Risk-Taking Tournaments –Theory and Experimental Evidence*

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Abstract

We study risk-taking behavior in a simple two person tournament in a theoretical model as well as a laboratory experiment. First, a model is analyzed in which two agents simultaneously decide between a risky and a safe strategy. In contrast to the previous literature we allow for all possible degrees of correlation between the outcomes of the risky strategies. We show that risk-taking behavior crucially depends on this correlation as well as on the size of a potential lead of one of the contestants. We find that the experimental subjects acted mostly quite well in line with the derived theoretical predictions.

Key Words: Tournaments, Competition, Risk-Taking, Experiment

JEL Classification: M51, C91, D23

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1 Introduction

In tournaments, agents compete against each other for a limited set of given prizes. The agent with the best performance receives the winner prize and the less successful agents only gets a lower loser prize. Tournament situations have been analyzed analytically within many different frameworks since the seminal article of Lazear and Rosen (1981). In practice tournament situations can be frequently observed. For instance, employees within firms compete for promotions, fund managers compete for their clients' capital, firms in R&D-intensive industries are engaged in patent races.

Most of the classical tournament literature focuses on the agents' optimal effort choices and effects of the spread between winner and loser prizes on effort (compare e.g. Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), Green and Stokey (1983)). Risk taking as a choice variable in tournaments was first discussed by Bronars (1986) who argued that the leading agent prefers a low risk strategy while his contestant prefers a high risk strategy as the leading agent does not want to endanger his leading position whereas his opponent can only win if he increases the risk. Recently Hvide (2002) or Kräkel and Sliwka (2004) considered two stage models where agents first can choose the risk and then decide about the effort level exerted and show that the risk choice also has an indirect impact as it affects the efforts exerted on the second stage. In these models, it is no longer clear that indeed the leading agent prefers a safe and his follower a risky strategy. Whereas these papers assume that the outcomes of the risky strategies of both players are uncorrelated, Taylor (2003) investigates a model in which these outcomes are perfectly correlated. He shows that only a mixed strategy equilibrium exists in this case in which the leading player chooses the riskier strategy more often than the trailing player.

There are now numerous examples of empirical studies on tournaments mainly investigating sports contests like car racing (Becker and Huselid (1992)), golf tournaments (Ehrenberg and Bognanno (1990a), Ehrenberg and Bognanno

(1990b), Orszag (1994)) or tennis (Sunde (2003)). Experimental investigations of effort incentives have for instance been conducted by Bull et al. (1987) and Orrison et al. (2004) or Harbring and Irlenbusch (2003).

In this paper we investigate risk taking behavior in tournaments in a controlled laboratory experiment which has to the best of our knowledge not been done so far. We focus on the analysis of pure risk taking behavior, i.e. the agents can only choose between a safe and a risky strategy. We allow for the possibility that one agent has a lead over the other in the beginning of the game.

The paper also makes a contribution to the theoretical literature on risk taking in tournaments as we analyze a model in which the performance outputs of the risky strategies can be correlated with any possible degree of correlation. Hence, our setting nests cases where the strategies are uncorrelated as in Hvide (2002) or Kräkel and Sliwka (2004)[1] or are perfectly correlated as in Taylor (2003). But we also consider intermediate cases leading to interesting results in the theoretical analysis as well as in the experiment.

In the experiment we study three different treatments, one in which the outcomes of the risky strategies are uncorrelated, a second in which they are perfectly correlated, and a third treatment in which the correlation coefficient between these outcomes is equal to $\frac{1}{2}$. Furthermore, within each treatment we varied the size of the lead. Hence, we can study how (i) the correlation between the outcomes of the risky strategies and (ii) the size of the lead affects risk taking behavior.

The remainder of the paper is organized as follows. In section 2 we introduce the model and analyze the possible Nash equilibria. Section 3 describes the experimental design and procedures. The hypotheses are shown in section 4 and in section 5 we present the experimental results. Section 6 concludes.

[1] In constrast to these papers we do not consider effort choices to set a clear focus on risk taking behavior.

2 Theoretical Analysis

2.1 The Model

We consider a simple tournament between two risk neutral agents A and B. We focus on the risk taking decisions of the contestants and assume that both agents simultaneously decide among a risky and a safe strategy, i.e. $d_i \in \{r, s\}$ for $i = A, B$. Each agent's decision affects the distribution of his performance y_i as

$$\begin{array}{ll} y_i = \mu_s & \text{when } d_{ti} = s \\ y_i = \tilde{y}_i \sim N\left(\mu_r, \sigma^2\right) & \text{when } d_{ti} = r \end{array}$$

where $\mu_r \geq \mu_s$ such that the risky strategy may have a higher expected performance than the safe strategy. We also allow for the possibility in which one of the agents initially has a lead which may for instance be due to differences in ability or the outcome of some prior stage in the competition. Without loss of generality we assume that agent A has a lead and wins the tournament when the sum of his performance y_A and the lead Δy_A exceeds his rival's performance y_B where $\Delta y_A \geq 0$. When $y_A + \Delta y_A = y_B$ each agent wins the tournament with probability $\frac{1}{2}$.

Note that the variance of the risky option is the same for both agents. We allow for the possibility that the performance outcomes $\tilde{y}_i$ are correlated with correlation coefficient ρ. Hence, we allow for the possibility that $\rho = 0$ as in Hvide (2002) and Kräkel and Sliwka (2004) or that $\rho = 1$ as in Taylor (2003) but also can consider intermediate cases. The winner of the tournament receives a prize normalized to 1 and the loser's payoff is zero.

