risky decision weights. For medium probability events, however, no difference is observable. Such results therefore confirm experimental results showing that attitude towards ambiguity depends on the location of the probability and, specifically that decision-makers tend to react more to ambiguity for extreme probability events. It is noteworthy that the highest sensitivity of decision weights for extreme probabilities we observed in this experiment is also in line with previous research on decision weights. Wu and Gonzalez (1996, 1999) in particular highlight that diminishing sensitivity (i.e. sensitivity decreases when the distance from the reference points "impossibility" and "certainty" increases) affects both decision weights under risk and uncertainty. As a consequence, it is more likely to observe significant changes in sensitivity near those reference points than for medium probability events. For such events indeed the distance from the reference points is higher and thus the sensitivity to changes in likelihood is smaller.

ii) What effects does ambiguity have on beliefs? Are beliefs less sensitive to ambiguity than to risk? Are beliefs equal to the average of the two end points of the range (or set) of probabilities?

Research on attitude towards ambiguity has speculated that nonneutrality to ambiguity (i.e. ambiguity aversion or ambiguity seeking) results from the fact that decision-makers probability judgments of ambiguous events are different from the precise probability of their risky counterpart (i.e., the midpoint of the range of probability). Budescu et al. (2002) for instance have suggested that decision-makers' probability judgments under ambiguity are a weighted combination of the two end points of the range of probability. To estimate participants' attitude to ambiguity, they estimated, for each participant, a single "probability vagueness coefficient". In the loss domain, for instance, if the estimated probability vagueness coefficient of a participant is below ½ (resp. above), this means that the participant gives more weight to the upper bound of the probability interval and then, is ambiguity averse (resp. ambiguity seeking). One limitation of that approach is that it cannot capture the common finding that attitude towards ambiguity depends on the location of the probability (Camerer and Weber 1992; Viscusi and Chesson 199). In this article, we therefore adopted a different viewpoint: we introduce the notion of revealed belief to allow the weighted combination of the two end points to vary along the probability interval. Our experimental data confirm the need for such an approach as they show that the weighted combination of the two end points depends on the location of the midpoint probability. In the A^c context for instance, revealed beliefs for very unlikely events are above the midpoint probability (i.e., more weight is given to the upper bound of the probability interval) but they are below the midpoint probability for very likely events (i.e. weight is given to the lower bound of the probability interval).

iii) Does the kind of ambiguity (i.e., imprecision or conflict) have an impact on decision weights and beliefs?

Until Smithson (1999), the experimental literature on ambiguity has assumed that the source of ambiguity (e.g., conflict, imprecision) does not matter. In this research, we experimentally tested this assumption and we compared revealed beliefs and decision weights under two different sorts of ambiguity commonly used in the literature: imprecise ambiguity (A^i) and conflicting ambiguity (A^c). Our experimental results support Smithson (1999) as they make clear that decision-makers disentangle the two kinds of ambiguity. We indeed found that the way extreme probabilities are weighted significantly depends on the kind of ambiguity. In particular, we observed that the participants give significantly more weight to very unlikely A^i losses (than to very unlikely A^c losses) but give significantly less weights to very likely A^i losses (than to very likely A^c losses). These findings suggest that participants have a preference for A^c over A^i (conflict seeking) for low probability negative outcomes but prefer A^i over A^c (conflict aversion) for high probability negative outcomes. Tests on the A^i and A^c revealed beliefs confirm these findings and strongly suggest that both the A^i and A^c revealed beliefs could be modelled as non-additive linear combinations of the upper and lower bounds of the probability set (or range): the A^i revealed belief function would tend to be inverse S-shape (sub-additive function) but the A^c revealed belief function would rather have an S-shaped form. Eventually, analysis of the pessimism and sensitivity indexes highlighted the fact that implementing ambiguity through imprecision decreases participants' discriminability and makes them more pessimistic while conflicting ambiguity generates "over-sensitivity". These results, all pointing in the same direction, therefore strongly suggest that ambiguity does not correspond to a unique, homogeneous set but congregates informational contexts that are differently treated by decision makers and induce different responses. In this article, by stressing the impact of the source of ambiguity (i.e., imprecision or conflict) on revealed beliefs we therefore contributed to further the analysis of source dependency (Tversky and Fox 1995, Tversky and Wakker 1995, Kilka and Weber 2001, Abdellaoui, Baillon and Wakker 2007).