2.2 Equilibrium Analysis

When both choose the safe option $d_A = d_B = s$ of course A always wins the tournament when Δy_A is strictly positive. When $\Delta y_A = 0$ each agent wins with probability $\frac{1}{2}$. When A plays safe agent B's only chance of winning is to choose the risky strategy. In this case A´s winning probability is

$$P_A^{sr} = \Pr\left(\Delta y_A + \mu_s > \tilde{y}_B\right) = \Phi\left(\frac{\Delta y_A + \mu_s - \mu_r}{\sigma}\right)$$

where $\Phi\left(.\right)$ is the cumulative distribution function of a standard normal distribution. When both agents choose the risky strategy player A wins with probability $P_A^{rr} = \Pr\left(\tilde{y}_B - \tilde{y}_A \leq \Delta y_A\right)$. Note that $\tilde{y}_B - \tilde{y}_A$ follows a normal distribution with mean 0 and variance $2\sigma^2\left(1-\rho\right)$. Hence,

$$P_A^{rr} = \begin{cases} \Phi\left(\frac{\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right) \text{ when } \rho < 1 \\ 1 \text{ when } \rho = 1. \end{cases}$$

Finally, when A plays risky and B plays safe, A's winning probability is

$$P_A^{rs} = \Pr\left(\Delta y_A + \tilde{y}_A > \mu_s\right) = 1 - \Phi\left(\frac{\mu_s - \Delta y_A - \mu_r}{\sigma}\right).$$

For ease of notation let $\Delta\mu = \mu_r - \mu_s$. It is instructive to start with the case that $\Delta y_A = 0$. In that case the following simple game is played.

	Risky	Safe
Risky	$\frac{1}{2}, \frac{1}{2}$	$\Phi\left(\frac{\Delta\mu}{\sigma}\right), \quad \Phi\left(\frac{-\Delta\mu}{\sigma}\right)$
Safe	$\Phi\left(\frac{-\Delta\mu}{\sigma}\right), \quad \Phi\left(\frac{\Delta\mu}{\sigma}\right)$	$\frac{1}{2}, \frac{1}{2}$

When $\Delta\mu = 0$ both players are indifferent between both strategies. But there is a unique Nash equilibrium in dominant strategies in which both agents choose the risky strategy when the risky strategy has a higher return, i.e. $\Delta\mu > 0$. Whatever the opponent's strategy, a player can always raise the probability of winning by deviating to the risky strategy.

Much more interesting is the case where one player has a lead, i.e. where w.l.o.g. $\Delta y_A > 0$. The agents then play the following zero sum game where the leading player A is the row and player B the column player.

	Risky	Safe
Risky	$\Phi\left(\frac{\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right), \quad \Phi\left(\frac{-\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right)$	$\Phi\left(\frac{\Delta y_A+\Delta\mu}{\sigma}\right), \quad \Phi\left(\frac{-\Delta y_A-\Delta\mu}{\sigma}\right)$
Safe	$\Phi\left(\frac{\Delta y_A-\Delta\mu}{\sigma}\right), \quad \Phi\left(\frac{-\Delta y_A+\Delta\mu}{\sigma}\right)$	1, 0

First, it is straightforward to see that $(risky, safe)$ and $(safe, safe)$ can never be Nash equilibria. In the first case, the leading player A wins for sure when deviating to the safe strategy. In the second, player B will always deviate to the risky strategy as he otherwise loses for sure.

When agent B plays risky the leading player A can indeed lose the tournament with a positive probability. It is interesting to investigate under what conditions he still prefers to stick to the safe strategy. He will do so when

$$\Phi\left(\frac{\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right) \leq \Phi\left(\frac{\Delta y_A - \Delta\mu}{\sigma}\right) \Leftrightarrow \frac{\Delta\mu}{\Delta y_A} \leq \left(1 - \frac{1}{\sqrt{2(1-\rho)}}\right).$$

As playing risky leaves player B the only chance to win the tournament, we can directly conclude:

Proposition 1 *A pure strategy Nash Equilibrium exists in which the leading player A plays the safe strategy and player B plays the risky strategy if and only if*

$$\frac{\Delta\mu}{\Delta y_A} \leq \left(1 - \frac{1}{\sqrt{2(1-\rho)}}\right). \tag{1}$$

Hence, higher values of the lead Δy_A and smaller values of $\Delta\mu$ tend to make it more likely that the leading player sticks to the safe strategy. To understand the result it is instructive first to consider the case where the performance outcomes of the risky strategies are uncorrelated (i.e. $\rho = 0$). In this case, condition (1) is equivalent to $\frac{\Delta\mu}{\Delta y_A} \leq 1 - \frac{1}{2}\sqrt{2}$. When the risky strategy does not lead to a higher expected performance such that $\Delta\mu = 0$ the leading agent A will then always stick to the safe strategy as playing the risky strategy will only raise the probability to forgo the leading position. The larger $\Delta\mu$ the more attractive it of course becomes to switch to the risky strategy. This will be the more so, the smaller the initial lead Δy_A as protecting a small lead is not worthwhile when the risky strategy becomes more attractive in terms of expected performance. But it is interesting that this picture changes when the outcomes of the risky strategies are correlated. Note that condition (1) is *always violated* when ρ tends to one.

The larger the correlation between the risky strategies the more attractive it becomes for player A to choose the risky strategy when B has done the same – even when his lead Δy_A is large and even when the risky strategy does not lead to a much higher expected performance. The reason is that with correlated performance outcomes, choosing the risky strategy becomes a means to protect the lead. Hence, we now have to check under which conditions a Nash equilibrium exists in which both agents play the risky strategy.

As analyzed above, when B plays risky the leading player A will prefer to play risky as well when condition (1) is violated, i.e.

$$\frac{\Delta\mu}{\Delta y_A} \geq \left(1 - \frac{1}{\sqrt{2\,(1-\rho)}}\right). \tag{2}$$

Player B then indeed also prefers the risky option when

$$\Phi\left(\frac{-\Delta y_A}{\sigma\sqrt{2\left(1-\rho\right)}}\right) \geq \Phi\left(\frac{-\Delta\mu-\Delta y_A}{\sigma}\right) \Leftrightarrow \frac{\Delta\mu}{\Delta y_A} \geq \left(\frac{1}{\sqrt{2\left(1-\rho\right)}}-1\right). \tag{3}$$

Hence we can conclude:

Proposition 2 *A pure strategy Nash Equilibrium exists in which both players choose the risky strategy if and only if*

$$\frac{\Delta\mu}{\Delta y_A} \geq \max\left\{1-\frac{1}{\sqrt{2\left(1-\rho\right)}}, \frac{1}{\sqrt{2\left(1-\rho\right)}}-1\right\} \tag{4}$$

Consider first again the case where the outcomes of the risky strategies are uncorrelated (i.e. $\rho = 0$). Condition (4) is now equivalent to $\frac{\Delta\mu}{\Delta y_A} \geq 1 - \frac{1}{2}\sqrt{2}$. Note that this is the opposite of condition (1) given in Proposition 1. The reason is that player B always prefers the risky strategy when $\rho = 0$ irrespective of A's decision. As already laid out, when A plays safe playing the risky strategy is the only way for player B to have at least a chance of winning. When, however, A plays risky, player B has such a chance already when playing safe, but can increase the odds by playing risky. Hence, for $\rho = 0$ only player A's considerations determine which equilibrium is played. Both play risky in this case if and only if $\frac{\Delta\mu}{\Delta y_A}$ is sufficiently large as only then it will be reasonable for player A to take the risk and not to protect the lead. As pointed out above, the reasoning is different if the outcomes of the risky strategies are correlated. As we have already seen, agent A has an incentive to imitate a risky strategy of his opponent if the correlation gets larger. To see that consider figure 1, in which the equilibrium conditions are mapped in the $\left(\rho, \frac{\Delta\mu}{\Delta y_A}\right)$-space. Condition (2) determines the downward sloping curve that separates the region in which agent A plays safe and agent B plays risky from that where both play risky. The higher ρ the more attractive it becomes for agent A to switch to the risky strategy as well. A special case

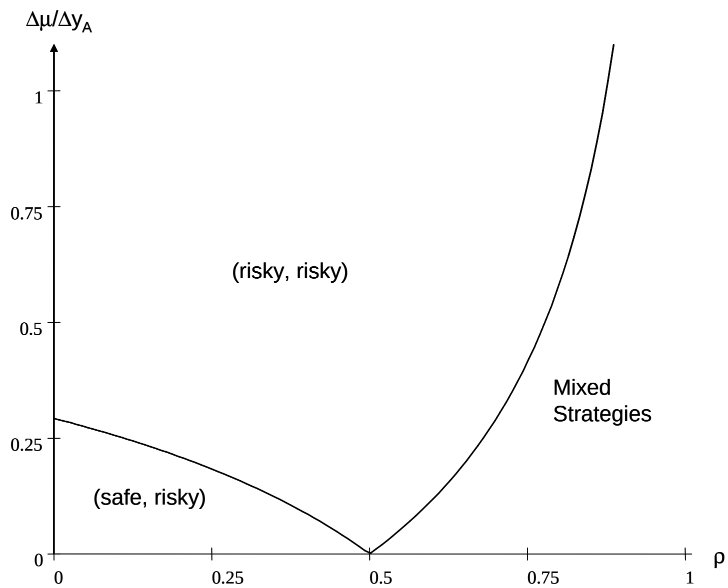

Figure 1: Nash Equilibria of the Game

is $\rho = \frac{1}{2}$. In this case condition (4) simplifies to $\frac{\Delta\mu}{\Delta y_A} \geq 0$ and, hence, the agents will always play $(risky, risky)$ no matter how large the initial lead or the expected performance is.

But when the correlation gets larger, choosing the risky strategy becomes less attractive for player B. The stronger the correlation the smaller is the probability for player B to overtake player A when both play risky. In the extreme, when $\rho = 1$, both agents will always attain the same outcome when playing the risky strategy and, hence, agent A would win for sure in this case. In that case, however, player B has an incentive to deviate to the safe strategy when player A plays risky. Playing safe leaves at least the possibility that A is unlucky and falls behind. But of course, when player A in turn knows that B chooses the safe strategy, he would again want to deviate and choose the safe strategy as well. Hence, we cannot have equilibria in pure strategies if $\rho = 1$ as has already been shown by Taylor (2003). But note that we can already conclude from Propositions 1 and 2 that a similar reasoning must hold for a larger set of parameters. As we already have checked the existence conditions for all potential pure strategy equilibria, when conditions (1) and (4) are both violated only mixed strategy equilibria can exist. Hence, we can show the following result:

Proposition 3 *A Nash Equilibrium in mixed strategies exists if and only if*

$$\frac{\Delta\mu}{\Delta y_A} \leq \frac{1}{\sqrt{2(1-\rho)}} - 1. \tag{5}$$

In any mixed strategy equilibrium, player A chooses the risky strategy with a larger probability than player B.

Proof: See appendix.

Hence, only mixed strategy equilibria exist in the area below the upward sloping curve in figure 1. The larger the correlation between the outcomes of the risky strategies and the smaller $\frac{\Delta\mu}{\Delta y_A}$ the more likely it is that a mixed strategy is played. In such an equilibrium, the leading player always chooses the risky strategy with a higher probability than his opponent.
Hence, it has turned out that the correlation between the outcomes of the risky strategies is an important parameter.

3 Experimental Design and Procedure

We implemented the simple risk taking tournament in a laboratory experiment. We ran three different treatments for each of which we conducted one session with 24 participants. In each of 23 periods two players were matched together randomly and anonymously. Hence, each participant played 23 times and each time with a different opponent. We varied the correlation coefficient of the risky strategy between the treatments. The first treatment had a correlation coefficient of zero, the second of one and the third of $\frac{1}{2}$. Furthermore, we varied the lead Δy_A between the periods such that we are able to investigate the effects of Δy_A on player's strategy choices.

The experiment was conducted at the Cologne Laboratory of Economic Research at the University of Cologne in January 2007. Altogether 72 students participated in the experiment. All of them were enrolled in the Faculty of Management, Economics, and Social Sciences and have completed their second year of studies. For the recruitment of the participants we used the online recruitment system by Greiner (2003). We used the experimental software z-tree by Fischbacher (1999) for programming the experiment.

At the outset of a session the subjects were randomly assigned to a cubical where they took a seat in front of a computer terminal. The instructions were handed out and read out by the experimenters.[2] After that the subjects had time to ask questions if they had any difficulties in understanding the instructions. Communication - other than with the experimental software - was not allowed.