5.2. Discussion and implications for further research

The experimental design used to study the properties of decision weights and revealed beliefs might raise some objections as it did not involve any real incentive mechanism. In addition to Camerer and Hogarth (1999)'s argument that for simple tasks (such as a certainty equivalent task without any performance measure) real incentives do not systematically make any difference, there is a simple reason for this methodological choice: in this study, the use of real incentives would have confounded the description of the informational contexts by introducing strategic interaction between the subject and the experimenter. Consider for instance an experiment in which a subject receives x€ as an initial endowment and then is asked for his/her certainty equivalent of the prospect [0.6; 0.8]:-x;0. The subject can anticipate that a rational experimenter facing his/her budgetary constraint will minimize the cost of the experiment by implementing the worst case. Consequently, the subject may consider [0.6; 0.8] as being 0.8 for sure. This kind of anticipations would have prevented us from studying the effects of ambiguity on decision weighs and beliefs.

The experimental design might raise a second critique: in this research, revealed beliefs are derived from certainty equivalents, whereas in Abdellaoui et al. (2005), choice-based probabilities are directly obtained by finding indifference between a risky and an uncertain prospect. Since revealed beliefs and choice-based probabilities should be equivalent assuming transitivity of preferences, it could be asked why the same technique was not applied here. The answer to that question is that during a pilot study, it appeared that asking participants for choice-based probability made them focus on the probability dimension (see Tversky, Sattath and Slovic 1988 for the effects on preferences of the response scale used). As a result, they tended to systematically compute the midpoint of the ambiguous probabilities [p-r;p+r] and {p-r;p+r}; and the averaging strategy ended up to be very common. Consequently, we introduced a certainty equivalents task to allow the participants to consider the two dimensions of the choice. It is noteworthy that this methodological strategy also contributes to prevent subjects from easily guessing what the main purpose of the experiment was.

A natural area of extension for future work concerns the aggregation of experts' probabilistic judgments and forecasts (e.g. Budescu et al. 2003; Clemen and Winkler 1999). This paper indeed develops a technology that is easily transferable to contexts where decision makers have to take a decision on the basis of probabilistic forecasts that are communicated to them. Combining expert judgments still constitutes an active part of the literature in decision analysis (see Clemen and Winkler 1999 for an overview of the literature). However, most descriptive studies about the aggregation of probability distributions use judged probability (e.g., Budescu et al. 2003). They study decision makers' beliefs without considering their choices and decisions. The revealed-preference approach of beliefs developed in this paper could therefore be useful for matching an analysis of beliefs (resulting from the aggregation of several probability distributions) to an analysis of decision-makers' effective actions and choices.

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Appendix

Table A1 (below) – based on Wakker (2004) – visually presents the indexes of sensitivity and pessimism and illustrates how the combination of the two different psychological processes combine together to create(s) a non additive revealed-belief exhibiting some elevation.