Each session started with 5 trial periods so that the players could get used to the game. In the trial rounds each player had the opportunity to simulate the game by choosing the strategies for both players and observing the outcomes. After that the 23 main periods started. All periods were identical but played with a different partner. In the beginning of each period the players were informed about their score of points which they had in the beginning and the score of their opponent. So they knew whether they were the player in lead and how large the difference between the scores was. The initial scores of points were drawn from a normal distribution with a mean of 150 points and a standard deviation of 42 points. Then the players had to decide whether they wanted to play a safe or a risky strategy. If a player chose the safe strategy he received 80 additional points for sure. When choosing the

[2]The full set of all our experimental instructions translated into English can be found in the appendix.

risky strategy the additional points awarded where determined by a random draw from a normal distribution with a mean of 100 points and a standard deviation of 20 points. In the first treatment the risky strategies of both players were uncorrelated. In the second treatment the risky strategies were perfectly correlated and in the third they were correlated with $\rho = \frac{1}{2}$. This information was common knowledge. The key concepts where explained in the introduction and the players had the opportunity to develop a "feel" for the distribution in the trial rounds. After each player made his decision they were informed about the additional points received and the final score of the game. The final score was the sum of the initial points of each player and his additional points won in the game. They were also informed which player was the winner of the period. They played 23 periods with different partners. In the end of the experiment one of the 23 periods was drawn by lot. Each player who won the tournament in which he participated in the drawn period earned 25 Euro each loser earned only 5 Euro. Additionally, all subjects received a show up fee of 2.50 Euro independent of their status as winner or loser. After the last period the subjects were requested to complete a questionnaire including questions on gender and age. The whole procedure took about one hour.

4 Hypotheses

First of all, based on the theoretical reasoning above, we expect that in the treatment without correlation the leader plays the safe strategy much more often than the trailing player (Hypothesis 1). But of course, the model makes a more precise prediction. Recall that the trailing player should always choose the risky strategy. The leader should play the safe strategy if and only if the lead is sufficiently large and the expected gains from playing risky are low. In our experiment the expected gains from playing risky were fixed for all treatments ($\Delta\mu = 20$). In other words the player in lead should choose the

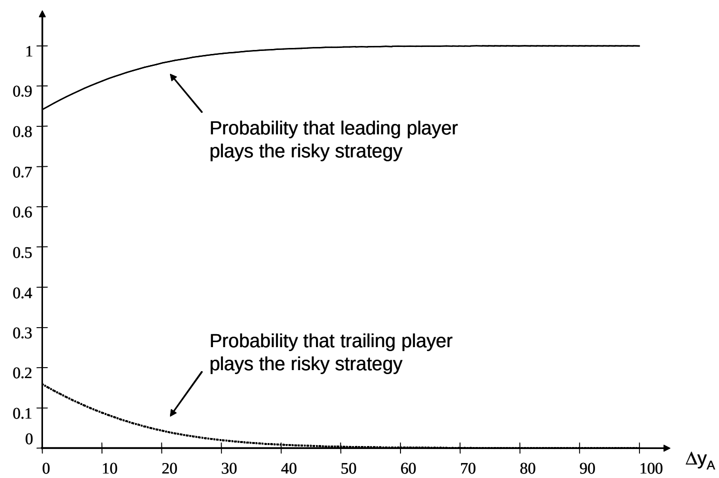

Figure 2: Equilibrium mixed strategies when $\rho = 1$

safe strategy if $\Delta y_A > \frac{20}{1-\frac{1}{2}\sqrt{2}} = 68.28$ and otherwise should prefer the risky strategy.

In the second treatment the performance outcomes of the risky strategies are perfectly correlated and therefore only an equilibrium in mixed strategies exists in the theoretical model. But the most important and testable implication is that – in contrast to the zero correlation case – we expect that the player in lead will play risky more often than his opponent (Hypothesis 2). Although we cannot expect that participants in the experiment are able to coordinate on the mixed strategies equilibrium perfectly, the data should at least be in line with some qualitative features of the equilibrium. Therefore it is useful to consider the probabilities with which the players choose the risky strategy derived in the proof of proposition 3. Figure 2 shows these probabilities as a function of Δy_A for the parameter values used in the experiment.Note that the leading player should choose the risky strategy in more than 80% and the trailing player in less than 20% of the cases. Furthermore we expect that the probability that the trailing player plays the risky strategy should decrease in Δy_A and the probability that the leader

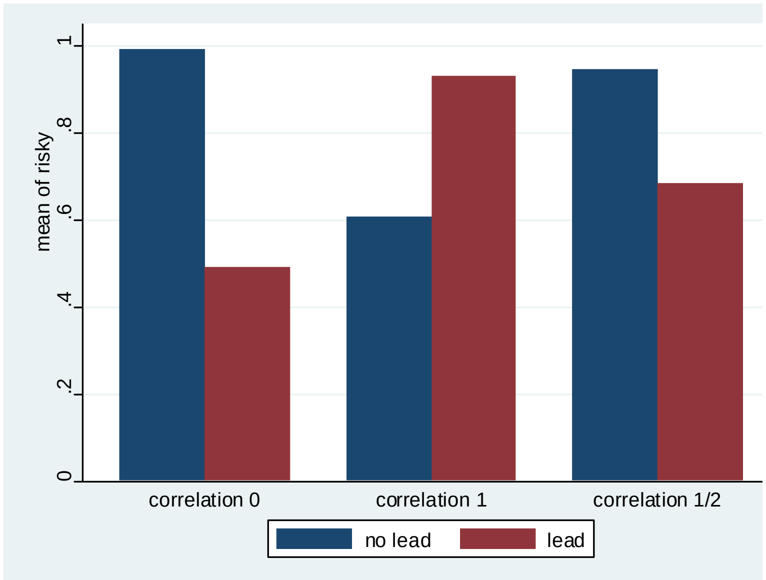

Figure 3: Overview

does the same should increase in his lead.
For the third treatment we predict that both players will always choose the risky option no matter how large the lead is (Hypothesis 3) or at least that they both learn during the course of the experiment that the risky strategy is beneficial.

5 Results

We now test these hypotheses with the data from our experiment. Figure 3 shows the fraction of rounds in which the players in each treatment chose the risky strategy depending on whether the player had a lead.[3]
We start by investigating the results from treatment 1 where the outcomes of the risky strategies were uncorrelated. Looking at figure 3 we see already

[3] Table A1 in the Appendix gives the precise values.

that the trailing player almost always chose the risky strategy when the risky strategies were uncorrelated but that the leading player chose the safe strategy in nearly 50% of the cases. Hence, these observations are well in line with hypotheses 1.

To analyze whether the lead had an effect on the choice of strategy for the leader we ran a binary probit regression. The dependent variable is the probability that the leading agent chooses the risky strategy.[4] The results are reported in table 1.[5] Regression (1) reports the results if we use the size of the lead and the period as independent variables. The period is included to check for time trends capturing possible learning effects.

	(1) Leading player	(2) Leading player
Lead	-0.0284^{***}	
	(0.0043)	
Lead > 68.28		-1.399^{***}
		(0.22)
Period	0.00588	0.00523
	(0.012)	(0.0095)
Constant	1.192^{***}	0.244
	(0.22)	(0.19)
Observations	276	276
Pseudo R^2	0.2858	0.1436

Robust standard errors in parentheses are calculated by clustering on subjects
$^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$

Table 1: Probit regression for leading players in treatment 1

We see that in line with the theoretical prediction, a larger lead makes it indeed more likely for the leader to choose the safe strategy. This effect is highly significant. Regression (2) uses a dummy variable which takes value

[4] The observations are not independent from each other as one subject plays the game 23 times. We control for that by computing robust standard errors and adjusting them for intraperson correlation.

[5] In the Appendix we report the marginal effects for all probit regressions.

one if the lead is larger than 68.28 and zero otherwise. The results are qualitatively similar to those of regression (1). Note that there are no significant time trends. Of course, the participants did not switch to the safe strategy precisely at the predicted cut-off value, but still they learned surprisingly well that playing safe is preferable when the lead gets larger as is also illustrated in figure 4. It shows the frequencies of the risky strategy choice for different leads in treatment 1 (interval size 5).

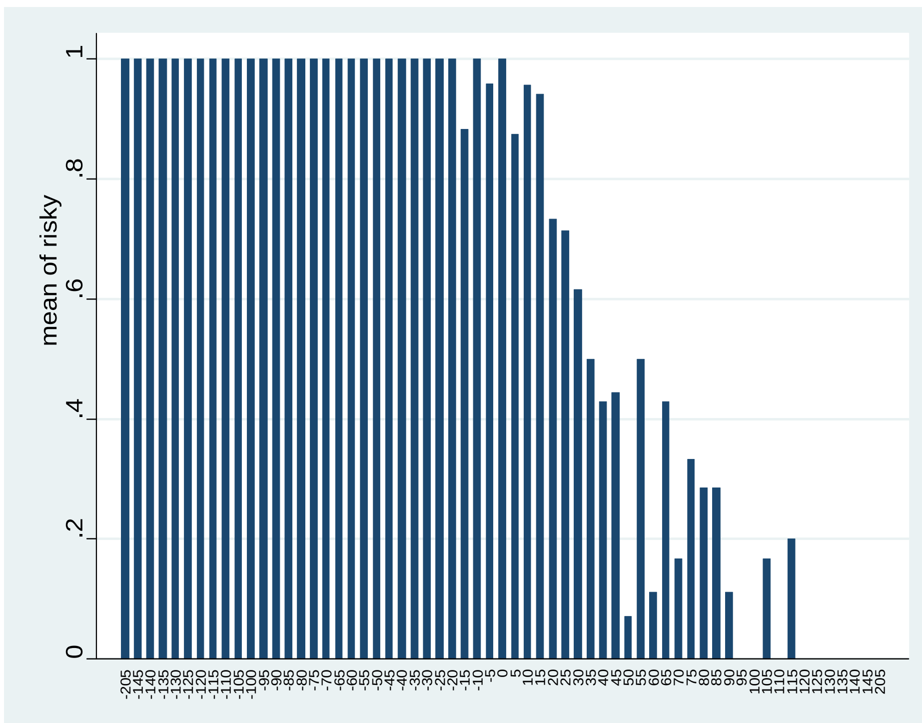

Figure 4: Choice of the risky strategy for different leads in treatment 1

We can summarize these observations as follows.

Result 1 ($\rho = 0$)**:** *When the outcomes of the risky strategies are uncorrelated the leading players choose the safe strategy more often than their opponents. The trailing players nearly always choose the risky strategy* (98.9%)*. The size* *of the lead has a strong influence on the probability that the leader chooses the safe strategy: The larger the lead, the more often the safe strategy is chosen.*

We now turn to the perfect correlation case in treatment 2. A look at figure 3 already indicates that the leading player picked the risky option more often than his opponent which is in stark contrast to the results from treatment 1 but well in line with the theoretical prediction.
Furthermore, as the theory predicts the leading players chose the risky strategy in more than 80% (92.8%) of the cases. But the trailing players also chose the risky option in 60.5% of the cases and not as we predicted in less than 20% of the cases. This behavior may be due to the false intuition that they had nothing to lose and therefore they preferred to gamble. The trailing players seemed to disregard at least partially that the leader may also want to play the risky strategy in which case the best reply is to play safe as only this leaves a chance to win the tournament.

	(1) Leading player	(2) Trailing player
Lead	-0.00540^{*}	0.00941^{***}
	(0.0031)	(0.0035)
Period	0.0775^{***}	-0.0332^{***}
	(0.016)	(0.011)
Constant	0.976^{***}	0.246
	(0.21)	(0.19)
Observations	276	276
Pseudo R^2	0.1161	0.0623

Robust standard errors in parentheses are calculated by clustering on subjects
$^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$

Table 2: Probit regression for treatment 2

Again, we ran probit regressions to test the predictions of the model. We first consider the leading players' behavior in model (1) and then that of the trailing players in model (2) of table 2. First note, that we have to reject our prediction concerning the effect of the lead in both cases. The theoretical model predicted that the leader plays the risky strategy more often the larger the lead and the trailing player plays risky less often for larger initial differences. The empirical analysis shows the opposite signs for both effects. It seems that initially the players followed the much more straightforward intuition from the case where the outcomes were uncorrelated, i.e. that the leader should protect his lead by playing safe and the trailing player can only 'attack' the leader by choosing the risky strategy. But note that we observe strong learning effects that seem to direct the players closer to the equilibrium prediction. Over the course of the experiment the leading players significantly increased the probability of playing the risky strategy and the trailing players reduced this probability. We can summarize:

Result 2 ($\rho = 1$): *When the outcomes of the risky strategies are perfectly correlated the leading players choose the risky strategy more often than their opponents. The leaders choose the risky strategy in* 92.8% *and the trailing players in* 60.5% *of the cases. Over the course the leading players increased the probability of playing the risky strategy, whereas the trailing players reduced this probability.*

Finally, we consider the results from the third treatment in which the correlation coefficient between the outcomes of the risky strategies was $\rho = \frac{1}{2}$. According to our theoretical predictions both players should always play the risky strategy regardless how large the lead is. As we see in figure 3 this prediction is true only for the trailing players. Leading agents chose the risky option only in 68.1% of the cases. To analyze learning effects and the effect of the lead on the choice of the strategy we use again a probit regression with the choice of strategy as dependent variable. The results of the regression are reported in table 3.

The regression shows that the lead indeed had an effect on the choice of the strategy. The probability that the leader played the safe option rises when

	Leading player
Lead	-0.0139^{***}
	(0.0034)
Period	0.0263^{***}
	(0.0094)
Constant	0.807^{***}
	(0.20)
Observations	276
Pseudo R^2	0.1088

Robust standard errors in parentheses are calculated by clustering on subjects, $^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$

Table 3: Probit regression for leading players in treatment 3

the lead got larger. This effect might occur because the leader thought that playing safe was an appropriate way to protect his leading position. During the experiment the leader learned that this assumption is not true and played risky more often. When we take a look at the decisions the leading players made in the last 5 periods, we see that $76,7\%$ of them preferred the risky strategy. We can conclude:

Result 3 ($\rho = \frac{1}{2}$)**:** *When the outcomes of the risky strategies are correlated with $\rho = \frac{1}{2}$ the trailing players play the risky strategy nearly in all cases* (94.6%)*. The leading players choose the risky strategy in only* 68.1% *of the cases but increase this probability over the course of the experiment.*

Hence, it seems to be the case that learning directed the players towards the equilibrium prediction.

6 Conclusion

We have investigated a simple tournament model in which two agents simultaneously choose between a risky and a safe strategy. We have shown that the equilibrium outcome strongly depends on the correlation between the outcomes of the risky strategy as well as the size of a potential lead of one of the players. We then tested the predictions from the model in a laboratory experiment. The key predictions have been confirmed: The leading players choose the safe strategy more often than the trailing players when outcomes are uncorrelated, but the contrary is true when the outcomes are perfectly correlated. However, in the latter case not all qualitative features of the predicted mixed strategy equilibrium were confirmed, as players reacted to changes in the lead differently than in the theoretical model. In line with our predictions, the risky strategies were chosen most often in the case where the correlation coefficient was equal to $\frac{1}{2}$ although this did occur not in all cases as the theory predicted. When we observed learning behavior, i.e. significant time trends affecting the probability that a player chooses the risky strategy, those trends always led the players' behavior closer to the equilibrium prediction.

There are many open questions for future research. For instance, we so far did not consider endogenous effort choices and focused only on risk-taking behavior. It would therefore be interesting to study the interplay between risk-taking and effort incentives in further tournament experiments.

7 Appendix

Proof of Proposition 3:

Both players will randomize only if they are indifferent between the payoffs of both strategies. Suppose that player A chooses the risky strategy with probability p and player B with probability q. Hence, we must have that

$$p \cdot \Phi\left(\frac{-\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right) + (1-p)\cdot\Phi\left(\frac{-\Delta y_A+\Delta\mu}{\sigma}\right) = p\cdot\ \Phi\left(\frac{-\Delta y_A-\Delta\mu}{\sigma}\right) + (1-p)\cdot 0 \Leftrightarrow$$

$$p = \frac{\Phi\left(\frac{-\Delta y_A+\Delta\mu}{\sigma}\right)}{\Phi\left(\frac{-\Delta y_A+\Delta\mu}{\sigma}\right) + \Phi\left(\frac{-\Delta y_A-\Delta\mu}{\sigma}\right) - \Phi\left(\frac{-\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right)}$$

and

$$q \cdot \Phi\left(\frac{\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right) + (1-q)\cdot\Phi\left(\frac{\Delta y_A+\Delta\mu}{\sigma}\right) = q\cdot\Phi\left(\frac{\Delta y_A-\Delta\mu}{\sigma}\right) + (1-q)\cdot 1 \Leftrightarrow$$

$$q = \frac{1-\Phi\left(\frac{\Delta y_A+\Delta\mu}{\sigma}\right)}{1+\Phi\left(\frac{\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right) - \Phi\left(\frac{\Delta y_A+\Delta\mu}{\sigma}\right) - \Phi\left(\frac{\Delta y_A-\Delta\mu}{\sigma}\right)}.$$

Player A will indeed choose the risky strategy with higher probability than player B when

$$\frac{\Phi\left(\frac{-\Delta y_A+\Delta\mu}{\sigma}\right)}{\Phi\left(\frac{-\Delta y_A+\Delta\mu}{\sigma}\right) + \Phi\left(\frac{-\Delta y_A-\Delta\mu}{\sigma}\right) - \Phi\left(\frac{-\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right)}$$

$$> \frac{1-\Phi\left(\frac{\Delta y_A+\Delta\mu}{\sigma}\right)}{1+\Phi\left(\frac{\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right) - \Phi\left(\frac{\Delta y_A+\Delta\mu}{\sigma}\right) - \Phi\left(\frac{\Delta y_A-\Delta\mu}{\sigma}\right)}$$