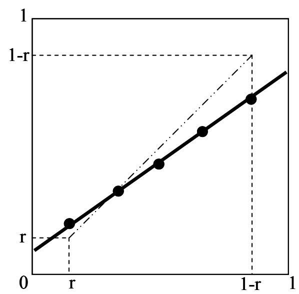
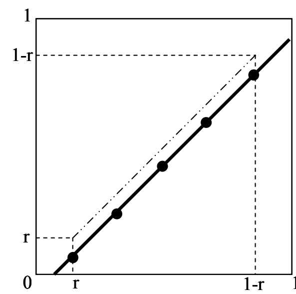
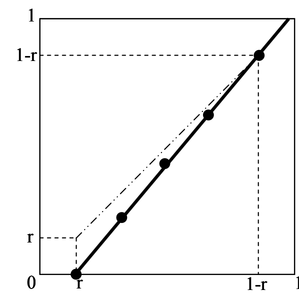
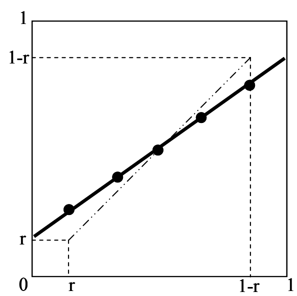
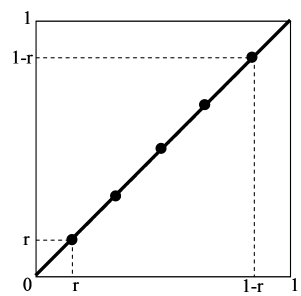
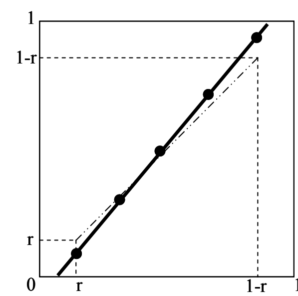
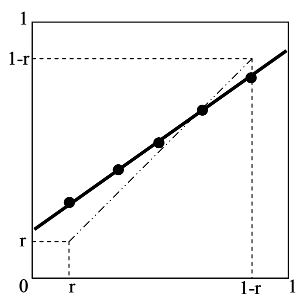
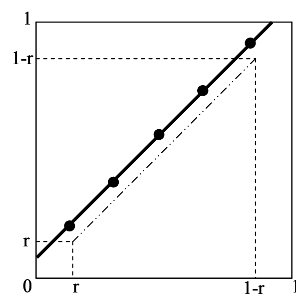
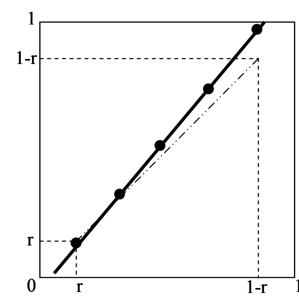
		Sensitivity Index (b)		
		Less sensitivity to ambiguity than to risk **b<1**	Same sensitivity to ambiguity as to risk **b=1**	More sensitivity to ambiguity than to risk **b>1**
	Optimism in the loss domain **a+b/2<1/2**			
Pessimism Index (a+b/2)	Neutrality **a+b/2=1/2**			
	Pessimism in the loss domain **a+b/2>1/2**			

Table A1: Visual representations of the degrees of sensitivity and optimism of revealed beliefs (losses)

The box in the middle of the table depicts a revealed-belief without any pessimism or optimism (neutrality) and with the same sensitivity to ambiguity as to risk. The rows above and below the neutrality row then depict the preference or dispreference for ambiguous lotteries (over risky lotteries) that could arise, independently of any effect of ambiguity on the ability to discriminate between different levels of likelihoods. The interpretation of the attractiveness obviously depends on the domain of the outcome. In the loss domain, a shift-down of the revealed belief ($a+b/2<1/2$) reflects ambiguity seeking because the revealed-belief for the ambiguous lottery is below the midpoint probability p at all levels. In that case, the participant is said to be optimistic. On the contrary a shift-up of the revealed belief ($a+b/2>1/2$) (in the loss domain) traduces the fact that the participant(s) considers the probability of losing with the ambiguous lottery is larger than the probability of losing with the risky lottery at all levels. The participant thus exhibits ambiguity/uncertainty aversion and is said to be pessimistic. The opposite interpretation holds in the gain domain. By moving now from the column in the middle to the left-hand column or the right-hand column, we consider another kind of deviation: b, the slope of the function q, measures the decision-maker's sensitivity to changes in probabilities. b equals 1 reflects the fact that the participant exhibits exactly the same sensibility to ambiguity as to risk: ambiguity does not affect the his/her ability to distinguish among various likelihood levels. On the contrary, when ambiguity affects the participant's discriminability, b is different from 1. In that case, the participant is said to have less sensibility to ambiguity than to risk when $b<1$ (right-hand column) and to have more sensibility to ambiguity than to risk when $b>1$ (left-hand column).