using that $\Phi(x) = 1 - \Phi(-x)$ this is equivalent to

$$\begin{aligned}
& \frac{\Phi\left(\frac{-\Delta y_A + \Delta\mu}{\sigma}\right)}{1 + \Phi\left(\frac{\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right) - \Phi\left(\frac{\Delta y_A - \Delta\mu}{\sigma}\right) - \Phi\left(\frac{\Delta y_A + \Delta\mu}{\sigma}\right)} \\
> & \frac{1 - \Phi\left(\frac{\Delta y_A + \Delta\mu}{\sigma}\right)}{1 + \Phi\left(\frac{\Delta y_A}{\sigma\sqrt{2(1-\rho)}}\right) - \Phi\left(\frac{\Delta y_A + \Delta\mu}{\sigma}\right) - \Phi\left(\frac{\Delta y_A - \Delta\mu}{\sigma}\right)} \\
\Leftrightarrow & \Phi\left(\frac{-\Delta y_A + \Delta\mu}{\sigma}\right) > 1 - \Phi\left(\frac{\Delta y_A + \Delta\mu}{\sigma}\right) \\
\Leftrightarrow & \Phi\left(\frac{-\Delta y_A + \Delta\mu}{\sigma}\right) > \Phi\left(\frac{-\Delta y_A - \Delta\mu}{\sigma}\right)
\end{aligned}$$

which is clearly always the case. ∎

	Correlation 0		Correlation 1		Correlation $\frac{1}{2}$	
	no lead	lead	no lead	lead	no lead	lead
safe	0.011	0.507	0.395	0.072	0.054	0.319
risky	0.989	0.493	0.605	0.928	0.946	0.681

Table A1: Distribution of strategy choices for all treatments

	(1) leading player	(2) leading player
Lead	-0.0113^{***}	
	(0.0016)	
Lead > 68.28		-0.483^{***}
		(0.062)
Period	0.00233	0.00208
	(0.0046)	(0.0038)
Observations	276	276
Pseudo R^2	0.2858	0.1436

Robust standard errors in parentheses are calculated by clustering on subjects
Marginal effects reported, $^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$

Table A2: Probit regressions for leading players in treatment 1

	(1) leading player	(2) trailing player
Lead	-0.000545^{*}	-0.00361^{***}
	(0.00041)	(0.0013)
Period	0.00783^{***}	-0.0127^{***}
	(0.0027)	(0.0043)
Observations	276	276
Pseudo R^2	0.1161	0.0623

Robust standard errors in parentheses are calculated by clustering on subjects, Marginal effects reported, $^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$

Table A3: Probit regressions for treatment 2

	leading player
Lead	-0.00483^{***}
	(0.0015)
Period	0.00918^{***}
	(0.0033)
Observations	276
Pseudo R^2	0.1088

Robust standard errors in parentheses are calculated by clustering on subjects, Marginal effects reported, $^{***}p < 0,01$, $^{**}p < 0,05$, $^{*}p < 0,1$

Table A4: Probit regression for leading players in treatment 3

Instructions for the experiment

(Note that the instructions had to be slightly different for each treatment. The parts that are only valid for one of the treatments are marked with (treatment 1,2 or 3) here.)

Welcome to this experiment!

Please read these instructions carefully. If you have any questions please raise your hand and ask us. Please note the following:

- There is no communication allowed.
- All decisions are anonymous. None of the other participants will learn the identity of the one who makes a certain decision.
- The payment is anonymous too. Nobody gets to know how high the payment of another participant is.
- This experiment consists of two parts. The first part will be explained now. The second part consists only of a short questionnaire.

Periods and partners

- The experiment consists of 23 periods.
- Before the experiment starts you will have the chance to get a better feel for it in 5 trial periods. This trial periods have no influence on your payment. Their only purpose is to help you develop a better understanding of the experiment.
- You will play each period with a different partner. The identity of your partner will not be revealed. We have ensured that you will never play with the same partner again.

Progress of one period

- At the beginning of each period you will be matched with one partner.
- At the start of each period you will learn your own score and that of your partner. From information you can infer whether you are have a lead or are lagging behind your partner. The initial scores are determined randomly.
- You have to decide between two strategies. Strategy A will give you 80 additional points for sure. Strategy B will give you additional points that are determined by a random draw from a normal distribution with a mean of 100 points and a standard deviation of 20 points. You will find a chart at the end of this instructions to clarify this distribution.
- Your partner can also choose between strategy A and B. There are 4 possible outcomes:
  - both players choose strategy A.
  - you choose strategy A and your partner chooses strategy B.
  - you choose strategy B and your partner chooses strategy A.
  - both players choose strategy B.
- (treatment 1) If both players choose strategy B with the random return you have to keep in mind that the outcomes are completely independent of each other. If you achieve a certain amount of additional points by choosing the risky strategy the additional points your partner achieves when also playing risky are completely independent of your points.
- (treatment 2) If both players choose strategy B with the random return you have to keep in mind that the outcomes are perfectly correlated.

If you achieve a certain amount of additional points by choosing the risky strategy your partner will always achieve the same amount of additional points if he chooses the risky strategy as well.

- (treatment 3) If both players choose strategy B with the random return you have to keep in mind that the outcomes are correlated, i.e. there is a relation between the outcomes. The correlation coefficient in this experiment is 0.5. It is a measure for the relationship between the points score of both players. If you play risky and achieve a high additional point score the probability is high that your partner will also attain a high score when playing the risky strategy as well. If you play risky and achieve a rather low additional point score the probability is high that your partner will also attain only a low score if he is also playing risky. We will give you more information about the correlation coefficient after these instructions.

- After both players have made their decision about the strategy the results of the period will be calculated. The final result is the sum of the initial points and the additional point score you attained in this game. Is your final score higher than that of your partner you are the winner of this period. Is your final score smaller than that of your partner you have lost this period. If the scores are equal the winner will be drawn by lot.

- After you and your partner have been informed about your final scores and the winner of the period you will be matched with a new partner. The game then starts again.

- You will play 23 periods with different partners. Please note that you will only play once with a certain partner.

- After the 23 periods one period is drawn by lot which determines your payment. If you are the winner of that period you will receive the winner prize of 25 Euro. If you have lost this period you will receive the loser prize of 5 Euro.

- The second part of the experiment starts. Please fill out the questionnaire which will appear on the screen.
- Please stay at your seat until we call your cubical number. Please bring along these instructions and your cubical number. Otherwise we cannot hand out the payment.

Your payment as winner =

winner prize 25 Euro + 2.50 Euro show-up fee = 27.50 Euro

Your payment as loser =

loser prize 5 Euro + 2.50 Euro show-up fee = 7.50 Euro

Good luck!

Additional comments:
Strategy B
Reminder:
The probability that the additional point score belongs to a certain interval corresponds to the size of the surface under the graph. For instance, 95.44% of the drawn points are located in the interval between 60 and 140.

(treatment 3)
Explanation of correlation:
The correlation coefficient is a measure for the linear relationship between two random variables. It can vary between -1 and 1. If it is 1 there is a perfect positive linear relationship between the random numbers. Graphically the

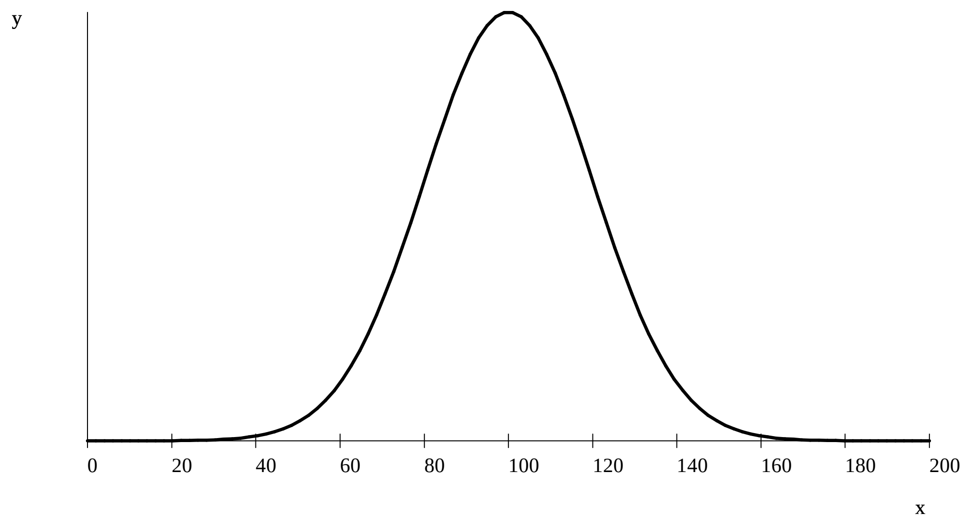

Figure 1: Chart of the normal distribution with a mean of 100 and a standard deviation of 20

scatterplot follows a straight line. If the correlation coefficient is zero the random numbers are independent from each other. In this experiment the correlation coefficient is 0.5.
We present you some scatterplots here:

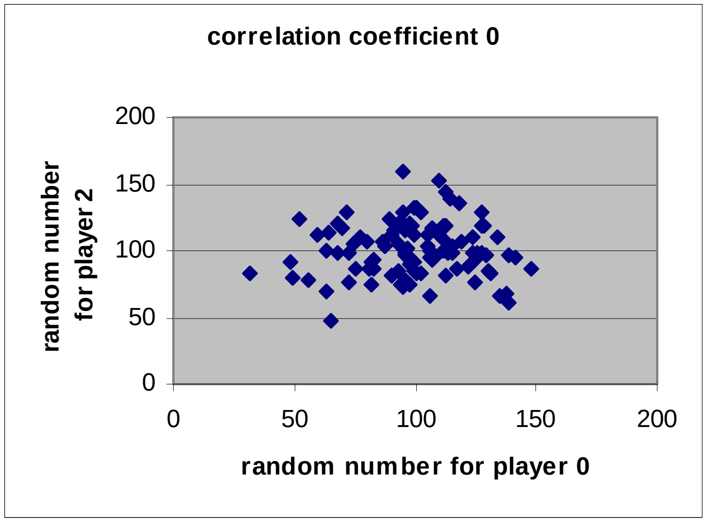

Figure 2: No correlation

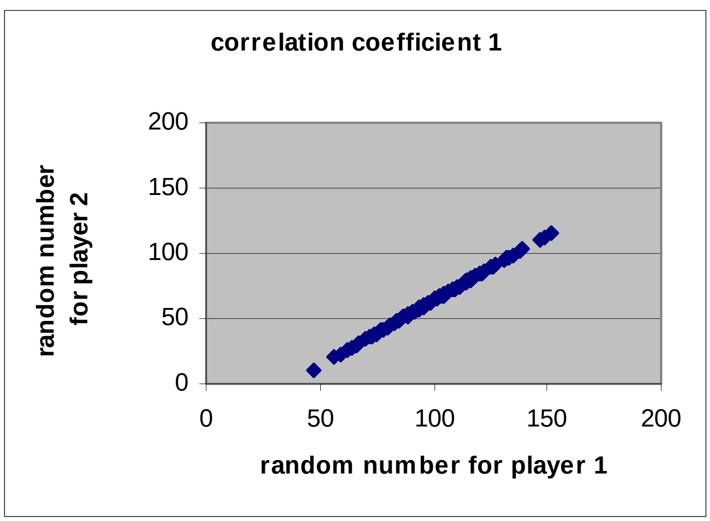

Figure 3: Perfect correlation

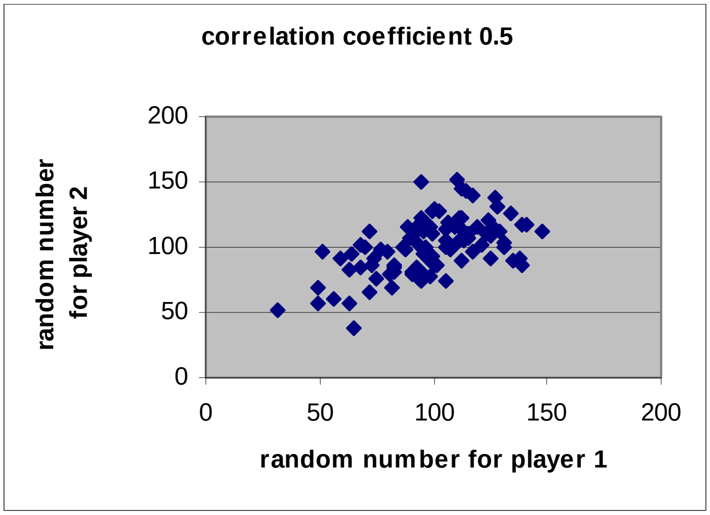

Figure 4: Positive correlation 0.5

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